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DEPARTMENT OF COMMERCE AND LABOR

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BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON Director

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VOLUME 1

(Numbers 1, 2, 3)

1904-05



WASHINGTON  
GOVERNMENT PRINTING OFFICE  
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S. W. STRATTON,  
*Director.*

WASHINGTON, D. C., *October 15, 1904.*



DEPARTMENT OF COMMERCE AND LABOR

Vol. 1

BULLETIN

No. 1

OF THE

# BUREAU OF STANDARDS

S. W. STRATTON, Director

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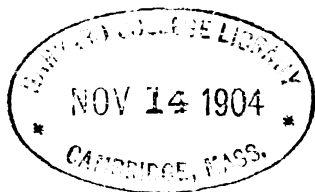
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WASHINGTON, D. C., *October 15, 1904.*



## RECOMPARISON OF THE UNITED STATES PROTOTYPE METER.

---

By LOUIS A. FISCHER.

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Through the courtesy of Doctor Benoît, Director of the International Bureau of Weights and Measures, advantage was taken of a visit by the writer to Paris in October, 1903, to compare U. S. Prototype Meter No. 27 with the standards of the International Bureau.

### U. S. METERS NOS. 27 AND 21.

U. S. Meter No. 27, like the prototypes of all the principal nations, and also like the international meter, is composed of 90 per cent platinum and 10 per cent iridium, with minute traces of other metals which compose less than 0.1 per cent of the total. It was intercompared at the International Bureau of Weights and Measures, in 1888, with the national prototypes above referred to and with the international meter; and shortly afterwards it was brought to this country by a special messenger, who certified that it had suffered no violent mechanical or temperature disturbance in transportation. Soon after its arrival in this country—or to be exact, on January 2, 1891—the standard was unpacked with considerable ceremony at the Executive Mansion in the presence of President of the United States, who accepted it as the national prototype meter.<sup>a</sup>

It was then immediately repacked, sealed in its metal case, and taken to the Office of Standard Weights and Measures, in the custody of which it remained until the formation of the Bureau of Standards, on July 1, 1901, when it was transferred to the Bureau with the other apparatus belonging to the Office of Standard Weights and Measures. It remained packed and sealed in its case until a few weeks before it was taken to Europe, when it was compared with Meter No. 21, which is exactly similar to No. 27, except that the lines and surfaces are not as perfect as those of No. 27, on account of its having been frequently packed in shaved ice.

---

<sup>a</sup>For a full description of the prototype meter, its transportation and acceptance, see Appendix 18, U. S. Coast and Geodetic Survey Report for 1890.

## COMPARISON OF No. 27 WITH No. 21.

This comparison was made solely to furnish a check on the length of No. 27 in case it should meet with accident in transporting it to and from Europe.

The comparisons were made on an improvised comparator installed in the subbasement of the Butler Building, in Washington, now occupied by the Bureau of Standards.

*The comparator.*—The comparator, which is a temporary structure, differs essentially from those used at the International Bureau of Weights and Measures, and elsewhere, and in consequence the results at present obtained with it are not as concordant as the results obtained at the international bureau. For general purposes, however, it offers decided advantages, and if properly constructed, it is believed that it will give results just as concordant.

The essential features of the comparator are two brick piers, A A, which support the ends of a heavy iron I beam, B, to which are clamped two heavy iron brackets, C C. These brackets support the microscopes D D. The microscope supports may be clamped anywhere on the I beam, and hence the comparator may be used for comparing bars having any length from 0.1 meter to 1 meter. Where the microscopes are fixed to the piers it is only possible to compare bars of a definite length. By properly protecting the I beam and microscopes against temperature changes, or by making them of the 36 per cent nickel-steel alloy, it is believed that the distance between the microscopes will be as constant, or more so, than the distance between microscopes mounted on independent piers. In addition to the I beam and its two supporting piers, two intermediate piers, E E, support steel rails upon which the carriage F moves transversely to the I beam. Mounted upon this carriage is a wooden box G covered with sheet copper, inside of which is a heavy sheet brass box in which the two bars to be compared were placed. The object of this arrangement was to secure uniform temperature within the inner box. Since the coefficients of expansion of the two bars were almost identical, an accurate knowledge of the true temperature was not important, though it was important that both have the same temperature. The box rested upon three adjusting screws used to focus the bars under the microscopes.

To protect the I beam and the microscopes and clamps from the heat of the observers' bodies they were covered with sheet asbestos.

*Thermometers.*—The temperatures of the two bars were determined by means of two Tonnelot thermometers previously studied at the International Bureau of Weights and Measures. One was placed upon each bar, the bulbs being placed in opposite directions, and the ther-



momometer scales being read through small openings in the brass and wooden inclosing boxes. As before stated, the question of the actual temperature was not of importance, because the expansions of the two bars were so nearly equal that an error of a whole degree in reading

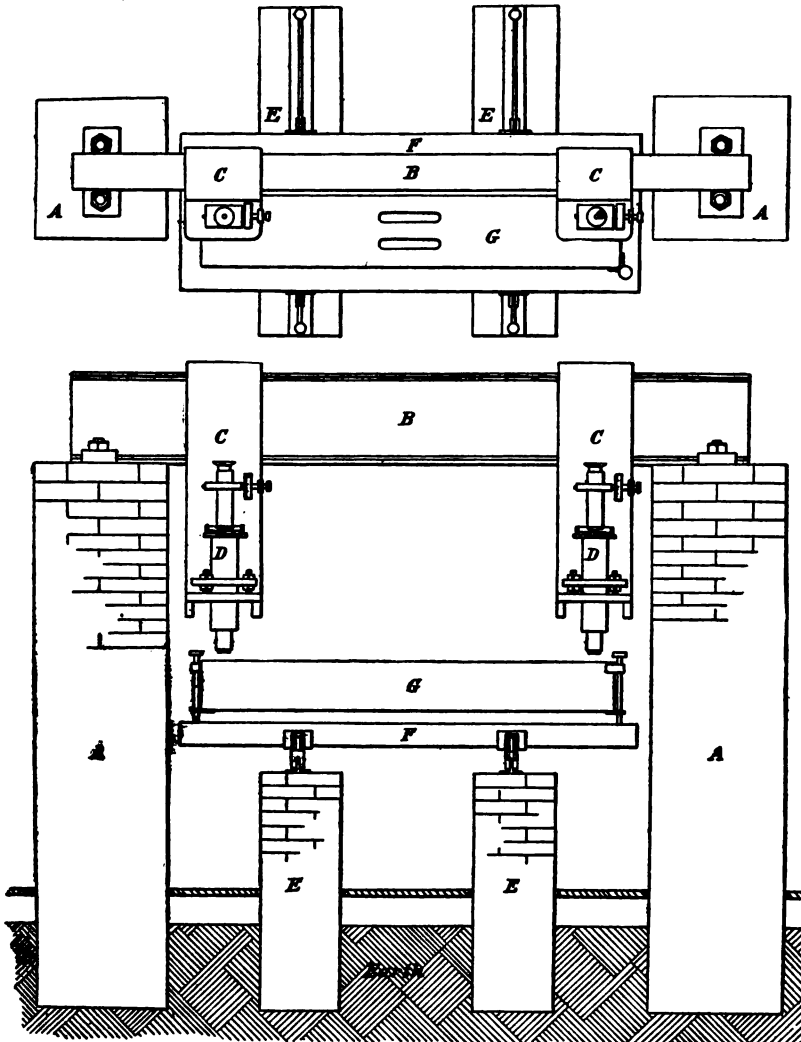


FIG. 1.—Arrangement of the comparator.

the temperature introduced an error in the final result of less than  $0.01\mu$ . Should, however, the temperature of one bar differ from that of the other by  $0.1^\circ$  it would, if not corrected for, introduce an error of

0.87 $\mu$ ; hence the necessity of a uniform temperature in the inside brass box.

*Microscopes.*—The microscopes have a magnifying power of very nearly 50 diameters, the objective and eyepiece each contributing equally to the power. The objectives were of the compound type, the illumination being secured by mounting small prisms in the principal focus of the lower lens of the objective. Diffused light from incandescent lamps was thrown through a screen of thin ground glass upon the prisms, from which the light was reflected vertically downward on the meter bars.

The micrometer screws were carefully studied for periodic errors about ten years ago, and a number of determinations of the screw values have been made since. They were again determined in August, 1903, and the values found were used to reduce the observations made at this period.

The values for one turn of the micrometer screws at different dates are tabulated below:

| Date.                        | Micrometer<br>No. 6. | Micrometer<br>No. 5. |
|------------------------------|----------------------|----------------------|
| September–October, 1893 .... | 75 $\mu$ .99         | 74 $\mu$ .69         |
| July, 1894 .....             | .98                  | .67                  |
| May–September, 1896 .....    | .97                  | .76                  |
| August, 1903.....            | .99                  | .66.                 |

The foregoing values indicate a very satisfactory agreement in the screw values at widely different dates. Only one, namely, that of micrometer No. 5, made in 1896, shows an appreciable deviation from the others. The reason for this unusually large value is not known, nor is it important for the present purpose.

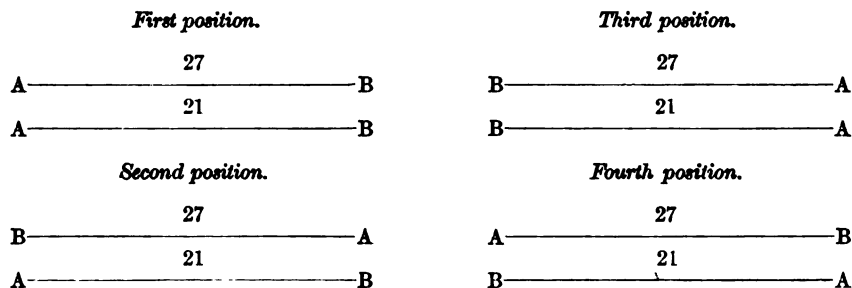
*Observations.*—An observation consisted of the following operations, which consumed about fifteen minutes:

1. Reading of thermometers in inner case.
2. Reading on No. 21.
3. Reading on No. 27.
4. Reading on No. 21.
5. Reading on No. 27.
6. Reading on No. 21.
7. Reading of thermometers in inner case.

Every time a bar was brought under the microscope the micrometers were read simultaneously four times, the observers exchanging places after the second reading to eliminate personal equation.

To insure thermal equilibrium at the beginning of the observations at least three hours was allowed to elapse between the observations.

A series consisted of eight observations, with the bars occupying as many different positions with respect to the observer and the microscopes. The positions were as follows:



The other four positions were obtained by substituting one bar for the other in each of the above diagrams.

By this procedure each bar was in front, and hence nearer to the observer during half of the observations; also each end of both bars was brought under each microscope twice. The result of a series of observations is, therefore, independent of any possible affect of peculiar conditions.

The observations were all reduced to zero centigrade by means of the differential expansion deduced from the equations of the meter bars as furnished by the International Bureau.

In the comparison now described three series of observations were made, making a total of twenty-four observations. The results will be found in the following table, in which the first column gives the number and the second the date of the observation. The third gives the reading of the left-hand microscope on No. 21; the fourth gives the reading of the same microscope, on No. 27; the fifth gives the difference, in revolutions, of the readings of the micrometer; the sixth, the difference in microns. The seventh column gives the reading of the right-hand microscope on No. 21; the eighth, the reading of the same microscope on No. 27; the ninth, the difference in the readings of the micrometers; the tenth, the difference in microns; the eleventh, the sum of the differences of the two microscopes, in microns; the twelfth gives the mean corrected reading of the two thermometers, and the thirteenth gives the residuals for the individual observations when referred to the mean temperature of observation, namely, 23.50° C. All of the observations at the Bureau of Standards were made with the bars in air.

FIRST COMPARISONS BETWEEN NOS. 21 AND 27.

| Observation. | Date,<br>1903. | No. 6.—1 revolution=75.99 $\mu$ . |         |                                    |                              | No. 5.—1 revolution=74.66 $\mu$ . |         |                                         |                              | Mean<br>tempera-<br>ture, C. | Residuals,<br>Obs.—cal. |  |
|--------------|----------------|-----------------------------------|---------|------------------------------------|------------------------------|-----------------------------------|---------|-----------------------------------------|------------------------------|------------------------------|-------------------------|--|
|              |                | No. 21.                           | No. 27. | Difference<br>in revolu-<br>tions. | Difference<br>in<br>microns. | No. 21.                           | No. 27. | Differ-<br>ence<br>in revolu-<br>tions. | Difference<br>in<br>microns. |                              |                         |  |
| 1            | Aug.           | 16.979                            | 16.939  | -0.040                             | -3.04                        | 18.386                            | 18.404  | +0.018                                  | +1.34                        | 24.127                       | -0.53                   |  |
| 2            |                | 16.417                            | 16.355  | -0.062                             | -4.71                        | 17.799                            | 17.797  | -0.002                                  | -0.15                        | 24.136                       | -0.71                   |  |
| 3            |                | 15.872                            | 15.865  | -0.007                             | -0.53                        | 17.238                            | 17.277  | +0.039                                  | +2.91                        | 24.225                       | +0.41                   |  |
| 4            |                | 15.866                            | 15.989  | +0.103                             | +7.83                        | 17.256                            | 17.412  | +0.156                                  | +11.65                       | 24.200                       | +0.03                   |  |
| 5            |                | 18.094                            | 18.176  | +0.082                             | +6.23                        | 19.458                            | 19.582  | +0.124                                  | +9.26                        | 24.250                       | +0.82                   |  |
| 6            |                | 18.769                            | 18.717  | -0.052                             | -3.95                        | 20.097                            | 20.105  | +0.008                                  | +0.60                        | 23.701                       | -0.70                   |  |
| 7            | Sept.          | 18.818                            | 18.824  | +0.006                             | +0.46                        | 20.150                            | 20.212  | +0.062                                  | +4.63                        | 23.639                       | -0.32                   |  |
| 8            |                | 18.789                            | 18.732  | -0.057                             | -4.33                        | 19.840                            | 19.848  | +0.008                                  | +0.60                        | 22.915                       | -1.08                   |  |
| 9            |                | 18.410                            | 18.396  | -0.014                             | -1.06                        | 19.706                            | 19.740  | +0.034                                  | +2.54                        | 22.876                       | +0.25                   |  |
| 10           |                | 18.315                            | 18.256  | -0.059                             | -4.48                        | 19.613                            | 19.606  | -0.007                                  | -0.52                        | 22.869                       | -0.11                   |  |
| 11           |                | 18.226                            | 18.312  | +0.086                             | +6.53                        | 19.516                            | 19.658  | +0.142                                  | +10.60                       | 22.690                       | -0.22                   |  |
| 12           |                | 18.124                            | 18.079  | -0.045                             | -3.42                        | 19.410                            | 19.402  | -0.008                                  | -0.60                        | 22.636                       | +1.03                   |  |
| 13           |                | 19.413                            | 19.456  | +0.043                             | +3.27                        | 20.705                            | 20.798  | +0.093                                  | +6.94                        | 22.969                       | +0.18                   |  |
| 14           |                | 19.522                            | 19.595  | +0.073                             | +5.55                        | 20.849                            | 20.968  | +0.119                                  | +8.88                        | 22.784                       | +0.52                   |  |
| 15           |                | 19.756                            | 19.880  | +0.124                             | +9.42                        | 21.084                            | 21.254  | +0.170                                  | +12.69                       | 22.800                       | +0.58                   |  |
| 16           |                | 19.622                            | 19.623  | +0.001                             | +0.01                        | 20.960                            | 21.010  | +0.050                                  | +3.73                        | 22.900                       | +0.13                   |  |
| 17           |                | 19.854                            | 19.750  | -0.104                             | -7.90                        | 20.879                            | 20.830  | +0.049                                  | +3.66                        | 22.806                       | -0.39                   |  |
| 18           |                | 19.406                            | 19.390  | -0.016                             | -1.22                        | 20.428                            | 20.456  | +0.028                                  | -2.09                        | 22.758                       | -0.54                   |  |
| 19           |                | 19.504                            | 19.376  | -0.128                             | -9.73                        | 20.536                            | 20.452  | -0.084                                  | -6.27                        | 22.604                       | +0.39                   |  |
| 20           |                | 19.614                            | 19.597  | -0.017                             | -1.29                        | 20.629                            | 20.658  | +0.029                                  | +2.16                        | 22.510                       | +0.40                   |  |
| 21           |                | 19.452                            | 19.411  | -0.041                             | -3.12                        | 20.442                            | 20.466  | +0.024                                  | +1.79                        | 22.422                       | -1.06                   |  |
| 22           |                | 19.385                            | 19.359  | -0.026                             | -1.98                        | 20.372                            | 20.403  | +0.031                                  | +2.31                        | 22.380                       | -0.44                   |  |
| 23           |                | 20.075                            | 20.003  | -0.072                             | -5.47                        | 21.092                            | 21.074  | -0.018                                  | -1.34                        | 22.176                       | -0.28                   |  |
| 24           |                | 20.209                            | 20.195  | -0.014                             | -1.06                        | 21.233                            | 21.264  | +0.031                                  | +2.31                        | 22.174                       | +0.48                   |  |
|              |                |                                   |         |                                    |                              |                                   |         |                                         |                              | 23.046                       |                         |  |
|              |                |                                   |         |                                    |                              |                                   |         |                                         |                              | -3.85                        |                         |  |
|              |                |                                   |         |                                    |                              |                                   |         |                                         |                              | $\pm 0.04$                   |                         |  |

The relation of the two bars, as determined in 1888, was

$$\text{No. 27} - \text{No. 21} = -4.00\mu \text{ at } 0.^{\circ}0 \text{ C.}$$

If we assume that the length of No. 27 has remained constant, then No. 21 shows a decrease in length of  $0.33\mu$ . The evidence is, however, too meager to draw reliable conclusions from, and hence a discussion of this question is postponed until further observations have been made.

After the above observations had been completed, No. 27 was carefully packed in its case and transported to the International Bureau of Weights and Measures. During the transportation the bar was handled with the greatest care and received no shocks, nor was it subjected to any sudden changes of temperature. Upon the arrival of No. 27 at the International Bureau of Weights and Measures, it was immediately placed in the Brunner comparator with meter No. 26, and M. Maudet, one of the assistants at the International Bureau, was delegated to assist in the comparisons, which were begun the following day.

#### THE BRUNNER COMPARATOR.

The essential features of the Brunner comparator, which is fully described in Vol. 4, *Travaux et Mémoires* of the International Bureau, are two massive stone piers on which are mounted the two micrometer microscopes, and a double-walled box in which are placed the bars and which can be displaced laterally so as to bring the ends of the two standards successively under the microscopes. The trough is supported on a foundation which is only connected to the microscope piers through the earth. None of the piers are in contact with the floors of the laboratory. All the comparisons at the International Bureau were made with the bars submerged in water, which was thoroughly stirred before each observation. The temperature of the water and of the bars was determined by means of four symmetrically disposed Tonnelot thermometers, which had been used in the 1888 comparisons. These thermometers, numbered 4246, 4247, 4248, and 4249, respectively, are perhaps better known in terms of the hydrogen scale than any other mercury thermometers in existence, though in this case, as before stated, the actual temperature was of little importance.

*Micrometers.*—The micrometers used are the regular micrometers belonging to the Brunner comparator, and they will be found described in the volume above referred to. Their values have been carefully determined from time to time, and the values adopted for this period by the International Bureau were used. Merely as a check a determination of the values was made by the writer after the observations had

been completed, and the results obtained agreed almost perfectly with those used.

#### STANDARDS OF THE INTERNATIONAL BUREAU.

The comparison of No. 27 was made with the two standards of the International Bureau, namely, No. 26 and  $T_s$ .

Meter No. 26, which is the principal standard, was included in the 1888 comparisons, and it was again directly compared with the international meter in 1892.  $T_s$ , which is the secondary standard, was compared directly with the international meter in 1892, and a number of times with No. 26 between 1892 and 1894.

It might be well at this point to give the equations of all of the bars involved in the comparisons, both at the Bureau of Standards at Washington, and at the International Bureau of Weights and Measures.

They are as follows:

$$\begin{aligned} \text{No. 21} &= 1\text{m} + 2.45 + 8.665T + 0.00100T^2 \\ \text{No. 27} &= 1\text{m} - 1.55 + 8.657T + 0.00100T^2 \\ &= 1\text{m} - 1.55 + 8.606t + 0.00170t^2 \\ \text{No. 26} &= 1\text{m} + 0.80 + 8.596t + 0.00170t^2 \\ T_s &= 1\text{m} + 1.50 + 8.583t + 0.00170t^2 \end{aligned}$$

The corrections to Nos. 21 and 27 at  $0^\circ \text{C.}$  are given one place further than the value furnished in their certificates, the values in the certificates having been rounded off to the nearest tenth of a micron. It was deemed advisable, in view of the small difference that might be looked for, to take the actual values found in 1888 to the nearest 0.01 of a micron. For the same reason the values of No. 26 and  $T_s$  were taken to the nearest 0.01 of a micron. The above values of No. 26 and  $T_s$  are the values at present accepted by the International Bureau as the result of the comparisons made in 1892 and 1894. In the first two equations  $T$  means temperature in degrees of the hydrogen scale. In the last three equations  $t$  means temperature in degrees of the hard glass scale of the International Bureau.<sup>a</sup> In the observation made at Washington the thermometer readings were reduced to the hydrogen scale, while the observation made at the International Bureau was referred to the hard glass temperature scale. The second equation of No. 27 was therefore used in the latter observations.

#### OBSERVATIONS.

Following the method in use at the International Bureau, an observation made there consisted of five pointings on one bar and four on the other, the pointings being alternately on the two, but always beginning and ending on the same bar. A series of observations was

<sup>a</sup> *Nouvelles déterminations des Metres Étalons: Travaux et Mémoires, Vol. 11, p. 6.*

similar to the series described in connection with the Washington observations, namely, it consisted of eight observations with the bars occupying as many different positions with respect to the observers and microscope.

The observers in these observations worked independently—that is to say, for any given position of the bars an observation was made at different times by each of the observers, the observations succeeding one another by intervals of half an hour or more. This short interval was made possible by reason of the bars being immersed in water. Two series of observations between Nos. 26 and 27 and one series between No. 27 and T<sub>1</sub> were made by each observer between the 1st and the 12th of October, 1903. The results of these observations are summarized on pages 12 and 13. Below is a record of the first observation made on October 1. It is given here simply to show how the observations were made and recorded.

## COMPARISON OF METERS NOS. 26-27.

Oct. 1, 1903.

Observ: Maudet.

A ——— 27 ——— B  
A ——— 26 ——— B

## OBSERVATION.

*Thermometers.*

|      | 4246          | 4247          | 4248          | 4249          | Means.        |
|------|---------------|---------------|---------------|---------------|---------------|
|      | 16.570        | 16.755        | 16.705        | 16.785        | 16.704        |
|      | .590          | .760          | .710          | .790          | .712          |
|      | <u>16.580</u> | <u>16.758</u> | <u>16.707</u> | <u>16.788</u> | <u>16.708</u> |
| CORR | + .161        | + .005        | .049          | — .035        | .045          |
|      | <u>16.741</u> | <u>16.763</u> | <u>16.756</u> | <u>16.753</u> | <u>16.753</u> |

*Microscope.**Differences.*

| Meter. | Left.          |                | Right.         |                | Left.          |  | Right.         |  |
|--------|----------------|----------------|----------------|----------------|----------------|--|----------------|--|
| 27     | 14.773         |                | 15.067         |                | —0.294         |  |                |  |
| 26     |                | 14.825         |                | 15.088         |                |  | —0.263         |  |
| 27     | .839           |                | .127           |                | —0.288         |  |                |  |
| 26     |                | .772           |                | .044           |                |  | —0.272         |  |
| 27     | .829           |                | .123           |                | —0.294         |  |                |  |
| 26     |                | .759           |                | .028           |                |  | —0.269         |  |
| 27     | .839           |                | .128           |                | —0.289         |  |                |  |
| 26     |                | 14.753         |                | 15.024         |                |  | —0.271         |  |
| 27     | 14.849         |                | 15.139         |                | —0.290         |  |                |  |
|        | <u>14.8258</u> | <u>14.7773</u> | <u>15.1168</u> | <u>15.0460</u> | <u>—0.2910</u> |  | <u>—0.2687</u> |  |
|        | rev.           |                | rev.           |                | rev.           |  |                |  |
|        | +0.0485        |                | +0.0708        |                | —0.0223        |  |                |  |
|        | +4.83 $\mu$    |                | +7.10 $\mu$    |                | —2.27 $\mu$    |  |                |  |

SUMMARY OF COMPARISON BY MAUDET.

| First series.<br>(27-26.) |              |             |       | Second series.<br>(27-26.) |              |             |       | Third series.<br>(27-T <sub>3</sub> .) |              |             |       |
|---------------------------|--------------|-------------|-------|----------------------------|--------------|-------------|-------|----------------------------------------|--------------|-------------|-------|
| Date.                     | Temperature. | Difference. | 0-C.  | Date.                      | Temperature. | Difference. | 0-C.  | Date.                                  | Temperature. | Difference. | 0-C.  |
| 1903.                     | °            | μ           | μ     | 1903.                      | °            | μ           | μ     | 1903.                                  | °            | μ           | μ     |
| Oct. 1                    | 16.753       | -2.27       | +0.24 | Oct. 5                     | 17.001       | -2.88       | -0.13 | Oct. 10                                | 16.931       | -3.06       | +0.01 |
| Oct. 1                    | .880         | -2.85       | -0.34 | Oct. 5                     | .097         | -2.80       | -0.03 | Oct. 10                                | .945         | -3.34       | -0.27 |
| Oct. 1                    | .921         | -2.54       | -0.03 | Oct. 5                     | .135         | -3.04       | -0.29 | Oct. 12                                | .409         | -2.94       | +0.13 |
| Oct. 2                    | .890         | -2.21       | +0.31 | Oct. 6                     | .060         | -2.67       | +0.08 | Oct. 12                                | .411         | -3.06       | +0.01 |
| Oct. 2                    | 17.014       | -2.58       | -0.07 | Oct. 6                     | .134         | -2.65       | +0.10 | Oct. 9                                 | 17.035       | -2.77       | +0.30 |
| Oct. 2                    | .031         | -2.43       | +0.08 | Oct. 6                     | .165         | -2.56       | +0.19 | Oct. 10                                | 16.924       | -2.69       | +0.38 |
| Oct. 3                    | 16.965       | -2.68       | -0.17 | Oct. 6                     | .191         | -2.89       | -0.14 | Oct. 10                                | .922         | -3.27       | -0.20 |
| Oct. 3                    | 17.065       | -2.54       | -0.03 | Oct. 7                     | 17.072       | -2.51       | +0.25 | Oct. 10                                | .924         | -3.42       | -0.35 |
|                           | 16.940       | -2.51       | +0.64 |                            | 17.107       | -2.75       | +0.54 |                                        | 16.813       | -3.07       | +0.83 |
|                           |              |             | -0.63 |                            |              |             | -0.59 |                                        |              |             | -0.82 |

## Computation.

$$\begin{aligned}
 (27-26) &= \frac{\mu}{26} = \frac{-2.51}{1^m + 0.80} \\
 \text{Reduction to } 0^\circ \text{ temp.} &= -0.17 \\
 \text{No. 27} &= 1^m - 1.88 \\
 \text{No. 27} &= 1^m - 2.12 \\
 \text{Mean value of No. 27} &= 1^m - 1.99\mu \text{ at } 0.^\circ 0.
 \end{aligned}$$

$$\begin{aligned}
 (27-T_3) &= \frac{\mu}{T_3} = \frac{-3.07}{1^m + 1.50} \\
 \text{No. 27} &= 1^m - 1.96
 \end{aligned}$$



## SUMMARY OF COMPARISON BY FISCHER.

| First series.<br>(27-28.) |                   |             |       | Second series.<br>(27-28.) |                   |             |       | Third series.<br>(27-T <sub>3</sub> .) |                   |                  |       |
|---------------------------|-------------------|-------------|-------|----------------------------|-------------------|-------------|-------|----------------------------------------|-------------------|------------------|-------|
| Date.                     | Tempera-<br>ture. | Difference. | 0-C.  | Date.                      | Tempera-<br>ture. | Difference. | 0-C.  | Date.                                  | Tempera-<br>ture. | Differ-<br>ence. | 0-C.  |
|                           | °                 | μ           | μ     | 1903.                      | °                 | μ           | μ     | 1903.                                  | °                 | μ                | μ     |
| Oct. 1                    | 17.091            | -2.82       | -0.21 | Oct. 5                     | 17.022            | -2.62       | +0.02 | Oct. 8                                 | 17.081            | -3.06            | +0.08 |
| Oct. 1                    | 16.857            | -2.51       | +0.10 | Oct. 5                     | .063              | -2.78       | -0.14 | Oct. 8                                 | .113              | -3.30            | -0.16 |
| Oct. 1                    | 16.937            | -2.87       | -0.26 | Oct. 5                     | .162              | -2.71       | -0.07 | Oct. 8                                 | .143              | -3.37            | -0.23 |
| Oct. 2                    | 16.931            | -2.85       | -0.24 | Oct. 5                     | .146              | -2.57       | +0.07 | Oct. 8                                 | .162              | -2.81            | +0.33 |
| Oct. 2                    | 16.981            | -2.25       | +0.36 | Oct. 7                     | .111              | -2.78       | -0.14 | Oct. 9                                 | .056              | -3.04            | +0.10 |
| Oct. 2                    | 17.056            | -2.31       | +0.30 | Oct. 7                     | .155              | -2.39       | +0.25 | Oct. 9                                 | .083              | -2.94            | +0.20 |
| Oct. 3                    | 17.007            | -2.56       | +0.05 | Oct. 7                     | .185              | -2.32       | -0.18 | Oct. 9                                 | .119              | -3.47            | -0.33 |
| Oct. 3                    | 17.043            | -2.74       | -0.11 | Oct. 7                     | 17.062            | -2.49       | +0.05 | Oct. 9                                 | 17.131            | -3.11            | +0.03 |
|                           | 16.986            | -2.61       | +0.81 |                            | 17.116            | -2.64       | +0.49 |                                        | 17.111            | -3.14            | +0.74 |
|                           |                   |             | -0.82 |                            |                   |             | -0.53 |                                        |                   |                  | -0.72 |

**Computation.**

|                                |                                |                                |
|--------------------------------|--------------------------------|--------------------------------|
| $\mu$                          | $\mu$                          | $\mu$                          |
| $(27-26) = -2.61$              | $(27-26) = -2.64$              | $(27-T_2) = -3.14$             |
| $26 = 1^m + 0.80$              | $26 = 1^m + 0.80$              | $T_2 = 1^m + 1.50$             |
| Reduction to 0° temp. = -0.17  | = -0.17                        | = -0.39                        |
| No. 27 = 1 <sup>m</sup> - 1.98 | No. 27 = 1 <sup>m</sup> - 2.01 | No. 27 = 1 <sup>m</sup> - 2.03 |

Mean value of No. 27 = 1<sup>m</sup> - 2.01  $\mu$  at 0. °C.

The values deduced from the six series are remarkably concordant, the total range being only  $0.24\mu$ . Further, the means of the four results depending upon No. 26 agree perfectly with the two results deduced from the observation between No. 27 and  $T_3$ . The question, therefore, of the relative weight of the values of No. 27 deduced from No. 26 and  $T_3$  is not important. The six results were, consequently, given equal weight. The probable error of a single observation is substantially the same for both observers; namely,  $0.15\mu$ . The probable error of the mean of the three series of each observer,  $0.03\mu$ .

For comparison the results of the two observers are repeated:

$$\begin{array}{r} \mu \\ \text{(Maudet) No. 27} = 1^m - 1.99 \text{ at } 0^\circ\text{C.} \\ \text{(Fischer) No. 27} = 1^m - 2.01 \text{ at } 0^\circ\text{C.} \\ \hline \text{Mean No. 27} = 1^m - 2.00 \text{ at } 0^\circ\text{C.} \end{array}$$

The computed probable error of the last result is  $\pm 0.02$ .

The value originally found for No. 27 in 1888 was:

$$\text{No. 27} = 1^m - 1.55\mu \text{ at } 0^\circ\text{C.}$$

The recent comparisons made at the International Bureau, therefore, show an apparent shortening of  $0.45\mu$  in the length of No. 27 with respect to the international meter, as represented by meters No. 26 and  $T_3$ . If the recent observations were the only evidence, there would be little doubt that a slight change had occurred in the length of No. 27, as only one observation of the 48 made gave a result as large as that previously assigned to No. 27, and moreover the probable error of the new result does not admit of a possible error greater than  $0.10\mu$ . On the other hand, irregular changes equally as large have been observed in other bars compared at different times at the International Bureau. The discrepancies referred to were likewise much greater than would be expected from the agreement of the observation upon which they depended.<sup>a</sup>

The same phenomenon has been observed in the comparison of the British imperial yard with its four copies.<sup>b</sup> These bars were compared in 1855, 1876, 1882, 1892, and 1902, so that their constancy relative to one another can be studied. On account of the coarse lines and other defects in the bars from a metrological standpoint, rather

<sup>a</sup> Nouvelles déterminations des Metres Étalons: Travaux et Mémoires, Vol. 11, p. 20.

<sup>b</sup> An Account of the Comparison of Four Parliamentary Copies of the Imperial Standards, 1902.

large variations might be expected, but those observed are too large to be accounted for in this manner.

The question naturally arises at this point as to whether the observed differences represent real changes in the lengths of the standards, or whether they are due to certain peculiar conditions of the observers and comparing apparatus. Leaving out of consideration the yards referred to, because they belong in an entirely different class, it may be said that if the lines on the standard meters were absolutely perfect and of the same width, and if, furthermore, the surfaces on which they are ruled were uniform, and in every case perpendicular to the axes of the bars, the method of observing would eliminate any possibility for a constant error in the final result.

The lines, however, are not perfect, but the edges appear slightly ragged, under the magnifying power used, and hence the estimation of the centers of the lines is dependent to some extent upon the variable judgment of the observer. Moreover, the surfaces are not in every case exactly perpendicular to the axes of the bar, and, in consequence, the illumination of the lines is not always perpendicular to the surfaces even if the microscopes of the comparator are in perfect adjustment. Undoubtedly, part of the slight differences noted are due to the defects pointed out, but to the writer it does not appear improbable that slight temporary differences are also due to the previous history of the meters, especially to their previous thermal history.

It was the original intention of the International Committee on Weights and Measures to recompare the various national prototypes with one another and with the international meter every ten years, but this plan has not been carried out, though preparations are now being made at the International Bureau to do this work. When this has been done, it will be interesting to see whether similar changes will be observed in other prototypes, in which case it will be time to speculate as to the causes.

#### COMPARISON OF NO. 27 AFTER ITS RETURN TO WASHINGTON.

A second comparison between No. 21 and No. 27 was made in April, 1904, the result showing conclusively that the length of No. 27 had not been altered appreciably by its transportation to and from Paris. The results of these observations will be found in the following table, which is arranged in the same manner as the observations made prior to taking No. 27 to Paris. The same apparatus was used under identical conditions as in the first series of comparisons, the only difference being that the observations in this comparison were made by the writer alone.

## SECOND COMPARISON BETWEEN NOS. 21 AND 27.

| Observation. | Date,<br>1904. | No. 6—1 revolution = 75.93 $\mu$ . |         | No. 5—1 revolution = 74.8 $\mu$ .  |                              |         |         | Mean<br>tempera-<br>ture, C. | Residuals.<br>Obs.—cal. |
|--------------|----------------|------------------------------------|---------|------------------------------------|------------------------------|---------|---------|------------------------------|-------------------------|
|              |                | No. 21.                            | No. 27. | Difference<br>in revo-<br>lutions. | Difference<br>in<br>microns. | No. 21. | No. 27. |                              |                         |
| 1            | Apr. 16        | 18.800                             | 18.802  | +0.002                             | +0.15                        | 19.076  | 19.129  | 15.619                       | +0.09                   |
| 2            | Apr. 16        | 18.460                             | 18.402  | -0.058                             | -4.41                        | 18.712  | 18.707  | 15.674                       | -0.14                   |
| 3            | Apr. 17        | 19.167                             | 19.160  | -0.007                             | -0.53                        | 19.399  | 19.439  | 15.238                       | +0.38                   |
| 4            | Apr. 17        | 19.263                             | 19.218  | -0.045                             | -3.42                        | 19.487  | 19.491  | 15.171                       | -0.04                   |
| 5            | Apr. 18        | 18.990                             | 18.843  | -0.147                             | -11.17                       | 19.217  | 19.122  | 15.146                       | -0.18                   |
| 6            | Apr. 18        | 18.863                             | 18.834  | -0.029                             | -2.20                        | 19.092  | 19.114  | 15.268                       | +0.06                   |
| 7            | Apr. 18        | 19.478                             | 19.453  | -0.025                             | -1.90                        | 19.729  | 19.760  | 15.475                       | -0.31                   |
| 8            | Apr. 18        | 19.531                             | 19.467  | -0.064                             | -4.86                        | 19.793  | 19.784  | 15.458                       | -0.29                   |
| 9            | Apr. 19        | 19.356                             | 19.254  | -0.102                             | -7.75                        | 19.638  | 19.588  | 15.432                       | -0.12                   |
| 10           | Apr. 19        | 19.229                             | 19.175  | -0.054                             | -4.10                        | 19.497  | 19.495  | 15.662                       | -0.05                   |
| 11           | Apr. 19        | 19.586                             | 19.534  | -0.052                             | -3.95                        | 19.850  | 19.844  | 15.741                       | +0.40                   |
| 12           | Apr. 19        | 19.698                             | 19.688  | -0.010                             | -0.76                        | 19.945  | 19.991  | 15.542                       | -0.07                   |
| 13           | Apr. 20        | 19.554                             | 19.506  | -0.048                             | -3.65                        | 19.783  | 19.784  | 15.462                       | +0.18                   |
| 14           | Apr. 20        | 19.289                             | 19.241  | -0.048                             | -3.65                        | 19.488  | 19.491  | 15.577                       | +0.03                   |
|              |                |                                    |         |                                    |                              |         |         | 15.462                       |                         |
|              |                |                                    |         |                                    |                              |         |         | $\pm 0.04$                   |                         |

No. 27—No. 21 = -3.90  $\mu$  at 15.°462  
 = -3.78  $\mu$  at 0.°0 C.  
 The result obtained prior to comparison at Paris: No. 27—No. 21 = -3.67  $\mu$  at 0.°0 C.  
 The mean of the two results given: No. 27—No. 21 = -3.72  $\mu$  at 0.°0 C.  
 The 1888 comparison: No. 27—No. 21 = -4.00  $\mu$  at 0.°0 C.

Difference: -0.28  $\mu$

The two comparisons made between No. 21 and No. 27 agree so well that the conclusion is justified that No. 21 is shorter with respect to No. 27 than has been previously assumed, though the magnitude of the change is not so certain. It is nevertheless interesting to see what the change in No. 21 is on the assumption that the new value of No. 27 represents its present relation to the international meter. For this purpose the values deduced from the recent comparisons of the International Bureau and at the Bureau of Standards are summarized below:

$$\begin{array}{rcl}
 \text{No. 21} & = & 27 + 3.72\mu \\
 \text{No. 27} & = & 1^m - 2.00\mu \\
 \text{hence..No. 21} & = & 1^m + 1.72\mu \\
 \text{Old value (1888)..No. 21} & = & 1^m + 2.45\mu \\
 \text{Difference} & = & \underline{0.73\mu}
 \end{array}$$

While the results of the new comparisons differ but slightly from the old, and are moreover not conclusive, they nevertheless introduce an uncertainty as to the lengths of Nos. 27 and 21 and make it desirable to recompare No. 27 directly with the international and other national prototypes. Until this has been done by the International Bureau the old value of No. 27 will be used.

It is probable that part of the differences observed are due to slight errors in the coefficients of expansion of No. 27 and No. 21, and in order to test this, and also to fix definitely for this period the relative values of these bars, further comparisons will be undertaken at once.

In conclusion, the writer wishes to express his obligations to Doctor Benoit, director, and Doctor Guillaume, assistant director, respectively, of the International Bureau of Weights and Measures, for placing at his disposal all the required apparatus of the Bureau and for many valuable suggestions; to M. Maudet, who shared equally in the work at the International Bureau, and who made most of the reductions; and also to Mr. L. G. Hoxton, of the Bureau of Standards, who assisted in the first series of observations made at Washington prior to the comparison at Paris.



## A STUDY OF THE SILVER VOLTAMETER.

By K. E. GUTHE.

Considering the important rôle which the silver voltameter plays in electrical measurements, relatively few investigations have recently been made with a view of improving the accuracy of this instrument. The older investigations of Kohlrausch,<sup>a</sup> Rayleigh and Sidgwick,<sup>b</sup> Gray,<sup>c</sup> Schuster and Crossley,<sup>d</sup> and of Glazebrook and Skinner,<sup>e</sup> proved the silver voltameter to be more reliable than the copper voltameter, and this led to the general use of the well-known form, the specifications of which have been adopted in connection with the legal definition of the ampere, and are practically identical in the United States,<sup>f</sup> England,<sup>g</sup> and Germany.<sup>h</sup> This type has consequently been used, sometimes with only slight modifications, in a large number of investigations; but as the demands upon the accuracy of the results increased, it became more and more apparent that it presented a number of peculiarities, and the thorough researches on the behavior of this voltameter by Kahle<sup>i</sup> and others soon created the impression that the results obtained with a silver voltameter can not lay any claim to a high degree of accuracy.

Until recently the attempts to find the source of trouble in the voltameter have been rather unsuccessful, different investigators arriving at different conclusions regarding the influence and nature of the disturbing factors. While it must be conceded that in two voltameters of the usual form, when arranged in series and treated in exactly the same manner, the same quantity of electricity will yield deposits differing in weight not more than 1 in 5,000, or even 1 in 10,000;

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<sup>a</sup> F. and W. Kohlrausch: Wied. Ann., 27, p. 1; 1886.

<sup>b</sup> Rayleigh & Sidgwick: Phil. Trans. Roy. Soc., 175, p. 411; 1884.

<sup>c</sup> Th. Gray: Phil. Mag., 22, p. 389; 1886.

<sup>d</sup> Schuster & Crossley: Proc. Roy. Soc., 50, p. 344; 1892.

<sup>e</sup> Glazebrook & Skinner: Phil. Trans. Roy. Soc., 183, p. 567; 1892.

<sup>f</sup> Nat. Acad. of Sci., Feb. 20, 1895.

<sup>g</sup> (London) Electrician, 27, p. 325; 1891.

<sup>h</sup> ZS. f. Instrk., 21, p. 180; 1901.

<sup>i</sup> Kahle: ZS. f. Instrk., 18, pp. 229, 267; 1898.

these same instruments, when used under slightly differing and frequently under apparently the same conditions, will show much larger differences in a series of independent experiments.

At this point Richards, Collins, and Heimrod,<sup>a</sup> and Leduc<sup>b</sup> took up the problem of improving the silver voltameter or coulometer, a name which Richards proposes for the instrument and which, I think, would be much more appropriate.

Since so far the research of Richards and his students is the only one in which a systematic comparison of various types has been made, it seemed of sufficient importance to partly repeat the work with such modifications as seemed advisable, and, after having found the most reliable form, to subject it to a severe test in order to determine the degree of accuracy which may be expected from it.

#### DESCRIPTION OF THE VOLTAMETERS AND THEIR TREATMENT.

The following types were investigated:

1. *The usual type*, consisting of a platinum bowl 10 cm in diameter and 5 cm deep as cathode and a silver plate held by a silver rod at its center as anode. The anode was surrounded by filter paper secured at the back with a little sealing wax. The electrolyte, unless otherwise stated, was a neutral freshly prepared 20 per cent solution of silver nitrate.

To test the neutrality of the solution, the silver was precipitated by NaCl solution and the filtrate tested with methylorange. Silver nitrate, obtained from Eimer and Amend as chemically pure, contained no acid. In a few cases the crystals were first fused to insure neutrality, but since it was found that the results obtained with a solution of the fused salt agreed closely with those where the solution was made from crystals, the latter was used in almost all experiments.

2. *The silver oxide voltameter*.—Patterson and Guthe<sup>c</sup> employed as electrolyte a solution saturated with  $Ag_2O$ . To allow the oxide to have as great an influence as possible the voltameter was constructed in such a manner that the current had to pass through a layer of the oxide. For this purpose the anode was placed inside a porous cup glazed at the side, at the bottom of which a layer of carefully prepared  $Ag_2O$  was placed. Since the glazing showed a tendency to crack, an ordinary porous cup was used in the later experiments, and a tight-fitting glass tube inserted in it, in order to prevent the current from

<sup>a</sup> Richards, Collins, and Heimrod: Proc. Am. Acad., 85, p. 123; 1899. Richards and Heimrod: Proc. Am. Acad., 87, p. 415; 1902.

<sup>b</sup> Leduc: J. de Phys., 1, p. 561; 1902.

<sup>c</sup> Patterson and Guthe: Phys. Rev., 7, p. 257; 1898.



passing through the sides. The liquid was a 20 per cent solution of silver nitrate kept in a bottle containing silver oxide. Either platinum bowls or platinum crucibles served as kathodes.

3. *The Richards voltameter*, consisting of a platinum crucible 10 cm high and 6 cm wide as kathode. The anode, a silver rod, was surrounded by a fine-grained porous cup, made from the lower end of a Pasteur filter. The electrolyte was the same as in type 1. During the experiment the liquid in the porous cup was always kept at a lower level than that outside by removing about every ten or fifteen minutes some of the liquid in the cup by means of a pipette. Once a siphon was employed to produce this difference in level, without, however, changing the result.

A slight and somewhat more convenient modification of this type will be described later.

4. *Large silver anode type*.—A platinum bowl formed the kathode. In the 20 per cent solution of silver nitrate a porous cup of 7 cm diameter was suspended, and, at the bottom of this, finely divided silver was placed, the current being led into the voltameter by means of a plate pressed upon the silver. At first the fine crystals deposited at the kathode in previous runs were used, but it was soon found that the silver became packed so closely that it acted like an ordinary plate; therefore, in the later, final, experiments granulated silver, prepared from the crystals, was substituted and upon this the plate was pressed.

5. *Leduc's form* differs from type 4 principally by the use of filter paper and muslin to hold the granules on the anode. Granulated silver was heaped upon a plate, 6 cm in diameter, then covered by pure filter paper and held tight on the plate by bleached millinet. This forms the anode. In all other respects this voltameter is identical with type 1.

The silver was either pure silver obtained from the United States Mint or silver bought from Eimer and Amend as chemically pure. While the results obtained with the two kinds agreed well with each other, the appearance of the anodes after the experiment showed a striking difference. While the finely divided silver which covers the pure anode after electrolysis is whitish and crystalline, the less pure silver from Eimer and Amend is perfectly black, the layer consisting mainly of silver. This difference in the appearance of the anode allows a good estimate of the purity of the silver used and has been noticed before in the work of the Reichsanstalt.<sup>a</sup>

The porous cups were treated with aqua regia, potassium cyanide solution, nitric acid, and finally with boiling water before being used

<sup>a</sup> ZS. f. Instrk., 22, p. 155; 1902.

in the voltameters. After a few runs the cups become dark on the outside, due to a thin layer of finely divided silver, which can, however, easily be removed by potassium cyanide. For the large silver anode type, closely grained cups of very convenient shape were obtained from the Akron Insulator and Marble Company.

The use of filter paper was carefully avoided in the preparation of the solution, since it seems that the organic substances act chemically upon neutral silver nitrate solution.<sup>a</sup> It was not filtered, but transferred from its bottle to the voltameters by means of a clean pipette. Only in types 1 and 5 was filter paper used during the experiment, since it is characteristic for these forms.

The treatment of the silver voltameters was in all cases identical. The platinum bowls, or crucibles, were washed successively with nitric acid, water, then heated to 160° C. in an air bath, and, after having cooled in a desiccator for an hour or two, carefully weighed. After the experiment the deposits were washed four or five times with distilled water, until the last wash water did not show a bluish color on addition of NaCl solution. Then the vessels were left over night filled with distilled water well covering the deposit, were emptied in the morning, heated to 160° C., cooled, and weighed as before. The weight of the deposit for each run was about 4 grams. Usually silver was deposited twice in the same vessel, after which most of it was removed by scraping and the rest dissolved in nitric acid.

It was not deemed necessary to wash the deposit with water at 80° C., as has been recommended by the Reichsanstalt. Though Kahle<sup>b</sup> and Merrill<sup>c</sup> have found that silver is slightly soluble in hot water, Richards, Collins, and Heimrod,<sup>d</sup> as well as Leduc,<sup>e</sup> have shown that after proper washing the weight of the deposit does not change upon continued digestion with water of room temperature, a conclusion fully corroborated by my experience.

Great care was taken to guard against any loss of silver during the washing of the deposit. The solution, as well as the wash water, was always first emptied into a clean beaker. With proper illumination even minute particles of silver could be seen very distinctly. If any were present they were carefully collected in a small filter and later added to the deposit.

Lord Rayleigh and Richards have found that in spite of careful washing some of the mother liquid remains included in the interstices between the crystals, shown by a slight loss of weight on heating to dull redness, but Gray<sup>f</sup> was unable to observe any loss in deposits

<sup>a</sup> ZS. f. Instrk., 22, p. 156; 1902.

<sup>b</sup> Loc. cit., p. 233.

<sup>c</sup> Merrill: Phys. Rev., 10, p. 170; 1900.

<sup>d</sup> Loc. cit., p. 139.

<sup>e</sup> Leduc: C. R., 185, p. 24; 1902.

<sup>f</sup> Gray: Phil. Mag. 22, p. 399; 1886.

properly washed. At any rate the amount included can only be very slight and must be rather constant, considering how accurate the results are with properly constructed voltameters. Therefore the deposit was not heated in my experiments. The weighings were made with a good chemical balance, having a sensitiveness of about 20 divisions for the load used. It was found advisable to keep two shallow dishes with calcium chlorid in the balance case in order to prevent condensation of moisture on the deposit. In each experiment four voltameters were arranged in series, namely, a set of two bowls, agreeing in weight within 4 mg (55.24 grams each), and a set of two crucibles, agreeing to within 7 mg (39.10 grams each). Each vessel of a set was first weighed separately by the method of double weighing, and then the difference between the two was found by the same method. The latter always agreed with the difference between the two separate determinations within 0.06 mg. From these data the probable weight of each vessel was calculated.

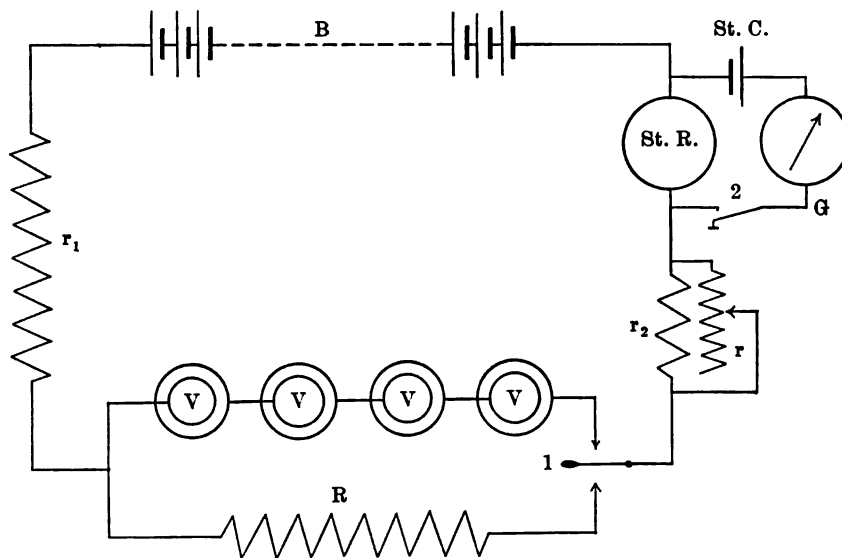
The vessels were counterpoised by platinum, so as to avoid troublesome corrections for the buoyancy of air, and the weights used were platinum weights, standardized by the Section of Weights and Measures of the Bureau.

#### DESCRIPTION OF THE METHOD.

The arrangement of the apparatus is represented by the figure. The current was furnished by a storage battery of 40 cells, *B*, and in series with it were placed a large variable resistance,  $r_1$ , the four voltameters, *V*, and standard resistance coil, *St. R*. The small resistance  $r_2$  was shunted by a Kelvin rheostat, *r*. This consists of a wire wound upon two cylinders, of which one is of metal and the other one insulating. By rotating the cylinders the wire is transferred from one cylinder to the other and the resistance of the rheostat varied in a continuous manner. By the use of this instrument slow variations of the current can easily be avoided. In a shunt to the standard resistance, a sensitive galvanometer, *G*, and a Weston cell, *St. C*., were placed, the latter so as to oppose the potential difference at the terminals of the standard resistance; equality between the two is indicated by the galvanometer remaining at rest. An unsaturated Weston cell was selected on account of its negligible temperature coefficient. The same cell was used throughout the investigation. For the determination of its *E. M. F.* in terms of the standard of the Bureau and for the measurement of the standard resistances and their temperature coefficient I am indebted to the Section of Resistance and Electromotive Force.

The current is first closed through the resistance  $R$ , which was made as nearly equal as could be estimated to that of the voltmeters, then adjustment was made for no deflection of the galvanometer by varying the resistance of the circuit, and finally, after the current had become practically constant, it was sent for a definite time through the voltmeters by throwing the switch 1. By rotating the Kelvin rheostat the current was kept constantly adjusted to its proper value, until the experiment was completed.

Standard resistances of 1, 2, 5, and 10 ohms were used in *St. R.*,



and once the 1 and the 2 ohms were placed in parallel. Thus the current was varied in the different experiments from 0.1 to 1.5 ampere. The current was usually kept closed long enough to allow one ampere hour to pass through the voltmeters—i. e., each time about 4 grams of silver were deposited. A change of the current as small as 1 in 100,000 could easily be seen on the scale. In the final determinations of the electrochemical equivalent only such experiments were used in which the variations of the current did not exceed 1 in 25,000.

The time was measured by an astronomical clock, connected with a chronograph, which also recorded the making and breaking of the current through the voltmeters. The rate of the clock was determined by means of the noon signals received from the Naval Observatory.

In a number of experiments the variations of the current exceeded

the required limit, due partly to the inconstancy of the storage battery, partly to variations in the resistance of the voltmeters or the circuit. In some cases the rate of the clock was not known to a sufficient degree of accuracy or the resistance of the voltmeters was not estimated correctly. Experiments in which such troubles occurred could not be used for the final calculation, but they allow relative comparisons of the different types of voltmeters.

For convenience the Weston cell was not placed directly in the shunt from the standard resistance, but a potentiometer was introduced, the Weston cell balanced over a resistance of 10,190 ohms and then the terminal potential difference of the standard resistance substituted for the cell. Of course the constancy of the auxiliary current through the potentiometer was frequently checked by reconnecting with the standard cell.

#### COMPARISON OF DIFFERENT VOLTAMETERS.

In the following tables the results obtained are given. Table I shows that two voltmeters of the same type and treated in precisely the same manner will yield deposits which will in general not differ from each other by more than 1 in 10,000.

TABLE 1.

| Richards type.          |          |                  | Usual type.       |          |                  |
|-------------------------|----------|------------------|-------------------|----------|------------------|
| 1.                      | 2.       | Difference.      | 1.                | 2.       | Difference.      |
|                         |          | <i>Per cent.</i> |                   |          | <i>Per cent.</i> |
| 4. 09927                | 4. 09911 | —0. 004          | 4. 09973          | 4. 09945 | —0. 007          |
| 4. 09955                | 4. 09959 | + . 001          | 4. 10003          | 4. 09961 | — . 010          |
| 4. 09851                | 4. 09870 | + . 007          | 4. 10154          | 4. 10098 | — . 014          |
| 4. 09927                | 4. 09941 | + . 003          | 4. 10100          | 4. 10247 | + . 036          |
| Ag <sub>2</sub> O type. |          |                  | Large anode type. |          |                  |
| 1.                      | 2.       | Difference.      | 1.                | 2.       | Difference.      |
|                         |          | <i>Per cent.</i> |                   |          | <i>Per cent.</i> |
| 4. 09913                | 4. 09935 | +0. 006          | 4. 10082          | 4. 09985 | —0. 001          |
| 4. 10382                | 4. 10392 | + . 002          | 4. 09891          | 4. 09888 | — . 001          |
|                         |          |                  | 4. 10172          | 4. 10167 | — . 001          |

The usual form does not show nearly as constant results as the other three. The last large difference may, however, be due to loss of silver in one bowl.

Tables 2 to 8 allow a comparison between the different types, when the same quantity of electricity is sent through them. In the first column of each table the current is given and in the last the percentage difference between the two forms. The asterisks denote that the deposit is on silver, i. e., that it is a second deposit.

TABLE 2.

| Current.     | Richards type. | Usual type. | Difference.      |
|--------------|----------------|-------------|------------------|
|              |                |             | <i>Per cent.</i> |
| 0.1          | 4.09863        | 4.10150*    | +0.070           |
| 0.2          | 4.09993*       | 4.10225     | + .057           |
| 0.5          | 4.09870        | 4.10126*    | + .062           |
| 1.0          | 4.09800        | 4.09973     | + .036           |
| 1.0          | 4.09927*       | 4.10100     | + .042           |
| 1.0          | 4.09941*       | 4.10247     | + .050           |
| 1.0          | 4.09881        | 4.10016     | + .030           |
| 1.0          | 4.09896        | 4.10056     | + .039           |
| Average..... |                |             | + .048           |

TABLE 3.

| Current.     | Richards type. | Ag <sub>2</sub> O type. | Difference.      |
|--------------|----------------|-------------------------|------------------|
|              |                |                         | <i>Per cent.</i> |
| 0.1          | 4.09863        | 4.10147                 | +0.069           |
| 0.5          | 4.09881*       | 4.10053                 | + .042           |
| 1.0          | 4.09881        | 4.10167                 | + .070           |
| 1.5          | 4.09897        | 4.10091*                | + .047           |
| Average..... |                |                         | + .055           |

TABLE 4.

| Current.      | Richards type. | Leduc type. | Difference.      |
|---------------|----------------|-------------|------------------|
|               |                |             | <i>Per cent.</i> |
| 0.2           | 4.09920        | 4.10245     | +0.057           |
| 0.5           | 4.09906*       | 4.10141*    | + .057           |
| Average ..... |                |             | + .057           |

TABLE 5.

| Current.      | Large anode type. | Ag <sub>2</sub> O type. | Difference.      |
|---------------|-------------------|-------------------------|------------------|
|               |                   |                         | <i>Per cent.</i> |
| 1.0           | 4.10172           | 4.10382*                | +0.051           |
| 1.0           | 4.10167           | 4.10392*                | + .055           |
| Average ..... |                   |                         | + .053           |

TABLE 6.

| Current.      | Usual type. | Ag <sub>2</sub> O type. | Difference.      |
|---------------|-------------|-------------------------|------------------|
|               |             |                         | <i>Per cent.</i> |
| 0.1           | 4.10150     | 4.10147                 | -0.001           |
| 0.2           | 4.10003*    | 4.09913                 | - .022           |
| 0.2           | 4.09961*    | 4.09935                 | - .006           |
| 1.0           | 4.10016     | 4.10167                 | + .038           |
| Average ..... |             |                         | + .002           |

TABLE 7.

| Current.      | Richards type. | Large anode type. | Difference.      |
|---------------|----------------|-------------------|------------------|
|               |                |                   | <i>Per cent.</i> |
| 0.2           | 4.09993*       | 4.09962           | -0.007           |
| 0.5           | 4.09881*       | 4.09889           | + .002           |
| 0.5           | 4.09906*       | 4.09919*          | + .003           |
| 1.0           | 4.10449        | 4.10468           | + .005           |
| 1.0           | 4.10461        | 4.10510           | + .012           |
| 1.0           | 4.09919        | 4.09912           | - .002           |
| 1.0           | 4.11888        | 4.11902           | + .004           |
| 1.0           | 3.47004*       | 3.47053           | + .014           |
| 1.5           | 4.09897        | 4.09880*          | - .004           |
| Average ..... |                |                   | + .003           |

TABLE 8.

| Current.      | Richards type. | Richards type (modified). | Difference.      |
|---------------|----------------|---------------------------|------------------|
|               |                |                           | <i>Per cent.</i> |
| 0.2           | 4.09920        | 4.09942                   | +0.005           |
| 0.5           | 4.09906*       | 4.09896*                  | - .002           |
| 1.0           | 3.47004*       | 3.46998*                  | - .003           |
| 1.0           | 4.09896        | 4.09881                   | - .004           |
| 1.0           | 4.11888        | 4.11857                   | - .008           |
| Average ..... |                |                           | - .002           |

From these tables it can be seen that there are two distinct classes of voltmeters, on one side the usual, Leduc's, and the silver oxide type, on the other the Richards and the large anode type, the latter class yielding deposits about 0.05 per cent smaller than the former.

The difference between the usual form and Richards's agrees very well with the final average of 0.050 per cent given by Richards.<sup>a</sup> A voltmeter in which the electrolyte was always kept saturated with  $\text{Ag}_2\text{O}$  has not been used before. Patterson and Guthe employed a solution saturated before the experiment was made, but in all other respects adhered to the legal specifications. Richards's comparison between his type and Patterson and Guthe's showed for the latter a

<sup>a</sup> Richards and Heimrod: Proc. Am. Acad., 87, p. 439; 1902.



deposit 0.112 per cent larger than for the former, besides a considerable variation in the percentage difference (from 0.039 to 0.230 per cent). We see from Table 3 that the form of  $\text{Ag}_2\text{O}$  voltameter used in the present investigation gives somewhat more concordant results and a much smaller difference between the two types. Possibly the presence of filter paper in the Patterson and Guthe voltameter may explain the greater value. A porous cup has doubtless the effect of decreasing the weight of the deposit.

It must be confessed that the results obtained with Leduc's voltameter were not expected. He has repeatedly asserted that no trouble arises when small anodic current density is employed. As will be seen from Table 4, in these comparisons small currents were used, but the deposit was in each case 0.057 per cent larger than that obtained with the Richards type. A comparison of Tables 3 and 4 verifies Leduc's statement<sup>a</sup> that a solution saturated with the oxide and a normal solution will give identical results. It was also noticed that even with these small currents the heavy anode liquid sinking to the bottom of the platinum bowl and there spreading produced a starlike figure of the deposit. This peculiar arrangement of the crystals, very characteristic for the usual type, has been studied by Behn,<sup>b</sup> who showed it to be due to the motion of the heavy anode liquid along the surface of the kathode.

With a porous cup around the anode the crystals are distributed perfectly evenly over the kathode without showing the striæ spoken of. Filter paper allows an easy passage to the anode solution, and should therefore be avoided, even if the chemical action upon neutral nitrate solution, to which attention was drawn above, will not affect the electro-chemical equivalent.

All previous researches on the subject have led to the conclusion that the source of trouble is to be looked for at the anode. The solution becomes acid during electrolysis, and therefore the presence of acid has been suggested as the reason for the inconstancy of the voltameter. Leduc<sup>c</sup> has shown, however, that a fresh solution, originally neutral, but afterwards slightly acidified, gives smaller deposits than a neutral solution. According to this observation, the presence of acid alone has not the effect of increasing the weight of the deposit. Another interesting observation by the same author shows that if the amount of acid surpasses a certain limit in the original solution, its density will decrease during electrolysis, a fact observed before by

<sup>a</sup> Leduc: *Rapp. Congr. intern.*, 2, p. 444; Paris, 1900.

<sup>b</sup> Behn: *Wied. Ann.*, 51, p. 105; 1894.

<sup>c</sup> Leduc: *J. de Phys.*, 1, p. 573; 1902.

Rodger and Watson.<sup>a</sup> The latter seem to be the first to have pointed out that during electrolysis a complex silver ion may be formed and that this may produce too heavy deposits. The thorough work of Richards and his students have strengthened this view and my own results seem all to point to the same explanation. The existence of such a complex ion, whatever it may be,<sup>b</sup> would easily enough explain the formation of the acid at the anode. It is well known that, during electrolysis of silver nitrate solution, crystals are formed at a platinum anode, which according to Sulc<sup>c</sup> and Mulder and Heringa<sup>d</sup> have the formula  $\text{Ag}_7\text{NO}_{11}$ . I have been able to obtain these bluish-black crystals mixed with silver at the cathode from a solution saturated with  $\text{Ag}_2\text{O}$ , using a large current density. In this case the deposit was abnormally heavy, 0.23 and 0.32 per cent greater than that obtained in a large silver anode voltameter. Complex ions are therefore present in such a solution and it seems reasonable to suppose that if they reach the kathode, as they will in voltameters in which the anode is surrounded by filter paper or in the  $\text{Ag}_2\text{O}$  type, the deposit will be too heavy, resulting in a wrong value for the electro-chemical equivalent of silver.

Richards's method of removing the heavy anode liquid from the bottom of the porous cup and preventing its diffusion toward the kathode by keeping the level inside lower than the outside is therefore correct, and will not give too small an equivalent, as Leduc asserts.

The large silver anode voltameter gives the same results as Richards's form, in spite of the fact that the solution is at the same level inside and outside. (See Table 7.) How can we explain this? A few experiments by Richards and Heimrod<sup>e</sup> show that a crucible on which silver had been deposited previously will increase in weight when filled with the anode solution. Apparently a chemical reaction takes place, resulting in a breaking up of the complex ion and a deposition of silver. In the large anode type the porous cup prevents the anode solution from diffusing rapidly, and the silver at the bottom of the cup breaks up the troublesome ion. This secondary chemical reaction, I believe, is the reason why the anode is always covered by a layer of finely divided silver—the "anode dust"—the anode itself acting, to a certain extent, chemically on the ion produced by electrolysis. The

<sup>a</sup> Rodger and Watson: *Phil. Trans.*, 186, p. 631; 1895.

<sup>b</sup> Richards suggests  $\text{Ag}_2\text{O}$ . See also the theoretical discussion between Revay and Küster and von Steinwehr: *ZS. f. Electroch.*, 4, pp. 313, 451; 1898.

<sup>c</sup> Sulc: *ZS. f. anorg. Chem.*, 12, p. 90; 1896.

<sup>d</sup> Mulder and Heringa: *Verh. Kon. Ak. Wet.*, 3, p. 37; 1896.

<sup>e</sup> Richards and Heimrod: *loc. cit.*, p. 431.

large anode type will therefore give as good results as the Richards type and is more convenient to use.

The above considerations led the author to the construction of a modified Richards voltameter. Instead of removing the liquid from the bottom, some of the silver crystals deposited by previous runs were placed in the bottom of the cup and the liquid was left undisturbed during the experiments. At first some inconvenience was experienced, due to the rapid growth of the silver toward the anode and consequent sudden changes in the resistance of the voltameter, which prevented the proper adjustment of the current to a constant value. But this trouble was overcome entirely by surrounding the lower part of the porous cup by a small glass cup cut from a test tube or a small beaker. The excellent agreement with Richards's form (see Table 8) shows that also this modification will give the correct silver equivalent.

It is therefore proposed to abandon in future experiments, especially for the absolute measurement of current, the usual form of the silver voltameter and employ either Richards's type or one of those proposed by the author. The differences in the weight of silver obtained by two of these voltameters in series show much smaller variations than those by the old form, and, as will be seen in the following, determinations made by means of independent experiments agree very closely.

#### SILVER NITRATE SOLUTION—DEPOSIT ON PLATINUM AND ON SILVER.

It has been shown by Kahle that an old solution will yield too heavy deposits, its effect increasing as more and more silver is electrolysed. The Reichsanstalt advises, therefore, to use the solution only until 3 grams of silver are deposited from 100 cm<sup>3</sup> of solution. It seemed to be of interest to see whether in the Richards form the solution outside of the porous cup could be used repeatedly. Eight grams of silver were deposited by two runs from the same solution, amounting to about 150 cm<sup>3</sup>, and the solution left standing for several weeks. In the experiments following two Richards voltameters were placed in series, one with a fresh solution, one with this old solution. The latter gave on the average 0.055 per cent more silver. Now, it was thought that it might be possible to obtain a good solution by leaving it in contact with silver crystals. After several weeks more this solution was tried again and gave on the average 0.025 per cent heavier deposits than the Richards form. While an improvement is to be observed, it is clear that keeping the solution simply in contact with silver will not entirely eliminate the complex ion. It is therefore advisable to use always freshly prepared silver nitrate solution. Variations in

concentration have been shown by former investigators to be of no influence upon the weight of the deposit. In our case the electrolyte was always a 20 per cent solution.

Gore<sup>a</sup> and Kahle found a somewhat larger deposit when the kathode was used a second time before removing the silver collected in a previous run; and Richards and Heimrod corroborate their statement so far as the filter-paper voltameter is concerned, but found no difference with the porous-cup voltameter. Patterson and Guthe obtained the same deposit on platinum and on silver. Table 9 gives the results of tests made with this point in view.

TABLE 9.

| Current.        | Richards type. |            |                  | Large anode type. |            |                  |
|-----------------|----------------|------------|------------------|-------------------|------------|------------------|
|                 | On platinum.   | On silver. | Difference.      | On platinum.      | On silver. | Difference.      |
| <i>Amperes.</i> |                |            | <i>Per cent.</i> |                   |            | <i>Per cent.</i> |
| 1.0             | 3.70837        | 3.70850    | +0.003           | 3.70879           | 3.70839    | -0.011           |
| 1.0             | 4.10461        | 4.10449    | -.003            | 4.10468           | 4.10510    | +.011            |

It is apparent that it makes no difference whether a clean platinum kathode is used or whether the silver from previous runs is left on it. Of course the deposit should not become too heavy, on account of the increasing danger of mother liquid remaining included in the interstices between the crystals.

The explanation for the heavier deposits on silver in the filter-paper voltameter is to be sought in the action of the silver upon the anode liquid.

#### ELECTRO-CHEMICAL EQUIVALENT OF SILVER.

In conclusion, the results are given of experiments which were suited for the calculation of the electro-chemical equivalent of silver in terms of the electrical standards of the Bureau. According to legal specifications the *E. M. F.* of a Clark cell at 15° C. is 1.434 volts. The unsaturated Weston cell was found to have an *E. M. F.* of 1.01954 volts. The resistance standards have been repeatedly intercompared with those of the Reichsanstalt.

In Table 10 the amount of silver deposited by one ampere hour and under the condition that the terminal potential difference of the resist-

<sup>a</sup>Gore: *Nature*, 27, p. 327; 1882.

ance was equal to the *E. M. F.* of the Weston cell is given for the different forms.

TABLE 10.

| Current.        | Richards type. | Richards type (modified). | Large anode type. | Usual type.   |
|-----------------|----------------|---------------------------|-------------------|---------------|
| <i>Amperes.</i> | <i>Grams.</i>  | <i>Grams.</i>             | <i>Grams.</i>     | <i>Grams.</i> |
| 0.5             | 4.09881        | .....                     | 4.09888           | 4.10154       |
| 0.5             | 4.09906        | 4.09896                   | 4.09891           | 4.10098       |
| 0.5             | 4.09870        | .....                     | .....             | .....         |
| 1.0             | 4.09881        | 4.09881                   | .....             | 4.10016       |
| 1.0             | 4.09896        | .....                     | .....             | 4.10056       |
| 1.5             | 4.09897        | .....                     | 4.09880           | .....         |
| Average ....    | 4.09888        | 4.09888                   | 4.09888           | 4.10081       |
| 0.2             | 4.09920        | 4.09942                   | 4.09952           | .....         |
| 0.2             | 4.09955        | .....                     | 4.09985           | .....         |
| 0.2             | 4.09959        | .....                     | 4.09962           | .....         |
| 0.2             | 4.09993        | .....                     | .....             | .....         |

It is seen that the agreement between these independent experiments is a surprisingly good one, the largest differences amounting to less than 1 in 10,000 in the first three types, as long as the current is equal or larger than 0.5 ampere. For 0.2 ampere the deposits are invariably greater. This is not surprising. The current in these cases was closed five hours, and in spite of all precautions some of the anode liquid probably diffuses through the porous cup during that time. This explanation seems the more plausible from the results obtained in the Richards form with 0.2 ampere. In the first experiment of the four the anode liquid was removed every ten minutes, in the next two every fifteen to twenty minutes, and in the last every half hour. The increase of the weight of the deposit with the opportunity of the liquid to penetrate the porous partition is apparent. The current should therefore be closed not longer than two hours, this of course depending to a certain extent upon the grain of the porous cup.

To obtain the electrochemical equivalent of silver we have to divide the deposit, 4.09888 grams, by  $1.01954 \times 3,600$ . This gives 1.11675 mg per coulomb; but the silver was weighed with platinum weights, and

therefore a correction of  $+0.0069$  per cent should be applied for the buoyancy of air. Thus the final result for the silver equivalent is

**1.11683 mg per coulomb.**

With a filter-paper voltameter the corresponding results would be 0.048 per cent higher, or 1.11736 mg. Kahle's comparison between the *E. M. F.* of the Clark cell and the electrochemical equivalent of silver reduced to the same value for the *E. M. F.* of the Clark cell would give 1.11726 mg.

In the following table we find a comparison of the values obtained by different observers, all being reduced to the same units. In the case of Perot and Fabry,<sup>a</sup> who used a Clark cell at  $0^{\circ}$  C. and found its *E. M. F.* to be 1.4522 volts, using 1.118 mg as the electrochemical equivalent, the difference of 0.0164 volt, given by the Reichsanstalt, has been used to reduce to  $15^{\circ}$  C., instead of the ratio given by them. If the latter is used their results are identical with von Ettinghausen's.

TABLE 11.

| Name.                       | Year. | Filter-paper<br>voltameter. | Porous-cup<br>voltameter. |
|-----------------------------|-------|-----------------------------|---------------------------|
|                             |       | <i>mg</i>                   | <i>mg</i>                 |
| Carhart.....                | 1882  | 1. 1172                     | 1. 1167                   |
| Rayleigh and Sidgwick ....  | 1884  | 1. 1183                     | 1. 1179                   |
| Von Ettinghausen.....       | 1884  | 1. 1180                     | 1. 1175                   |
| Glazebrook and Skinner .... | 1892  | 1. 11831                    | 1. 11778                  |
| Perot and Fabry .....       | 1898  | 1. 1193                     | 1. 1188                   |
| Kahle .....                 | 1898  | 1. 11726                    | 1. 11673                  |
| Guthe .....                 | 1904  | 1. 11736                    | 1. 11683                  |

The large differences in these results can hardly all be due to the silver voltameter alone. No doubt the Clark cell comes in for its share. Carhart's value is based on only two comparisons, with very small silver deposits, and Rayleigh's is the result of only one experiment.

Richards has made an attempt to reduce the absolute measurements of an electric current to the same basis by calculating the electrochemical equivalent under the supposition that the porous cup voltameter had been used, and finds good agreement between the results

<sup>a</sup> Perot and Fabry: *Annal. de fac. de sci., Marseille*, 8, p. 201; 1898.

of Lord Rayleigh and Mrs. Sidgwick, Fr. and W. Kohlrausch, Kahle, and Patterson and Guthe. They all agree within 3 in 10,000 and give the mean value of 1.1175 mg.

Since that time two more absolute determinations have been made. The recent one by van Dijk and Kunst using the filter paper voltameter gives 1.1182 (or 1.1177) mg for the silver equivalent; but Pellat and Luduc obtained 1.1195 mg, or, reduced to the same basis, 1.1188 mg. Thus, the latest determinations of the electrochemical equivalent still show a difference larger than 1 in 1,000.





## THE SO-CALLED INTERNATIONAL ELECTRICAL UNITS.<sup>a</sup>

By FRANK A. WOLFF.

As one of the most important questions likely to be considered by the St. Louis International Electrical Congress will be that of redefining the fundamental electrical units, it may therefore not be out of place to briefly review the efforts which have thus far been made to bring about international uniformity.

The need of a definite and universal system of electrical units was early recognized, and became a necessity as soon as industrial applications of electricity were made. At first the principal measurements were those of resistance (line resistance, insulation resistance, measurements for the location of faults, etc.). These were expressed in terms of some entirely arbitrary standard, such as the resistance of a given length of an iron or copper wire of given cross section. This naturally led to a great multiplicity of units, none of which ever gained general acceptance.

In 1848 Jacobi pointed out that it would be more satisfactory to adopt as a universal standard the resistance of a certain piece of wire, copies having the same resistance being easily constructed. Jacobi carried this suggestion into practice by sending copies of his standard, since known as "Jacobi's Étalon," to the leading physicists of that period.

In 1860 Werner von Siemens proposed as a standard of resistance the resistance, at 0° C., of a column of mercury of a uniform cross-section of 1 square millimeter and 1 meter in length.

In 1861 a committee composed of the most eminent English physicists was appointed by the British Association to consider the question of standards of electrical resistance. The leading foreign physicists were invited to offer suggestions, and various special investigations of the problems with which the committee was confronted were undertaken by its members.

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<sup>a</sup> A paper presented at the International Electrical Congress, St. Louis, 1904.

It was decided that the unit of resistance should be defined in terms of the Gauss-Weber absolute system of electromagnetic units, which had already received such well merited recognition; but since this unit was inconveniently small it was decided to define the practical unit as an integral decimal multiple of the same.

The value of the unit depends upon the units of length, mass, and time adopted as the basis of the system. Those chosen by Gauss and Weber were the millimeter, milligram, and second. In England efforts were being made to establish an absolute system for the definition of all physical units, for which the fundamental units of Weber were of inconvenient magnitude, and for which the centimeter, gram, and second were finally adopted (the c. g. s. system).

The practical unit of resistance in this system was defined as  $10^9$  c. g. s. electromagnetic units, and while this definition fixes the unit theoretically, it can only be applied in practice by the measurement of some particular resistance in absolute measure. This requires the construction of especially designed apparatus, with which measurements lying within a very limited range may be made; the determination of its instrumental constants most frequently involving tedious mathematical approximations, and the elimination of errors of observation. With all possible precautions the errors of such methods exceed, even to-day, a hundredfold the relative errors in resistance comparisons.

Investigations were therefore made to determine whether the absolute unit of resistance could be accurately defined in terms of the resistance of a definite portion of a definite substance. The electrical properties of alloys and pure metals in the solid and liquid states were studied with this end in view. On account of the excessive influence, on the resistance, of even small quantities of impurities in metals of the highest obtainable purity, and of small variations in the compositions of alloys, the choice was greatly limited. It was found, in addition, that solid metals had to be rejected on account of the marked influence of physical changes produced by annealing, hardening, drawing, bending, etc.

Mercury, already recommended by Siemens, was therefore the only material to be further considered, but was also rejected for two reasons, viz, the large differences found to exist between coils supposedly adjusted to different German mercurial standards, and differences between a number of mercurial standards constructed by members of the committee.

The committee therefore recommended the alternative method of

constructing material standards adjusted with reference to the absolute unit. In this connection a special form of resistance standard known as the B. A. type was designed, and after an investigation of the constancy of a number of new alloys in addition to many already in use, one containing two parts by weight of silver to one part by weight of platinum was finally selected as best meeting all requirements.

In 1863 and 1864 the values of certain coils were determined in absolute units by one of the methods proposed by Weber, and from these measurements the B. A. unit was derived. A number of copies were issued, gratis, by the association, and in addition arrangements were made for supplying others at a moderate price. The B. A. unit soon gained general acceptance in the English-speaking countries, while the Siemens unit still retained its supremacy on the Continent.

No action was at that time taken by the British association committee to define the units of current and electromotive force further than in terms of the c. g. s. system. The currents to be measured were all relatively small, and were usually measured by means of a tangent galvanometer with a sufficient accuracy. Electromotive forces were seldom measured, and then usually in terms of the Daniell cell. In 1872 Latimer Clark brought to the attention of the committee the superiority of the cell which now bears his name, recommending it as a suitable standard of electromotive force, but no definite action was taken by the committee.

In 1878 it was shown by Prof. H. A. Rowland that the B. A. unit was in error by more than 1 per cent, and soon after the existence of a discrepancy of this magnitude was verified by a number of other investigators.

In 1881 a call was issued by the French Government for an International Electrical Congress, to be held in connection with the first International Electrical Exposition at Paris, for the purpose of adopting definitions of the electrical units which might serve as a basis for legislative enactments. In the meantime a number of mercurial standards had been constructed and had been found to be in satisfactory agreement; moreover, the results of most of the absolute determinations had been referred either directly or indirectly to the Siemens unit.

The Paris Congress, therefore, recommended that the practical electrical units be defined in terms of the units of the c. g. s. system of electromagnetic units, and that the unit of resistance be represented by a column of mercury 1 square millimeter in cross section, at the temperature of  $0^{\circ}$  C., of a length to be determined by an international

commission appointed for this purpose, as appears in the following resolutions:

RESOLUTIONS OF THE INTERNATIONAL CONGRESS OF ELECTRICIANS,  
PARIS, 1881.

- (1) That the c. g. s. system of electromagnetic units be adopted as the fundamental units.
- (2) That the practical units, the ohm and the volt, preserve their previous definitions,  $10^9$  and  $10^8$  c. g. s. units, respectively.
- (3) That the unit of resistance, the ohm, be represented by a column of mercury 1 square millimeter in cross section at the temperature of  $0^\circ$  C.
- (4) That an international commission be charged with the determination, by new experiments, of the length of the mercury column 1 square millimeter in cross section, at a temperature of  $0^\circ$  C., representing the ohm.
- (5) That the current produced by a volt in the ohm be called an ampere.
- (6) That the quantity of electricity produced by a current of 1 ampere in one second be called a coulomb.
- (7) That the unit of capacity be called a farad, which is defined by the condition that a coulomb in a farad raises the potential 1 volt.

The Congress<sup>a</sup> also recommended the employment of the carcel as the standard for photometric comparisons.

The international commission, appointed in accordance with paragraph 4 of the resolutions of the Paris Congress of 1881, met at Paris in 1882, but definite action was deferred until two years later, when the following definitions were unanimously recommended:

The legal ohm is the resistance of a column of mercury 1 square millimeter in cross section and 106 centimeters in length, at the temperature of melting ice.

The ampere is equal to one-tenth of a c. g. s. unit of the electro-magnetic system.

The volt is the electro-motive force which will maintain a current of 1 ampere in a conductor of which the resistance is a legal ohm.

The value adopted for the length of the mercurial column was taken as 106 centimeters, notwithstanding that most of the best results were very close to 106.3; as it was thought advisable to adopt a value known to be true to the nearest centimeter for a period of ten years. On account of this uncertainty, no steps were actually undertaken by the various governments represented.

The conference also adopted as the unit of light of any color the quantity of such light emitted in a perpendicular direction by 1 square centimeter of molten platinum at the temperature of solidification; and as the practical unit of white light the total quantity of light emitted perpendicularly by the same source.

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<sup>a</sup>For the sake of completeness, the recommendations of the various international electrical congresses on photometric standards are included in the summary.

In 1889 a second international congress of electricians was held at Paris, by which the following definitions were adopted:

The joule, the practical unit of energy, is equal to  $10^7$  c. g. s. units. It is equal to the energy disengaged as heat in one second by a current of 1 ampere flowing through a resistance of 1 ohm.

The practical unit of power is the watt. The watt is equal to  $10^7$  c. g. s. units, and is the power of 1 joule per second.

The practical unit of self-inductance is the quadrant, which is equal to  $10^9$  centimeters.

The Congress recommended that the power of machines be expressed in kilowatts instead of in horse-power.

It adopted also, as the photometric standard, the "bougie decimal," defined as one-twentieth of the Violle platinum standard adopted by the Conference of 1884.

The following definitions were also adopted:

The period of an alternating current is the duration of a complete oscillation.

The frequency is the number of periods per second.

The mean intensity is defined as the mean value of the current during a complete period, without reference to its sign.

The effective intensity is the square root of its mean-squared value.

The effective electromotive force is the square root of its mean-squared value.

The apparent resistance is the factor by which the effective current must be multiplied to obtain the effective electromotive force.

The positive pole of a storage cell is that which is connected to the positive pole of a dynamo in charging, and which is the positive pole during its discharge.

In addition, the question of defining and naming practical magnetic units was discussed. The definition proposed for the unit of field intensity was the intensity of a uniform field which would produce an electro-motive force of 1 volt in a conductor 1 centimeter in length normally cutting the lines of force with a velocity of 1 centimeter per second. The name proposed for this unit was the "Gauss;" and as the unit which is equal to  $10^8$  c. g. s. units does not correspond to field intensities ordinarily dealt with, the micro-Gauss was suggested for ordinary use.

The Weber, defined as  $10^8$  c. g. s. units, was proposed as the unit of magnetic flux.

No definite action was, however, taken by the Congress on either of these units.

The increased accuracy obtainable by the use of apparatus of improved construction, and by refinements in the methods employed, led to a much closer agreement of the various redeterminations of the absolute electrical units, and their relation to the Siemens unit, the Clark cell, and the electro-chemical equivalent of silver in terms of

which many measurements were made. The rapid development of the electrical industries also called for a redefinition of the units and the legalization of such definitions.

In December, 1890, a committee was appointed by the English Board of Trade to consider what action should be taken by the Board with a view to causing new denominations of standards for the measurements of electricity for use for trade to be made and duly verified. The members of this committee consisted of two representatives each of the Board of Trade, the General Post-Office, the Royal Society, the British Association, and the Institute of Electrical Engineers.

A set of resolutions embodying the proposals which appeared to be desirable were drafted, and copies of the same were submitted to the various interests for criticism. These resolutions also embodied proposals for standards of resistance, current, and electromotive force.

In 1891 a committee was appointed by the American Institute of Electrical Engineers to report on units and standards. The report of the committee, made in June, 1891, which deals mainly with magnetic units, is as follows:

Your committee, considering that authorized and recognized names for four practical electromagnetic units, at present unentitled, are needed by electrical engineers in this as well as in other countries, for dealing conveniently with magnetic circuits in analysis, discussion, and design, recommends to the Institute the four units as appended in detail, of magnetomotive force, reluctance, flux, and flux-density, in the hope that if favorably considered, the Institute may further the endeavors of the next International Electrical Congress toward securing for them universally recognized titles.

\* \* \* \* \*

First. *Magnetomotive force*; or difference of magnetic potential.

Simple definition.—The analogue in a magnetic circuit of voltage in an electric circuit.

Strict definition.—The magnetomotive force in a magnetic circuit is  $4\pi$  multiplied by the flow of current linked with that circuit.

The magnetomotive force between two points connected by a line is the line integral of magnetic force along that line. Difference of magnetic potential constitutes magnetomotive force.

Electromagnetic dimensional formula,  $L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$ .

The absolute unit of m. m. f. is  $\frac{1}{4\pi} \times$  unit current of one turn.

The practical unit is  $\frac{1}{4\pi} \times$  ampere of one turn, or one-tenth of the absolute unit—i. e., 0.0796 ampere-turns give the unit. The prefix kilo would perhaps be occasionally used for practical applications.

Second. *Magnetic flux*.

Simple definition.—Total number of lines of force or total field.

Strict definition.—The magnetic flux through a surface bounded by a closed curve is the surface integral of magnetic induction taken over the bounded surface, and when produced by a current is also equal to the line integral of the vector potential of the current taken around the boundary.

The uniform and unit time rate of change in flux through a closed magnetic circuit establishes unit electromotive force in the circuit.

Electromagnetic dimensional formula,  $L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$ .

The absolute unit is 1 c. g. s. line of induction.

The practical unit is  $10^9$  c. g. s. lines.

Fluxes range in present practical work from 100 to 100,000,000 c. g. s. lines, and the working units would perhaps prefix milli- and micro-.

Third. *Magnetic intensity*, or induction density.

Simple definition.—Flux per square centimeter.

Strict definition.—The induction density at a point within an element of surface is the surface differential of the flux at that point.

Electromagnetic dimensional formula,  $L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$ .

Absolute unit, 1 c. g. s. line per square centimeter.

Practical unit,  $10^9$  c. g. s. lines per square centimeter.

In practice, excluding the earth's field, intensities range from 100 to 20,000 lines per square centimeter, and the working unit would perhaps have the prefix milli- or micro-.

Fourth. *Magnetic reluctance*.

Definition.—Unit reluctance in a magnetic circuit permits unit magnetic flux to traverse it under the action of unit magnetomotive force.

Dimensional formula,  $L^{-1}M^0T^0$ .

The practical unit is  $10^{-9}$  the absolute unit.

Reluctances vary in present practical work from 100,000 to 100,000,000 of these practical units, so that the working unit would perhaps employ the prefix mega-.

There were considerable differences of opinion manifested in the discussion following the presentation of the report, and definite action thereon was postponed.

At the Frankfort International Electrical Congress, in September, 1891, the question of naming and defining the magnetic units was brought up. The names Gauss and Weber, for field intensity and flux, respectively appeared to meet with general approval, but there was considerable disagreement as to what their numerical values should be,  $10^9$  being apparently preferred for both.

Owing to the limited time allowed for consideration, no definite action was taken.

In connection with the British Association meeting in Edinburgh, in 1892, a conference was held, attended by Helmholtz, Guillaume, Carhart, and others, to discuss the Board of Trade report which was submitted at the meeting. It was resolved to adopt for the length of the mercurial column 106.3 centimeters and to express the mass of the column of constant cross-section instead of the cross-sectional area of 1 square millimeter. Final action was deferred to await the decision of the Chicago International Electrical Congress, arrangements for which had then been made.

This Congress, to which the various governments were invited to send delegates, met in 1893. The Governments represented were

United States, Great Britain, France, Italy, Germany, Mexico, Austria, Switzerland, Sweden, and British North America. Professor von Helmholtz was made honorary president of the Congress, and Prof. H. A. Rowland president of the Chamber of Delegates, composed of the official delegates of the various Governments represented. After six days' deliberation the following resolutions were adopted:

RESOLUTIONS OF THE INTERNATIONAL ELECTRICAL CONGRESS,  
CHICAGO, 1893.

*Resolved*, That the several Governments represented by the delegates of this International Congress of Electricians be, and they are hereby, recommended to formally adopt as legal units of electrical measure the following:

As a unit of resistance, the *international ohm*, which is based upon the ohm equal to  $10^9$  units of resistance of the c. g. s. system of electro magnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of the length of 106.3 centimeters.

As a unit of current, the *international ampere*, which is one-tenth of the unit of current of the c. g. s. system of electromagnetic units, and which is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, and in accordance with accompanying specifications, deposits silver at the rate of 0.001118 of a gramme per second.

As a unit of electromotive force, the *international volt*, which is the electromotive force that, steadily applied to a conductor whose resistance is 1 international ohm, will produce a current of 1 international ampere, and which is represented sufficiently well for practical use by  $\frac{1}{1.018}$  of the electromotive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of  $15^{\circ}$  C., and prepared in the manner described in the accompanying specifications.

As a unit of quantity, the *international coulomb*, which is the quantity of electricity transferred by a current of 1 international ampere in one second.

As a unit of capacity, the *international farad*, which is the capacity of a condenser charged to a potential of 1 international volt by 1 international coulomb of electricity.

As a unit of work, the *joule*, which is equal to  $10^7$  units of work in the c. g. s. system and which is represented sufficiently well for practical use by the energy expended in one second by an international ampere in an international ohm.

As a unit of power, the *watt*, which is equal to  $10^7$  units of power in the c. g. s. system, and which is represented sufficiently well for practical use by work done at the rate of 1 joule per second.

As the unit of induction, the *henry*, which is the induction in a circuit when the electro-motive force induced in this circuit is 1 international volt, while the inducing current varies at the rate of 1 ampere per second.



## SPECIFICATIONS.

In the following specifications the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time, the time average of the current, or if the current has been kept constant, the current itself can be deduced.

In employing the silver voltameter to measure currents of about 1 ampere, the following arrangements should be adopted:

The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 cm in diameter and from 4 to 5 cm in depth.

The anode should be a plate of pure silver some 30 cm<sup>2</sup> in area and 2 or 3 mm in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling onto the kathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

## SPECIFICATIONS FOR THE CLARK CELL.

A committee, consisting of Messrs. von Helmholtz, Ayrton, and Carhart, was appointed to prepare specifications for the Clark cell. (Owing to the death of Von Helmholtz no report was ever made by this committee.)

A motion was made and carried that for magnetic units the c. g. s. system be commended, and that for the present no names be given to these units.

Magnetic units.  
Photometric standards.

A resolution was adopted as follows:

*Resolved*, That this committee while recognizing the great progress realized in the standard lamp of Von Heffner-Alteneck and the very important researches made at the Reichsanstalt, also recognizes that other standards have been proposed and are now being tried, and that there are serious objections to every kind of standard in

which an open flame is employed. It is, therefore, unable to recommend the adoption at the present time of either the Von Heffner lamp or the pentane lamp, but recommends that all nations be invited to make researches in common on well-defined practical standards and on the convenient realization of the absolute unit.

In March, 1900, the following resolution was adopted by the American Institute of Electrical Engineers:

*Moved*, That the committee on units and standards be requested to investigate and report at the ensuing meeting in regard to the advisability of the following:

1. The giving of names to the absolute units of the electrostatic and electro-magnetic systems.
2. The denotations, by means of prefixes, of multiples of such units.
3. The rationalization of the present system by means of taking the absolute unit of magnetism as equal to the present magnetic line, and the absolute unit of difference of magnetic potential as equal to the present absolute unit of current-turn.
4. The advisability of taking up any or all of the above matters at the Congress to be held in Paris this year.

In May, 1900, the following report of the committee was adopted by the Institute:

1. We consider that there is need for names for the absolute c. g. s. units in the electrostatic and the electromagnetic systems; also for suitable prefixes to denote decimal multiples and submultiples of these units in supplement and addition to those already in common use.
2. That the International Electrical Congress convening this year at Paris should be urged to bestow the above-mentioned names and create said decimal prefixes.
3. That much advantage would accrue to a universal "rationalization" of electric and magnetic units, and that the Congress be requested to consider the means and advisability of such "rationalization."
4. That we recommend that the whole subject should be brought up as a topic for general discussion at the approaching general meeting of the Institute in Philadelphia.

(Signed)

F. B. CROCKER.  
W. E. GEYER.  
G. A. HAMILTON.  
W. D. WEAVER.  
A. E. KENNELLY, *Chairman*.

#### PARIS CONGRESS, 1900.

The last official Congress was held at Paris in August, 1900.

A committee of Section 1, to consider questions in reference to the units, reported as follows:

The committee will only take into consideration propositions not involving modifications of the decisions of previous congresses.

The committee believes that there is no actual need of giving names to all the electro-magnetic units.

However, owing to the employment, in practice, of apparatus giving directly field intensities in c. g. s. units, the committee recommends giving the name "Gauss" to this c. g. s. unit.

The committee recommends giving to the unit of magnetic flux, the value of which is subsequently to be fixed, the name "Maxwell."

The report adopted by the Section, after a spirited discussion, was as follows:

1. The Section recommends giving the name "Gauss" to the c. g. s. unit of magnetic field intensity.
2. The Section recommends giving the name "Maxwell" to the c. g. s. unit of magnetic flux.

These units were given an international character and standing by their adoption at the general meeting of the official delegates of the various governments, after a stormy debate.

#### THE LEGALIZATION OF THE ELECTRICAL UNITS BY THE VARIOUS GOVERNMENTS.

Notwithstanding that the resolutions of the Chicago Congress were adopted with practical unanimity, and might, therefore, have been considered as in a sense binding on the various governments, up to this date only six governments, United States, Great Britain, Canada, Germany, Austria, and France, have legislated on this subject, and only a few of these have acted strictly in accordance with the resolutions of the Chicago Congress.<sup>a</sup>

#### DISCUSSION OF LEGISLATION.

Strictly speaking, no two countries have defined the electrical units in the same way. This naturally suggests that there must be good and sufficient reasons, which may in part be traced to the insufficiency of the Chicago definitions.

(1) It is evident that all three of the units should not be defined in terms of concrete standards, connected as they are by Ohm's law so that only two of the three are independent, and hence the third should be defined in terms of the other two.

(2) The two units adopted as fundamental should be defined only in terms of concrete standards, and not in terms of the absolute units.

(3) The specifications for the silver voltameter were shown to be inadequate.

(4) Redeterminations of the electro-motive force of the Clark cell at 15° C. in absolute measure indicated that this value was nearer 1.433 volts than 1.434 volts.

However, the variations introduced in the definitions by some of the governments lead to confusion and are in violation of the principles laid down at the Chicago Congress.

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<sup>a</sup> For a copy of the laws see Appendix.

## THE UNIT OF RESISTANCE.

Taking the fundamental units up in turn, it will be found that the unit of resistance legalized by the United States, Germany, France, and Canada, and the definitions in the proposed Belgian and Swiss laws, are essentially the same as those adopted at Chicago, differing only in that no reference is made to the unit of resistance being based on  $10^9$  c. g. s. units in case of the German and French laws and in the proposed Swiss and Belgian laws. In fact, it must be admitted that this statement may be regarded as superfluous.

In Austria, on the other hand, the unit of resistance is defined as

**Austria.**  $10^9$  c. g. s. units of the electro-magnetic system, which  
“*for practical purposes* is to be considered equal to the resistance offered at the temperature of melting ice by a column of mercury 106.3 centimeters in length and having a mass of 14.4521 grams.” The uniformity of cross section is, curiously, not specified.

In England, finally, the ohm is defined both as *having the value* of  $10^9$

**England.** in terms of the centimeter and the second of time, and  
as *being represented* by “the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14.4521 grams in mass of a constant cross-sectional area and of a length of 106.3 centimeters.” In addition, a distinction is made between the *unit* of resistance and the *standard* of resistance, and for the latter purpose a particular platinum-silver coil preserved at the Board of Trade Electrical Standardizing Laboratory, in London, and adjusted to represent the unit on an assumed relation between the standards of the British Association and the mercurial unit, is legalized.

It will thus be seen that the unit of resistance has been defined—

- (1) In terms of the absolute c. g. s. unit.
- (2) In terms of the mercurial column.
- (3) In terms of the resistance of a particular coil.
- (4) In terms of combinations of the above.

The objections to the first method have been recognized as long as the subject has been under discussion. For, while the unit is theoretically fixed, resort must in practice be had to material standards, in the absolute measurement of which, errors amounting to at least 0.01 per cent are introduced. Errors several times as great are even met with in different series of observations with the same apparatus, and the differences of the results obtained by different methods may differ still more.

To overcome this objection a suggestion was made in 1893, by Profs. W. E. Ayrton and A. V. Jones, that the unit of resistance be defined

in terms of a particular Lorenz apparatus preserved in a National Physical Laboratory, but even then an uncertainty of at least 0.01 per cent would remain if this practice were adopted by a single government, whereas its general introduction would certainly introduce greater differences.

Another objection to this method lies in the limited range within which accurate measurements of resistance may be made with a given apparatus, so that in practice the measurements would have to be referred to material standards the constancy of which might from time to time be checked to within the above stated limit of accuracy.

Notwithstanding these objections we find that a number of governments have defined the unit of resistance in terms of the absolute unit, indicating that the above principles are not fully appreciated.

The accuracy with which resistance comparisons can be made has for a long time far exceeded the above limits, and the need of an accurate standard, reproducible at any time and at any place to a higher degree of accuracy, has been recognized, as this would enable measurements the world over to be expressed in terms of the *same unit*—a result of much greater importance than in *absolute measure*, with its limited accuracy. In the definition of the concrete standard it is only necessary to assume for it a value in accordance with the best absolute measurements. This once done with a sufficient approximation, the definition of the concrete standard need not be modified.

The meter was originally intended to represent the  $\frac{1}{10,000,000}$  part of the earth's quadrant; but in the actual construction errors of measurement were introduced, which will, however, not affect the *international* meter defined in terms of a particular platinum-iridium bar, of which accurate copies exist the world over, and to which all linear measurements are referred. In a similar manner, the kilogram was intended to represent the mass of a cubic decimeter of water at the temperature of its maximum density; but the *international* kilogram is the mass of a particular cylinder of platinum-iridium, to which all measurements of mass are referred.

It has, therefore, been generally recognized that *reproducibility* should be the first requirement for any international standard, and this qualification is fulfilled to an eminent degree by the mercurial unit, as defined by the Chicago Congress.

When this definition was adopted it was generally assumed that such mercurial standards would be constructed by the various governments represented; but this has been done by only two—Germany and England—each of which is provided with an institution equipped

to undertake this task. The construction of primary mercurial standards is also to be undertaken by the Bureau of Standards, more recently organized, and no doubt by other institutions.

The mercurial standards at the Reichsanstalt agree with one another to within a few parts in 100,000, as do those of the English National Physical Laboratory; and it is, in addition, most gratifying to know that the standards of the two institutions agree with each other almost equally well.

There is, however, one criticism which might still be made of the definition of the unit of resistance in terms of the mercurial unit. Some form of terminal must be applied to the tube to connect it to the circuit containing the resistance standard with which it is to be compared. The method used at the Reichsanstalt consists in employing spherical bulbs, each provided with a current and potential lead, which necessitates the application of a correction, the value of which can be calculated approximately, as Lord Rayleigh has shown, or may be experimentally determined. Unfortunately, the value experimentally found is less than the minimum limit according to Lord Rayleigh's calculations, so that a different result will be obtained according to the correction factor employed.

In addition, there is another method, which might and has been used, of applying potential terminals to the extremities of the tube, which is provided with prolongations previously continuous with the same. This method also introduces a correction the value of which would depend upon a number of conditions.

In any case, however, this source of uncertainty, although slight, could be eliminated by specifying the approximate cross-section or length of the tube representing the unit, the nature of the terminals, and the magnitude of the correction factor to be applied.

#### THE UNIT OF CURRENT.

The Chicago definition of the ampere as one-tenth of the c. g. s. unit has been followed almost verbatim by the United States, Canada, France, and Austria, and the specifications for the silver voltameter are essentially the same except in Austria, where no specifications have been legalized.

In Germany the ampere is simply defined in terms of the electrochemical equivalent of silver, and in addition the specifications for the silver voltameter are considerably modified.

The proposed Swiss law has been copied from the German law.

The Belgian law differs from the German law only in that the ampere is defined, not as being equal to, but as being sufficiently well

represented for practical purposes by "the intensity of a constant current which precipitates in one second 0.001118 grams of silver from an aqueous solution of silver nitrate."

In England it is defined both as one-tenth of a c. g. s. unit, and *as being represented by* "the unvarying electric current which, when passed through a solution of nitrate of silver in water in accordance with the specification appended hereto and marked 'A,' deposits silver at the rate of 0.001118 of a gram per second." In addition, a distinction is made between the *unit* of current and the *standard* of current, the latter being defined in terms of a particular standard ampere balance preserved in the Board of Trade Electrical Standardizing Laboratory.

It will thus be seen that the ampere is defined in three distinct ways, and in some cases the same country has defined it in two or more ways. As has been pointed out, if the ampere is selected as the second fundamental unit it should not be defined in terms of the absolute unit, but simply in terms of the silver voltameter, for which, according to a number of investigations since 1893, the specifications are insufficient, as differences amounting to more than 0.1 per cent may be obtained. The cause of these variations was first shown by Kahle at the Reichsanstalt to be due to secondary reactions in the voltameter, as indicated by differences when freshly prepared silver-nitrate solutions and solutions previously used are employed. Richards, Collins, and Heimrod have traced this influence to the secondary reactions at the anode, and have shown that they may be reduced and possibly eliminated by surrounding the same with a porous cup in which the solution is always kept at a lower level than outside, to prevent diffusion.

This subject has been further investigated by Dr. K. E. Guthe, of the Bureau of Standards, who confirmed the results of Richards, Collins, and Heimrod, and who constructed a more convenient form of voltameter by using a large anode in addition to a porous cup, the best results being obtained with a silver plate in contact with granulated silver. Variations in the results have been attributed to the filter paper, with which in the older forms the anode is surrounded. That the silver nitrate acts upon the paper can not be questioned, but its influence on the solution can hardly explain the results.

The filter paper, however, fails to prevent the secondary products formed at the anode, the exact nature of which has not been established, from reaching the cathode, while the porous cup prevents it almost entirely. The results obtainable with the Richards and the modified forms seem to be reproducible to within about 1 part in

20,000, so that unit current may be defined in terms of the electrochemical equivalent of silver to within this order of accuracy.

The two arguments most frequently advanced in favor of concretely defining the ampere, instead of the volt, are as follows:

(1) According to Faraday's laws of electrolysis, the amount of a given metal deposited in a given time by a given current is constant. As seen above, complications are introduced by secondary reactions at the electrodes, which vary with the metal and the current density employed, making it necessary to specify the form and manner of employment of the voltameter to obtain constant results.

(2) Current intensity can be determined in absolute measure directly by the electro-dynamometer, while electromotive force can only be measured directly in absolute units of the electrostatic system, and the accuracy with which the results can be reduced to electromagnetic units depends upon the accuracy with which " $v$ ," the ratio of the units of the two systems or the velocity of light is known and this is uncertain by possibly as much as 1 part in 1,000.

While this argument would have considerable weight if the fundamental units were to be defined in terms of their absolute values, a practice which, as pointed out above, should be abandoned entirely, it has little bearing on the definition of either current or electromotive force in terms of a concrete standard, for which a value may be adopted which agrees with the best results of absolute current measurement.

The objections which might be urged against defining the ampere, instead of the volt, are as follows:

(1) With a given silver voltameter the range is limited and only currents lying within certain narrow limits can be accurately measured.

(2) Enough time must be allowed for the deposit of at least several grams of silver, so that accurate weighings may be made.

(3) The duration of the experiment must be at least one-half hour, in order that the time may be accurately measured.

(4) During the experiment the current must be kept constant by continuous regulation, or the variations from its mean value must be determined at frequent intervals, so that the average value may be calculated.

(5) Tedious double weighings must be made to determine the amount of silver deposited.

(6) The result finally obtained applies to the average value of the current employed during the experiment and can not be utilized for the accurate measurement of other currents except by reference to a standard cell and a standard resistance, or to some form of apparatus



for current measurement, such as the electrodynamo meter, in which case the accuracy is not as great as that obtained by direct reference to a standard cell.

#### UNIT OF ELECTROMOTIVE FORCE.

The definition of the volt adopted at Chicago has been legalized almost verbatim by the United States, Canada, and France. In Germany and Austria it is defined simply in terms of the ohm and ampere, as is also the case in the proposed Swiss and Belgian laws.

In England the volt is defined as  $10^8$  c. g. s. units, in terms of the ohm and ampere, and in terms of the Clark cell. In addition, a distinction is made between the *unit* of electromotive force and the *standard* of electromotive force, the latter being defined as the  $\frac{1}{10}$  part of the pressure producing a certain deflection of a Kelvin electrostatic voltmeter of the multicellular type preserved at the Electrical Standardizing Laboratory of the Board of Trade.

Here again the definitions legalized differ considerably. Of the various definitions only two need be considered—that in terms of the ohm and ampere, if these units are taken as fundamental, and the definition in terms of the standard cell, if the ohm and volt are taken as the fundamental units.

The arguments in favor of the latter alternative may be briefly summarized as follows:

(1) The facility with which *any* voltage may be directly measured in terms of the standard cell by the potentiometer.

(2) The accuracy with which such measurements may be made, which is practically limited only by the accuracy with which resistance is measurable and by the reproducibility of the cell.

(3) The accuracy with which the standard cell can be reproduced, which even to-day exceeds all practical requirements and which may be still further increased by specifying more precisely the manner of purification and preparation of the materials employed, etc.

(4) The resulting definition of the ampere in terms of the ohm and volt, which corresponds to the actual method employed in precision measurements of current intensity by the potentiometer method.

(5) The facility and accuracy with which any current may be thus measured.

These considerations have led to the adoption of the Clark cell in Germany as the practical standard of electromotive force, notwithstanding that the ampere is legally defined in that country in terms of the electrochemical equivalent of silver and the volt in terms of the ohm and ampere.

For the electromotive force of the Clark cell, the value 1.4328 volts was adopted in Germany as equivalent to the legalized definitions of the ohm and ampere.

In the United States the legalized value of the electromotive force of the Clark cell is 1.434 volts at 15° C. Either this value had to be taken or that of the electrochemical equivalent of silver. The latter seemed less desirable, owing to the insufficiency of the legalized specifications for the silver voltameter and the large variations reported.

The specifications for the Clark cell legalized in the United States were drawn up by the National Academy of Sciences, and refer to the A type; while those in England, Canada, and France are essentially those drawn up by the Board of Trade committee and refer to the Board of Trade type.

#### STANDARD CELLS.

Of the standard cells proposed, only the Clark and Weston cells are to be considered at present. In the former the electrodes consist of zinc amalgam covered with a layer of zinc sulphate crystals and pure mercury in contact with a paste of mercurous sulphate, zinc sulphate crystals, and metallic mercury, the electrolyte being a saturated aqueous solution of zinc sulphate and mercurous sulphate.

In the Weston cell the electrodes consist of cadmium amalgam covered with a layer of cadmium sulphate crystals, and pure mercury in contact with a paste of mercurous sulphate, cadmium sulphate crystals, and metallic mercury, the electrolyte being a concentrated aqueous solution of cadmium sulphate and mercurous sulphate.

The investigations thus far reported indicate that differences between individual cells of either type, set up from materials obtained from various sources and at various times, agree with each other to within 0.0002 volts, corresponding to a slight advantage in favor of the Clark cell on account of its higher electromotive force. The constancy and reproducibility of both types have also been established by the constancy of the ratio between them.

While sharing with the Clark cell these most essential qualities, the Weston cell has a number of marked advantages.

(1) The higher temperature coefficient of the Clark cell is a serious obstacle to measurements of the highest precision, while that of the Weston cell at ordinary temperatures is less than one-twentieth as great, so that errors due to temperature uncertainties are correspondingly reduced.

(2) Clark cells are subject, particularly when a number of years old, to large hysteresis effects attending temperature variations. In the Weston cell the error due to this cause can only amount to a small

fraction of that in the Clark cell, owing to the relatively slight influence of temperature on the solubility of the cadmium sulphate.

(3) The average life of Clark cells is quite short, owing to the tendency of the cell to crack at the point where the platinum terminal is fused into the amalgam limb. This objection might be obviated by suitable modifications in the construction, as have been suggested, but not without introducing some complication. No such tendency has been observed with Weston cells.

(4) In Clark cells a layer of gas is formed at the amalgam surface, even when carefully neutralized solutions are employed, which may interrupt the circuit, thus rendering the cell useless. In the Weston cell no gas is, apparently, formed.

Owing to these marked advantages, the Weston cell is certain to displace the Clark cell in the laboratory, and no doubt many advocates of the adoption of the former as the standard of electromotive force will be found among the delegates to the St. Louis International Congress.

#### SPECIFICATIONS FOR THE STANDARD CELL.

If either the Clark or the Weston cell be adopted as the standard of electromotive force, the specifications will have to be to some extent redrawn if the highest accuracy of reproduction is sought, as the differences between individual cells set up with different materials at present far exceed the relative errors made in current and electromotive force measurements by the potentiometer method. It, therefore, seems desirable, as stated above, to specify more precisely the methods of purification and preparation of the materials employed.

Fortunately, the metals entering into the composition of Clark and Weston cells,—mercury, zinc, and cadmium,—are among the few which can be obtained by special methods so pure that the foreign metals in them do not exceed more than 0.001 per cent. Zinc sulphate and cadmium sulphate can be obtained from the specially purified metals and pure sulphuric acid. Even considerable quantities of the impurities usually accompanying the above materials, when purchased as “chemically pure,” exert a relatively small and even insignificant influence on the electromotive force of the cell. In defining the standard cell, however, the method of preparation or purification and the degree of purity of the materials should certainly be specified.

The principal source of variation of the standard cell has lately been shown to be due to differences in the electromotive properties of the mercurous sulphate. The “chemically pure” mercurous sulphate of commerce contains, besides nitrates, etc., basic mercurous sulphate,

mercuric sulphate, basic mercuric sulphate, and possibly sulphites. According to the Chicago specifications, since generally adopted, the mercurous sulphate is washed a number of times with distilled water, which converts the mercuric sulphate into basic mercuric sulphate, which is not removed. Moreover, the water hydrolizes the mercurous sulphate, converting part of it into basic mercurous sulphate. Both these materials having a definite solubility in the zinc sulphate and cadmium sulphate solutions, must exert an influence on the electromotive force of the cell. The basic mercurous sulphate, when present in excess, will exert an influence on the electromotive force, while the basic mercuric sulphate is gradually decomposed and eliminated, thus introducing a variable factor.

Pure mercurous sulphate, however, may be obtained from pure mercury and sulphuric acid, by an electrolytic method independently devised by Carhart and Hulett, and the author, and the results already obtained indicate that the agreement of cells set up with this material is within a few parts in 100,000, but further work is desirable.

It is, therefore, most important, if the unit of electromotive force is defined in terms of the standard cell, to specify the manner in which this material is to be prepared, and to modify some of the specifications relating to its treatment.

Besides new specifications for the ampere or volt in terms of the electrochemical equivalent of silver, or the electromotive force of some particular standard cell, respectively, it will be necessary to adopt a new value for one of these constants. This may be based either on the absolute determinations already made, applying to the accepted values, corrections determined by the modifications in the specifications which may be adopted and a correction in order to bring the unit into closer agreement with the absolute value upon which it is based, or by new absolute determinations. If the latter alternative is decided upon, considerable delay would probably ensue, and in addition, not much could be gained, owing to the relatively large errors of all absolute measurements and the differences likely to be found between the results obtained by different investigators using different methods and apparatus.

Two determinations of the electromotive force of the Clark cell in absolute measure, made by Kahle at the Reichsanstalt, and by Carhart and Guthe, indicate that the value adopted by the Chicago Congress, 1.434 volts, is too large by about 1 millivolt; and in addition, several redeterminations of the mechanical equivalent of heat in electrical units give values for the latter which can only be brought into accord with the values determined by the direct mechanical methods if the

electromotive force of the Clark cell be taken as 1.433. If this value be adopted for the Clark cell, or the equivalent value for the Weston cell, the international units, would be defined with a quite sufficient absolute accuracy, as the above value is most probably known to at least 1 part in 2,000, and as at the present time a much higher absolute accuracy can hardly be predicted. It seems, on the other hand, that the main question is to define the international units with the prime object of *reproducibility* to the highest order of accuracy, and it is hoped that in this an accuracy of a few parts in 100,000 will be realized.

#### DERIVED UNITS.

It will be generally agreed that the units of capacity, inductance, power, energy, and any others that the St. Louis Congress may decide to include, should be defined in terms of the definitions adopted for the fundamental units.

The joule and watt have, however, been defined in terms of the c. g. s. units by some countries, and objections will probably be raised to defining them in terms of the electrical units. Such objections could be met by making a distinction between the absolute joule and international joule, and the absolute watt and the international watt, a distinction already used to some extent in distinguishing between the absolute units and the international electrical units. As the system becomes established the designation international will gradually be dropped. Moreover, if the values adopted for the international units agree with the absolute values upon which they are based to within even 1 part in 1,000, as will be the case, the objections will be mainly theoretical, as all practical requirements will be met.

#### MAGNETIC UNITS.

The only official action thus far taken in defining the magnetic units is that of the Paris Congress of 1900 by which the c. g. s. units of magnetic field intensity and magnetic flux were adopted, the names Gauss and Maxwell being assigned to them.

The St. Louis Congress may, however, consider the adoption of additional units of magnetomotive force and magnetic reluctance and the definition of all the magnetic units in harmony with the practical system of electromagnetic units. None of the units, either of the practical or c. g. s. system, is of a convenient magnitude, but this should, of course, not determine the choice. If there is any need of decimal multiples or submultiples, these can be supplied by the use of a suitable prefix.

It must, however, be emphasized that the definition of the magnetic units directly in terms of the c. g. s. units or in terms of multiples or submultiples of the same is open to the serious objection that it would lead to an inconsistency if the fundamental electrical units are defined in terms of concrete standards and that it would be equivalent to a redefinition of these electrical units, interconnected as they are with the magnetic units in terms of the units of the c. g. s. system.

This can only be avoided by defining the magnetic units in terms of the fundamental international electrical units—the ohm, volt, and ampere—and any resulting ambiguity might be removed by designating the units thus defined as international magnetic units.

## APPENDIX.

### LEGAL DEFINITION OF THE ELECTRICAL UNITS IN THE UNITED STATES.

[Act approved July 12, 1894.]

*Be it enacted, &c.,* That from and after the passage of this act the legal units of electrical measure in the United States shall be as follows:

First. The unit of resistance shall be what is known as the international ohm, which is *substantially equal* to one thousand million units of resistance of the centimeter-gram-second system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice fourteen and four thousand five hundred and twenty-one ten-thousandths gram in mass, of a constant cross-sectional area, and of the length of one hundred and six and three-tenths centimeters.

Second. The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electro-magnetic units, and is *the practical equivalent* of the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths of a gram per second.

Third. The unit of electro-motive force shall be what is known as the international volt, which is the electro-motive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of an international ampere, and is *practically equivalent* to one thousand fourteen hundred and thirty-fourths of the electro-motive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of fifteen degrees centigrade, and prepared in the manner described in the standard specifications.

Fourth. The unit of quantity shall be what is known as the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one second.

Fifth. The unit of capacity shall be what is known as the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

Sixth. The unit of work shall be the joule, which is equal to ten million units of work in the centimeter-gram-second system, and which is practically equivalent to the energy expended in one second by an international ampere in an international ohm.

Seventh. The unit of power shall be the watt, which is equal to ten million units of power in the centimeter-gram-second system, and which is practically equivalent to the work done at the rate of one joule per second.

Eighth. The unit of induction shall be the henry, which is the induction in a circuit when the electro-motive force induced in this circuit is one henry. international volt while the inducing current varies at the rate of one ampere per second.

SEC. 2. That it shall be the duty of the National Academy of Sciences to prescribe and publish, as soon as possible after the passage of this act, such specifications of detail as shall be necessary for the practical application of the definitions of the ampere and volt hereinbefore given, and such specifications shall be the standard specifications herein mentioned.

#### SPECIFICATIONS.

In conformity with section 2 of the above act, the National Academy of Sciences, on February 9, 1895, accepted and unanimously adopted the following report:

#### REPORT.

In the preparation of this report, in order to have the specifications accord with international usage, free use has been made of the English Government specifications and of certain papers prepared by Dr. K. Kahle, of Germany, and Prof. H. S. Carhart, of this country.

#### SPECIFICATIONS FOR THE PRACTICAL APPLICATION OF THE DEFINITIONS OF THE AMPERE AND VOLT.

##### SPECIFICATION A.—*The ampere.*

In employing the silver voltameter to measure currents of about 1 ampere, the following arrangements shall be adopted:

The kathode on which the silver is to be deposited shall take the form of a platinum bowl not less than 10 centimeters in diameter, and from 4 to 5 centimeters in depth.

The anode shall be a disk or plate of pure silver some 30 square centimeters in area and 2 or 3 millimeters in thickness.

This shall be supported horizontally in the liquid near the top of the solution by a silver rod riveted through its center. To prevent the disintegrated silver which is formed on the anode from falling upon the kathode, the anode shall be wrapped around with pure filter paper, secured at the back by suitable folding.

The liquid shall consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

*Method of making a measurement.*—The platinum bowl is to be washed consecutively with nitric acid, distilled water, and absolute alcohol; it is then to be dried at 160° C., and left to cool in a desiccator. When thoroughly cool it is to be weighed carefully.

It is to be nearly filled with the solution and connected to the rest of the circuit by being placed on a clean insulated copper support to which a binding screw is attached.

The anode is then to be immersed in the solution so as to be well covered by it and supported in that position; the connections to the rest of the circuit are then to be made.



Contact is to be made at the key, noting the time. The current is to be allowed to pass for not less than half an hour, and the time of breaking contact observed.

The solution is now to be removed from the bowl and the deposit washed with distilled water and left to soak for at least six hours. It is then to be rinsed successively with distilled water and absolute alcohol and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is to be weighed again. The gain in mass gives the silver deposited.

To find the time average of the current in amperes, this mass, expressed in grams, must be divided by the number of seconds during which the current has passed and by 0.001118.

In determining the constant of an instrument by this method, the current should be kept as nearly uniform as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated from the voltameter results, corresponds to this reading.

The current used in this experiment must be obtained from a battery, and not from a dynamo, especially when the instrument to be calibrated is an electro-dynamometer.

#### SPECIFICATION B.—*The volt.*

*Definition and properties of the cell.*—The cell has for its positive electrode, mercury, and for its negative electrode, amalgamated zinc; the electrolyte consists of a saturated solution of zinc sulphate and mercurous sulphate. The electromotive force is 1.434 volts at 15° C., and between 10° C. and 25° C., by the increase of 1° C. in temperature, the electromotive force decreases by 0.00115 of a volt.

1. *Preparation of the mercury.*—To secure purity, it should be first treated with acid in the usual manner and subsequently distilled in vacuo.

2. *Preparation of the zinc amalgam.*—The zinc designated in commerce as “commercially pure” can be used without further preparation. For the preparation of the amalgam, 1 part by weight of zinc is to be added to 9 parts by weight of mercury, and both are to be heated in a porcelain dish at 100° C., with moderate stirring until the zinc has been fully dissolved in the mercury.

3. *Preparation of the mercurous sulphate.*—Take mercurous sulphate, purchased as pure; mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water and repeat the process at least twice. After the last washing, drain off as much of the water as possible. (For further details of purification, see Note A.)

4. *Preparation of the zinc sulphate solution.*—Prepare a neutral saturated solution of pure recrystallized zinc sulphate, free from iron, by mixing distilled water with nearly twice its weight of crystals of pure zinc sulphate and adding zinc oxide in the proportion of about 2 per cent by weight of the zinc sulphate crystals to neutralize any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised must not exceed 30° C. Mercurous sulphate, treated as described in 3, shall be added in the proportion of about 12 per cent by weight of the zinc sulphate crystals to neutralize the free zinc oxide remaining, and then the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

5. *Preparation of the mercurous sulphate and zinc sulphate paste.*—For making the paste, 2 or 3 parts by weight of mercurous sulphate are to be added to 1 by weight of

mercury. If the sulphate be dry, it is to be mixed with a paste consisting of zinc sulphate crystals and a concentrated zinc sulphate solution, so that the whole constitutes a stiff mass, which is permeated throughout by zinc sulphate crystals and globules of mercury. If the sulphate, however, be moist, only zinc sulphate crystals are to be added; care must, however, be taken that these occur in excess and are not dissolved after continued standing. The mercury must, in this case also, permeate the paste in little globules. It is advantageous to crush the zinc sulphate crystals before using, since the paste can then be better manipulated.

*To set up the cell.*—The containing glass vessel, \* \* \*, shall consist of two limbs closed at the bottom and joined above to a common neck fitted with a ground-glass stopper. The diameter of the limbs should be at least two centimeters and their length at least 3 centimeters. The neck should be not less than 1.5 centimeters in diameter. At the bottom of each limb a platinum wire of about 0.4 millimeter diameter is sealed through the glass.

To set up the cell, place in one limb pure mercury and in the other hot liquid amalgam, containing 90 parts mercury and 10 parts zinc. The platinum wires at the bottom must be completely covered by the mercury and the amalgam, respectively. On the mercury place a layer 1 centimeter thick of the zinc and mercurous sulphate paste described in 5. Both this paste and the zinc amalgam must then be covered with a layer of the neutral zinc sulphate crystals 1 centimeter thick. The whole vessel must then be filled with the saturated zinc sulphate solution, and the stopper inserted so that it shall just touch it, leaving, however, a small bubble to guard against breakage when the temperature rises.

Before finally inserting the glass stopper it is to be brushed around its upper edge with a strong alcoholic solution of shellac and pressed firmly in place. (For details of filling the cell, see Note B.)

#### NOTES TO THE SPECIFICATIONS.

(A) *The mercurous sulphate.*—The treatment of the mercurous sulphate has for its object the removal of any mercuric sulphate which is often present as an impurity.

Mercuric sulphate decomposes in the presence of water into an acid and a basic sulphate. The latter is a yellow substance—turpeth mineral—practically insoluble in water; its presence, at any rate in moderate quantities, has no effect on the cell. If, however, it be formed, the acid sulphate is also formed. This is soluble in water and the acid produced affects the electromotive force. The object of the washings is to dissolve and remove this acid sulphate, and for this purpose the three washings described in the specification will suffice in nearly all cases. If, however, much of the turpeth mineral be formed, it shows that there is a great deal of the acid sulphate present, and it will then be wiser to obtain a fresh sample of mercurous sulphate, rather than to try by repeated washings to get rid of all the acid.

The free mercury helps in the process of removing the acid, for the acid mercuric sulphate attacks it, forming mercurous sulphate.

Pure mercurous sulphate, when quite free from acid, shows on repeated washing a faint yellow tinge, which is due to the formation of a basic mercurous salt distinct from the turpeth mineral, or basic mercuric sulphate. The appearance of this primrose-yellow tint may be taken as an indication that all the acid has been removed; the washing may with advantage be continued until this tint appears.

(B) *Filling the cell.*—After thoroughly cleaning and drying the glass vessel, place it in a hot-water bath. Then pass through the neck of the vessel a thin glass tube reaching to the bottom, to serve for the introduction of the amalgam. This tube

should be as large as the glass vessel will admit. It serves to protect the upper part of the cell from being soiled with the amalgam. To fill in the amalgam, a clean dropping tube about 10 centimeters long, drawn out to a fine point, should be used. Its lower end is brought under the surface of the amalgam, heated in a porcelain dish, and some of the amalgam is drawn into the tube by means of the rubber bulb. The point is then quickly cleaned of dross with filter paper, and is passed through the wider tube to the bottom and emptied by pressing the bulb. The point of the tube must be so fine that the amalgam will come out only on squeezing the bulb. This process is repeated until the limb contains the desired quantity of amalgam. The vessel is then removed from the water bath. After cooling, the amalgam must adhere to the glass, and must show a clean surface with a metallic luster.

For insertion of the mercury, a dropping tube with a long stem will be found convenient. The paste may be poured in through a wide tube reaching nearly down to the mercury and having a funnel-shaped top. If the paste does not move down freely it may be pushed down with a small glass rod. The paste and the amalgam are then both covered with the zinc sulphate crystals before the concentrated zinc sulphate solution is poured in. This should be added through a small funnel, so as to leave the neck of the vessel clean and dry.

For convenience and security in handling, the cell may be mounted in a suitable case, so as to be at all times open to inspection.

In using the cell, sudden variations of temperature should, as far as possible, be avoided, since the changes in electromotive force lag behind those of temperature.

Respectfully submitted.

HENRY A. ROWLAND,  
*Chairman.*

HENRY L. ABBOT,  
GEORGE F. BARKER,  
CHARLES S. HASTINGS,  
ALBERT A. MICHELSON,  
JOHN TROWBRIDGE,  
CARL BARUS,

*Committee.*

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## LEGAL DEFINITION OF THE ELECTRICAL UNITS IN GREAT BRITAIN.

[Order in council, August 23, 1894.]

Whereas by "The Weights and Measures Act, 1889," it is, among other things, enacted that the Board of Trade shall from time to time cause such new denominations of standards for the measurement of electricity as appear to them to be required for use in trade to be made and duly verified;

And whereas it has been made to appear to the Board of Trade that new denominations of standards are required for use in trade, based upon the following units of electrical measurement, viz:

1. The ohm, which has the value  $10^9$  in terms of the centimeter and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.3 centimeters.

2. The ampere, which has the value one-tenth in terms of the centimeter, the gram, and the second of time, and which is represented by the unvarying electric current, which, when passed through a solution of nitrate of silver in water, in accordance with the specification appended hereto and marked "A," deposits silver at the rate of 0.001118 of a gram per second.

3. The volt, which has the value  $10^8$  in terms of the centimeter, the gram, and the second of time, being the electrical pressure that, if steadily applied to a conductor whose resistance is 1 ohm, will produce a current of 1 ampere, and which is represented by 0.6974 ( $\frac{1}{144}$ ) of the electrical pressure at a temperature of  $15^{\circ}$  C. between the poles of the voltaic cell known as Clark's cell, set up in accordance with the specification appended hereto and marked "B."

And whereas they have caused the said new denominations of standards to be made and duly verified.

Now, therefore, Her Majesty, by virtue of the power vested in her by the said act, by and with the advice of Her Privy Council, is pleased to approve the several denominations of standards set forth in the schedule hereto as new denominations of standards for electrical measurement.

C. L. PEEL.

## SCHEDULE.

### I.—STANDARD OF ELECTRICAL RESISTANCE.

A standard of electrical resistance denominated 1 ohm, being the resistance between the copper terminals of the instrument marked "Board of Trade Ohm Standard, verified 1894," to the passage of an unvarying electrical current when the coil of insulated wire forming part of the aforesaid instrument and connected to the aforesaid terminals is in all parts at a temperature of  $15.4^{\circ}$  C.

### II.—STANDARD OF ELECTRICAL CURRENT.

A standard of electrical current denominated 1 ampere, being the current which is passing in and through the coils of wire forming part of the instrument marked "Board of Trade Ampere Standard, verified 1894," when on reversing the current in the fixed coils the change in the forces acting upon the suspended coil in its sighted position is exactly balanced by the force exerted by gravity in Westminster upon the iridio-platinum weight marked "A" and forming part of the said instrument.

### III.—STANDARD OF ELECTRICAL PRESSURE.

A standard of electrical pressure denominated 1 volt, being one-hundredth part of the pressure which, when applied between the terminals forming part of the instrument marked "Board of Trade Volt Standard, verified 1894," causes that rotation of the suspended portion of the instrument which is exactly measured by the coincidence of the sighting wire with the image of the fiducial mark A before and after application of the pressure, and with that of the fiducial mark B during the application of the pressure, these images being produced by the suspended mirror and observed by means of the eyepiece.

In the use of the above standards the limits of accuracy attainable are as follows:

For the ohm, within one-hundredth part of 1 per cent.

For the ampere, within one-tenth part of 1 per cent.

For the volt, within one-tenth part of 1 per cent.

The coils and instruments referred to in this schedule are deposited at the Board of Trade Standardising Laboratory, 8 Richmond Terrace, Whitehall, London.

## SPECIFICATIONS REFERRED TO IN THE FOREGOING ORDER IN COUNCIL.

## SPECIFICATION A.

In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant, the current itself can be deduced.

In employing the silver voltameter to measure currents of about 1 ampere the following arrangements should be adopted: The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 centimeters in diameter and from 4 to 5 centimeters in depth.

The anode should be a plate of pure silver some 30 square centimeters in area and 2 or 3 millimeters in thickness.

This is supported horizontally in the liquid near the top of the solution by a *platinum wire* passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the kathode the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

## METHOD OF MAKING A MEASUREMENT.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution and connected to the rest of the circuit by being placed on a clean copper support to which a binding screw is attached. This copper support must be insulated.

The anode is then immersed in the solution so as to be well covered by it and supported in that position; the connections to the rest of the circuit are made.

Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in amperes, this weight, expressed in grams, must be divided by the number of seconds during which the current has been passed, and by 0.001118.

The result will be the time average of the current, if during the interval the current has varied.

In determining by this method the constant of an instrument the current should be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

SPECIFICATION B.—*On the preparation of the Clark cell.*

DEFINITION OF THE CELL.

The cell consists of zinc, or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess.

PREPARATION OF THE MATERIALS.

1. *The mercury.*—To secure purity it should be first treated with acid in the usual manner and subsequently distilled in vacuo.

2. *The zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

3. *The mercurous sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice. After the last washing, drain off as much of the water as possible.

4. *The zinc sulphate solution.*—Prepare a neutral saturated solution of pure ("pure recrystallized") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent by weight of the zinc sulphate crystals to neutralize any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30° C. Mercurous sulphate treated as described in 3 should be added in the proportion of about 12 per cent by weight of the zinc sulphate crystals to neutralize any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

5. *The mercurous sulphate and zinc sulphate paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to insure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30° C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case, add more crystals from the stock bottle and repeat the whole process.

This method insures the formation of a saturated solution of zinc and mercurous sulphates in water.

TO SET UP THE CELL.

The cell may conveniently be set up in a small test tube of about 2 centimeters diameter and 4 or 5 centimeters deep. Place the mercury in the bottom of this tube, filling it to a depth of, say, 0.5 centimeter. Cut a cork about 0.5 centimeter thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can

pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1 centimeter through the cork.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red-hot, and insert it in the mercury in the test tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste and introduce it without contact with the upper part of the walls of the test tube, filling the tube above the mercury to a depth of rather more than 1 centimeter.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least twenty-four hours before sealing, which should be done as follows:

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

In using the cell sudden variations of temperature should as far as possible be avoided.

The form of the vessel containing the cell may be varied. In the H form the zinc is replaced by an amalgam of 10 parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass.

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## LEGAL DEFINITION OF THE ELECTRICAL UNITS IN CANADA.

[Assented to July 23, 1894, 57-58 Vict., c. 38.]

Her Majesty, by and with the advice and consent of the Senate and House of Commons of Canada, enacts as follows:

1. This act may be cited as *The Electrical Units Act*.

2. The units of electrical measure for Canada shall be the following:

(a) As a unit of resistance, the ohm, which is based upon the ohm equal to  $10^9$  units of resistance of the centimeter-gramme-second system of electro-magnetic units, and is represented by the resistance offered

Ohm.

to an unvarying electric current by a column of mercury, at the temperature of melting ice 14.4521 grammes in mass, of a constant cross-sectional area and of the length of 106.3 centimeters.

(b) As a unit of current, the ampere, which is one-tenth of the unit of current of the centimeter-gramme-second system of electro-magnetic units, and is represented sufficiently well for practical use by the unvarying current, which, when passed through a solution of nitrate of silver in water, and in accordance with the specification contained in schedule 1 to this act, deposits silver at the rate of 0.001118 of a gramme per second.

(c) As a unit of electromotive force, the volt, which is the electromotive force that, steadily applied to a conductor whose resistance is 1 ohm, will produce a current of 1 ampere, and which is represented sufficiently well for practical use by  $\frac{1}{1000}$  of the electromotive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of 15° centigrade and prepared in accordance with the specification contained in schedule 2 to this act.

(d) As a unit of quantity, the coulomb, which is the quantity of electricity transferred by a current of 1 ampere in one second.

(e) As a unit of capacity, the farad, which is the capacity of a condenser charged to a potential of 1 volt by 1 coulomb.

(f) As a unit of work, the joule, which is equal to  $10^7$  units of work in the centimeter-gramme-second system, and is represented sufficiently well for practical use by the energy expended in one second by 1 ampere in 1 ohm.

(g) As a unit of power, the watt, which is equal to  $10^7$  units of power in the centimeter-gramme-second system, and is represented sufficiently well for practical use by the work done at the rate of 1 joule per second.

(h) As a unit of induction, the henry, which is the induction in a circuit when the electro-motive force induced in that circuit is 1 volt, while the inducing current varies at the rate of 1 ampere per second.

3. The units of electrical measure described in the next preceding section, or such standard apparatus as is necessary to produce them, shall be deposited in the Department of Inland Revenue and shall form part of the system of standards of measure and weight established by *The Weights and Measures Act*.

#### SCHEDULE ONE.

In the following specification, the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment; and by noting this time, the time average of the current, or, if the current has been kept constant, the current itself, can be deduced.

In employing the silver voltameter to measure currents of about 1 ampere, the following arrangements should be adopted: The cathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 centimeters in diameter and from 4 to 5 centimeters in depth. The anode should be a plate of pure silver 30 square centimeters in area and 2 or 3 millimeters in thickness. This is supported horizontally in the liquid near the top of the solution by a *platinum wire* passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling onto the cathode, the



anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

#### SCHEDULE TWO.

The cell consists of zinc and mercury in a saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess, and is conveniently contained in a cylindrical glass vessel.

*The mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled in vacuo.

*The zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass paper, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

*The zinc sulphate solution.*—Prepare a saturated solution of pure ("pure recrystallized") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent by weight of the zinc sulphate crystals to neutralize any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30° C. Mercurous sulphate treated as hereinafter described should be added in the proportion of about 12 per cent by weight of the zinc sulphate crystals and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

*The mercurous sulphate.*—Take mercurous sulphate, purchased as pure, and wash it thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice. After the last washing, drain off as much of the water as possible.

Mix the washed mercurous sulphate with the zinc-sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to insure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30° C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool, continuing to shake it occasionally while cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass. If this is not the case, add more crystals from the stock bottle, and repeat the whole process. This method insures the formation of a saturated solution of zinc and mercurous sulphates in water.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed in a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal; the other end and a portion of the glass tube dip into the mercury.

## LEGAL DEFINITION OF THE ELECTRICAL UNITS IN GERMANY.

[Law of June 1, 1898, R. G. Bl., p. 905.]

SECTION 1. The legal units for electrical measurements are the ohm, ampere, and volt.

SEC. 2. The unit of electrical resistance is the ohm. It is represented by the resistance of a column of mercury, at the temperature of melting ice, of uniform cross section, practically equivalent to 1 square millimeter, of a length of 106.3 centimeters, and of a mass of 14.4521 grammes.

Ohm.

SEC. 3. The unit of current is the ampere. It is represented by the unvarying electric current which in passing through an aqueous solution of silver nitrate deposits in one second 0.001118 grammes of silver.

Ampere.

SEC. 4. The unit of electro-motive force is the volt. It is represented by the electro-motive force which when applied to a conductor having a resistance of 1 ohm produces a current of 1 ampere.

Volt.

SEC. 5. The Bundesrath is empowered—

(a) To fix the conditions under which the silver is to be deposited, in the definition of the ampere. (Sec. 3.)

(b) To fix designations for the units of electric quantity, energy, power, capacity, and inductance.

(c) To prescribe designations for the multiples and submultiples of the electrical units.

(d) To fix the manner in which the strength, electro-motive force, energy, and power of alternating currents is to be calculated.

SEC. 6. According to this paragraph instruments used in the measurement of electrical power for commercial purposes must have their indications based on the legal units. The use of incorrect measuring instruments is prohibited. The Bundesrath is empowered to fix the limits of tolerance for such apparatus, after giving a hearing to the Physikalisch-Technische Reichsanstalt.

The Bundesrath is empowered to issue regulations concerning the official verification and periodic reverification of measuring apparatus.

SEC. 7. The Physikalisch-Technische Reichsanstalt is directed to construct primary mercurial resistance standards and assume responsibility for their control and safe custody at different places. It is also to reverify the resistance of standards of solid metals used in the intercomparisons by an annual recomparison with the mercurial standards.

SEC. 8. The Physikalisch-Technische Reichsanstalt is to provide for the issue of officially certified standard resistances and standard cells for the measurement of current and electro-motive force.

SEC. 9. The official testing and certification of electrical measuring instruments shall be carried out by the Physikalisch-Technische Reichsanstalt. The Imperial Chancellor may intrust this authority elsewhere. All standards and measuring apparatus employed in official verifications must be certified to by the Physikalisch-Technische Reichsanstalt.

SEC. 10. The Physikalisch-Technische Reichsanstalt is to assume the responsibility that the official testing and verification of electrical measuring apparatus in the German Empire shall be made in a uniform manner. It is to assume the technical supervision of the inspection service, and to prescribe all technical specifications concerning the same. It is especially to determine what kind of measuring instru-

ments shall be admitted to official verification, to adopt regulations concerning the material, construction, or designation of the apparatus, to regulate the methods employed in the testing and verification, and to fix the fees and specify the seal to be employed.

SEC. 11. Measuring apparatus verified in accordance with this law may be used in trade in any part of the Empire.

SEC. 12. Whoever, engaged in the industrial supply of electrical energy, does not comply with section 6, or the regulations based thereon, will be subjected to a fine not exceeding 100 marks, or imprisonment not to exceed four weeks. In addition, the incorrect instruments, or the instruments not complying with the regulations, shall be subject to seizure.

SEC. 13. This law and the regulations adopted in accordance with sections 6 and 12 shall take effect January 1, 1902; the remainder on the date of its promulgation.

#### REGULATIONS FOR CARRYING OUT THE LAW· ISSUED BY THE BUNDESRATH.

[Reichsgesetzblatt No. 16, 1901, June 1, 1898.]

In accordance with paragraph 5 of the law defining the electrical units of measurement (Reichsgesetzblatt, p. 905), the following specifications are adopted:

1. *Conditions under which the silver is to be deposited in the specification of the ampere.* (Sec. 5 a).—The solution shall consist of from 20 to 40 parts by weight of pure silver nitrate in 100 parts of distilled water free from chlorine. It shall not be used after 3 grammes of silver are deposited from 100 cubic centimeters of solution.

All parts of the anode in contact with the solution shall consist of pure silver. The cathode shall consist of platinum. When the deposited silver exceeds 0.1 gramme per square centimeter the silver is to be removed.

The current density at the anode shall not exceed one-fifth, and at the cathode one-fiftieth ampere per square centimeter.

Before weighing, the cathode is first to be rinsed with distilled water free from chlorine until the addition of a drop of hydrochloric acid to the wash water produces no opalescence. The cathode is then to be soaked for ten minutes in distilled water at 70° to 90° C., and is to be finally washed with distilled water. The last wash water after cooling must not become opalescent upon the addition of hydrochloric acid. The cathode is dried by the aid of heat and kept in a desiccator until it is weighed, which shall not be done less than ten minutes after cooling off.

2. *Designation of the electrical units (sec. 5b).*—(a) The quantity of electricity flowing through the cross section of a conductor in one second when the current in the same is equal to 1 ampere is called an ampere-second (coulomb), and the quantity flowing in one hour an ampere hour.

(b) The power corresponding to an ampere in a conductor having a potential difference of 1 volt between its terminals is called a watt.

(c) The work done in one hour when the power is equal to 1 watt is called a watt hour.

(d) The capacity of a condenser which is charged by an ampere-second to 1 volt is called a farad.

(e) The self-inductance of a conductor in which 1 volt is induced by a uniform change in the current of 1 ampere per second is called a henry.

3. *Designations for the multiples and submultiples of the electrical units (sec. 5c).*—The following prefixes to the name of a unit shall have the following meanings:

|                    |                    |
|--------------------|--------------------|
| Kilo .....         | 1,000 times        |
| Mega (meg) .....   | 1,000,000 times    |
| Milli .....        | One one-thousandth |
| Micro (mikr) ..... | One-millionth      |

#### EXPLANATION OF THE SPECIFICATIONS. (SEC. 5a.)

1. On account of its fundamental importance in defining unit current, the silver voltameter must be employed under prescribed conditions in order that the amount of silver deposited will give the correct value for the current measured. In this connection the fact is to be considered that the silver solution gradually experiences a yet unexplained change by the long-continued passage of an electric current, by which small variations in the amount of silver deposited are produced. Moreover, in order to obtain compact deposits of silver the concentration of the solution and the current density at the electrodes must be confined within certain limits. Finally, on account of the sensitiveness of the silver solution to impurities it appeared necessary to prescribe regulations concerning the water employed in washing, and in order to weigh only the deposited silver to specify the manner of washing.

In these specifications the experiences of the most reliable experimenters have been considered. They may be considered not only entirely sufficient for their purpose, but in addition they are possibly somewhat too rigorous for practical needs. Since, however, it is really not necessary, except in rare cases, to refer current measurements to the silver voltameter, as much simpler methods are available, the specifications could be rigorously drawn.

### LEGAL DEFINITION OF THE ELECTRICAL UNITS IN AUSTRIA.

[Ordinance No. 176, Ministry of Commerce, of July 4, 1900. Concerning the testing and certifying of electrical supply meters.]

The electrical units are derived from the fundamental metrical units of length, mass, and time, according to the electro-magnetic system of measurement, taking the centimeter as the unit of length, the gramme as the unit of mass, and the mean solar second, of which there are 86,400 in a mean solar day, as the unit of time. The resulting units are designated as units of the c. g. s. electro-magnetic system (centimeter-gramme-second system).

The unit of resistance is the ohm, which is equal to  $10^9$  units of resistance of the electro-magnetic c. g. s. system. For commercial purposes the ohm may be considered equal to the resistance offered to an unvarying current by a column of mercury having a mass of 14.4521 grammes, a length of 106.3 centimeters, at the temperature of melting ice. (No reference is made to a uniform cross section of the mercurial column.)

The unit of current is the ampere, which is equal to the one-tenth part of the electro-magnetic unit of current of the c. g. s. system. For commercial purposes the ampere may be considered equal to the value of an unvarying current which when passing through an aqueous solution of silver nitrate deposits 0.001118 gramme silver per second.

The unit of electromotive force is the volt, which is equal to that electromotive force which acting steadily on a conductor having a resistance of 1 ohm produces in the same a current of 1 ampere.

**Volt.**

The unit of power is the watt, which is equal to  $10^7$  units of power of the c. g. s. system, or equal to the power corresponding to a current of 1 ampere at an electromotive force of 1 volt (voltampere).

**Watt.**

The coulomb is equal to the quantity of electricity flowing in one mean solar second through a conductor carrying a current of 1 ampere. One ampere-hour corresponds to 3,600 coulombs.

**Coulomb.**

The work done in 3,600 seconds in a conductor in which the power is 1 watt is equal to 1 watthour. One hundred watthours are equal to 1 hectowatthour. One thousand watthours are equal to 1 kilowatthour.

**Watthour.**

## LEGAL DEFINITION OF THE ELECTRICAL UNITS IN FRANCE.

[Decree of the President, April 25, 1896.]

The unit of electrical resistance—the ohm—is the resistance offered to an unvarying current by a column of mercury at a temperature of melting ice, having a mass of 14.4521 grammes, a constant cross section, and a length of 106.3 centimeters.

**Ohm.**

The unit of current—the ampere—is the one-tenth of the electro-magnetic unit of current. It is represented sufficiently well for practical purposes by the unvarying current which deposits in one second 0.001118 grammes of silver.

**Ampere.**

The unit of electro-motive force—the volt—is the electro-motive force which produces a current of 1 ampere in a conductor having a resistance of 1 ohm. It is represented sufficiently well for practical purposes by 0.6974, or  $(\frac{1}{1434})$  of the electro-motive force of the Latimer Clark cell.

**Volt.**

### SPECIFICATIONS FOR THE SILVER VOLTAMETER.

The specifications for the silver voltameter are the same as those adopted by the Chicago congress, with the exception that the anode is to be supported by a silver rod instead of platinum wires, the latter having been found to be a source of variations. In addition, the directions for making a measurement are the same as those given in the English law.

### SPECIFICATIONS FOR THE STANDARD CELL.

The specifications for the standard cell are the Board of Trade specifications, which are legalized in England.

## PROPOSED LAW DEFINING THE ELECTRICAL UNITS FOR USE IN BELGIUM.

There is established for the Kingdom a single system of electrical units having as its base the ohm, the ampere, and the volt.

The ohm is the resistance offered to an unvarying current, by a column of mercury  
at the temperature of melting ice, having a mass of 14.4521  
grammes, a constant cross section, and a length of 106.3 centimeters.

The ampere is represented sufficiently well for practical purposes by the intensity  
of a constant current which precipitates in one second 0.001118  
grammes of silver from an aqueous solution of silver nitrate.

The volt is represented by the electro-motive force which produces a current of 1 ampere in a conductor of which the resistance is 1 ohm.

The denominations of the derived electrical units, especially the units of energy  
and power, shall be fixed by royal decree. Multiples and sub-  
multiples of the legal units may likewise be fixed.

Within two years after the promulgation of the law practical standards in conformity with the legal system of units shall be established by a special commission named by the King.

The custody and periodic verification of these standards shall be entrusted to the minister of industry and labor.

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## PROPOSED LAW DEFINING THE ELECTRICAL UNITS IN SWITZERLAND.

The law proposed by the Swiss Electro-Technische Verein, and drawn up by the president of the commission on inspection service and units of measurement, is substantially the same as the German law.

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## OTHER COUNTRIES.

In Mexico no electrical units have been legalized, but a bill is under consideration defining the electrical units, which at the suggestion of the Bureau of Standards will be so worded that any modifications adopted at St. Louis may be made.

In Norway, Sweden, Denmark, Netherlands, Portugal, Italy, Japan, and Russia no units have been legalized.

No definite information has been obtained from Spain.

## THE SPECTRA OF MIXED GASES.

By P. G. NUTTING.

Early in spectroscopic work it was observed that mixed gases frequently gave the spectrum of a heavy metallic component more strongly than that of a lighter component. In 1878 E. Wiedemann<sup>a</sup> described experiments with mercury and sodium in hydrogen and nitrogen. He proved that the intensities of the metallic spectra were out of all proportion to the relative amounts of metallic vapor present. It is a matter of common experience in preparing Plücker tubes to exhibit metallic spectra that as soon as the tube is heated sufficiently to vaporize the contained metal, the spectrum of the lighter gas filling the tube disappears. Recently Professor Lewis has shown<sup>b</sup> that when mercury vapor is present in a tube of hydrogen it will reduce the hydrogen spectrum to about half its original intensity when but one molecule of mercury to three thousand of hydrogen are present. It is well known that the cadmium spectrum will swamp that of hydrogen long before the cadmium is even melted. The red lines of the two spectra are equal at a temperature of about 200° C.

At first thought we should say in explanation that the vapor of a metal would be a better electrical conductor than that of a nonmetal, and, carrying the most of the current, would show the brightest spectrum. But this hypothesis is clearly untenable. Although gas conductivities have not yet been accurately determined nor yet even defined, it is well known that under the same conditions metallic vapors do not differ widely from nonmetallic vapors and the permanent gases in conducting power. And even then, why should the vast majority of lighter molecules be left idle as soon as a few of the heavier molecules are present?

Taking up the problem at this point, it was quickly shown that metallic character has little if anything to do with spectral predominance. Sulphur and iodine are nearly as effective in swamping

<sup>a</sup> E. Wiedemann: Wied. Ann., 5, pp. 500-524; 1878.

<sup>b</sup> P. Lewis: Astroph. J., 10, pp. 137-163; 1899. Ann. d. Phys., 2, pp. 447-458; 1900.

hydrogen and nitrogen as are mercury and cadmium. Further, a nonmetallic vapor may swamp a metallic. This was shown by a combination of sodium with bromine and iodine vapors. Iodine was found to be much more effective than chlorine, and this suggested that atomic weight might be the ruling factor. This view was confirmed by all the evidence obtained. Even the persistent mercury spectrum may be displaced by the slightly heavier thallium, iodine easily swamps chlorine and sulphur, while any of the heavier metals displaces sodium, itself so persistent in lighter gases. In all, about eighty combinations of the fifteen available vaporizable elements were tested without any contradictions to the atomic weight law being discovered; but several combinations like cadmium and indium, oxygen and nitrogen, of elements of nearly equal atomic weight would require a more accurate determination of relative pressure and spectral energy to be used in evidence. The law may be stated thus: In the spectrum of a mixture of gases, other things being equal, *the spectrum of the gas of greater atomic weight will be brighter*. This applies to such simple, moderate, fairly homogeneous excitation as we have in a Plücker tube containing gases under a pressure of from 0.1 mm to 10 mm carrying a current of not more than 10 milliamperes. Outside these limits, the atomic-weight effect is always less—i. e., the intensities of the spectra are more nearly in proportion to the relative amounts of the gases present. With excessive excitation, say with more than one-twentieth microfarad capacity in parallel with the tube of gas, secondary spectra are produced and the relative intensity of secondary spectra appears to depend only on the relative numbers of the two kinds of atoms present. In the luminous portions of the arc and spark, conditions of pressure, current density, etc., are so complex that a test is almost hopeless, but there are many indications that the atomic-weight law of spectral partition holds here as well.

The greater part of the work here described was done with small Plücker tubes excited by a 2,000-volt alternating current. Sometimes a small induction coil was used. Tests of easily vaporizable substances were repeated with tubes having external electrodes. Other tests were repeated with small electrodeless tubes excited by electric waves from a Seibt quarter wave resonance apparatus. Spectra were examined with a small direct vision spectroscope. On account of the greater ease in identifying and comparing spectra, a low dispersion is preferable. Except in a few instances, the effects to be observed were so pronounced that no accurate photometry nor measurements of the relative amounts of gases present were necessary. Sometimes the two spectra were brought to equality and relative pressure estimated, and



sometimes the tube was filled with nearly equal quantities of the two vapors and relative spectral energies estimated. A list of the tests made is given below. A plus sign indicates that tests were made and that the results obtained confirmed the atomic weight law. A parenthesis indicates that more accurate measurements of pressure and spectral energy are necessary to decide spectral predominance for equal proportions. Where spaces are left blank no tests were made.

| At. wt.  | El | Tl  | Hg | I | Te | In  | Cd | Br | Se | Zn | Cl  | S | Na | O   | N |
|----------|----|-----|----|---|----|-----|----|----|----|----|-----|---|----|-----|---|
| 1.....   | H  | +   | +  | + | +  | +   | +  | +  | +  | +  | +   | + | +  | +   | + |
| 14.....  | N  | +   | +  | + | +  | +   | +  | +  | +  | +  | +   | + | +  | (+) |   |
| 16.....  | O  | +   | +  |   |    |     | +  |    |    |    | (+) | + | +  |     |   |
| 23.....  | Na |     | +  | + | +  | +   | +  | +  | +  | +  | +   | + |    |     |   |
| 32.....  | S  | +   | +  | + | +  | +   | +  | +  | +  | +  | (+) |   |    |     |   |
| 35.....  | Cl | +   | +  | + |    |     | +  | +  |    |    |     |   |    |     |   |
| 65.....  | Zn | +   | +  |   |    |     | +  |    |    |    |     |   |    |     |   |
| 79.....  | Se | +   | +  | + | +  | +   | +  |    |    |    |     |   |    |     |   |
| 80.....  | Br | +   | +  | + |    |     | +  |    |    |    |     |   |    |     |   |
| 112..... | Cd | +   | +  | + |    | (+) |    |    |    |    |     |   |    |     |   |
| 114..... | In | +   | +  | + |    |     |    |    |    |    |     |   |    |     |   |
| 125..... | Te |     |    |   |    |     |    |    |    |    |     |   |    |     |   |
| 127..... | I  | +   | +  |   |    |     |    |    |    |    |     |   |    |     |   |
| 200..... | Hg | (+) |    |   |    |     |    |    |    |    |     |   |    |     |   |
| 204..... | Tl |     |    |   |    |     |    |    |    |    |     |   |    |     |   |

In addition to these results may be cited the work of Collie and Ramsay<sup>a</sup> on mixtures of argon and helium. They found that a little argon mixed with helium showed the spectrum of the former strongly, while a small percentage of helium in argon showed but a faint helium spectrum.

Chemical combination occurs in the vast majority of cases, but compound spectra occur only in the case of a few halides. Chemical combination, however, appears to have no effect on the relative spectral energies of the components, just as though the electrical excitation in all cases produces a completely nascent state of each atom before luminescence occurs. If we start with, say, hydrogen and sulphur, vaporize the sulphur and then allow the tube to cool while the discharge is passing, we get no spectrum other than those of hydrogen and sulphur during the process, and no trace of  $H_2S$  on opening the tube. The result is the same when we fill the tube with  $H_2S$  at the start. Similarly with black mercury sulphide, except that a heavy deposit of sulphide remains. Even with the cadmium-oxygen combination, where the product can not be kept vaporized, the two spectra may be observed superposed during combination. But with the

<sup>a</sup>Proc. Roy. Soc., 59, pp. 257-270; 1896.

mercury halides a strong band appears, brighter even than the mercury spectrum. The halogen spectrum remains visible as well as the mercury, evidently in proportion to the atomic weights of these substances, but the compound spectrum is brighter than either, possibly in proportion to the molecular weight of the compound. It would be in accord with theory if the spectra of compounds followed a molecular weight law, but too few compound spectra exist to give it much weight. Varying the pressure over quite a range does not generally affect the preponderance of one spectrum over another. But at very high or very low pressure the spectrum of the lighter gas is stronger in proportion than at intermediate pressures. Changes in temperature have little effect so long as all components and products remain vaporized.

Changes in current density over quite a wide range do not affect the spectral preponderance.<sup>a</sup> But an excessive current density was found to increase the relative intensity of the lighter gas in every case observed. This test was made with some special Plücker tubes having a third bulb in the middle of the capillary. Though the central bulb had several hundred times the cross section of the neighboring capillary the spectra of the two portions never differed greatly. Excessive current density of course produces an excessive electrical excitation of the atom much as the addition of capacity does. Hence heavy currents tend to produce secondary spectra and hence a division of spectral energy in proportion to the relative numbers of the two kinds of atoms present. The effect of introducing inductance or resistance with the capacity appears to be merely to partly neutralize the effect of the capacity, i. e., to favor the preponderance of the spectrum of the gas of greater atomic weight.

Frequently portions of one spectrum will swamp the nearest lines of the other spectrum first. This effect is most marked in combinations of some metallic vapor with nitrogen or a halogen. Combinations with the sulphur group, however, show no trace of this effect. Thallium appears to weaken the *distant* (red) oxygen lines most.

No attempt at quantitative work has been made, nor any attempt to establish a quantitative relation between spectral preponderance and atomic or molecular weight. Judging by the work here described, relative spectral energy might well be proportional to atomic weight; it certainly is not less but may be more than proportional.

In obtaining pure spectra, then, impurities of high atomic weight are to be particularly avoided. In working with the spectra of gases or vapors of great atomic weight, the presence of impurities

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<sup>a</sup> See Ferry: Phys. Rev., 7, p. 305; 1898.

of low atomic weight will be of little consequence. For detecting spectroscopically small amounts of vapors of large atomic weight, the primary spectrum is most favorable. If, however, it is desired to detect a small amount of a light gas mixed with a heavier, it is best to work with secondary spectra.

In conclusion it may be well to call attention to the simple explanation of the phenomena observed afforded by modern theories of gas conduction. Consider a current consisting of a convection of charged particles. Consider luminous and ultra violet radiation as due to vibrations of the electrons composing the atom. Excitation would be due the impact of electrons, largely negative. The larger atoms would not only be hit most frequently, but would shield the smaller atoms from impact by the moving negative electrons. At very low pressures the shielding action would be less in proportion. Irregular heat motion involving atomic and molecular impact would have little effect on either total or relative excitation. Increasing the current would increase the number of exciting electrons and hence the luminosity in proportion. The relative excitation (of large and small atoms) would remain unchanged, while the total excitation would be proportional to the current. While with moderate currents probably but a single electron (if any) is torn from an atom, excessive currents might still further break up the atom and so produce a more and more continuous spectrum. In the rare instances of molecules so stable as to withstand having a fourth of their charge suddenly torn away, the electrons in the combined atoms might still be able to vibrate but with altered and greatly damped vibrations. The large molecules would receive greater excitation than any atoms present. Hence compound spectra, when they do occur, are banded and relatively intense. Oscillating discharges would excite the smaller atoms relatively more than continuous currents.

If the exciting negative electrons are all alike no matter from what atoms they are torn, one can not properly speak of the current being carried by one gas of a mixture more than by another. Distribution of spectral energy would not then be a matter of distribution of current.



## ON SECONDARY SPECTRA AND THE CONDITIONS UNDER WHICH THEY MAY BE PRODUCED.

By P. G. NUTTING.

Since early in the development of spectroscopy<sup>a</sup> it has been well known that some of the elementary gases electrically excited give two entirely different spectra. One of these, called by Plücker and Hittorf the *primary* spectrum, usually consists of a large number of broad lines well distributed over the whole visible and neighboring ultra violet spectrum. The secondary spectrum, on the other hand, consists usually of but few lines, and these very prominent. It is the spectrum obtained when capacity is connected in parallel with the tube. The two spectra are easily obtained with an ordinary Plücker tube having not too long nor too fine a capillary and containing nitrogen, sulphur, or iodine vapor at a pressure of from 2 to 10 millimeters. With a large condenser (at least one-twentieth microfarad) in parallel, such a tube shows a secondary spectrum; without the condenser it shows the primary spectrum if the current be not excessive. Some gases which exhibit secondary spectra also show anode and cathode glows having quite different spectra. The present investigation was undertaken to separate, in the Mendeleef system, those elements which show multiple spectra from those that do not, and to determine what conditions govern the production of secondary spectra.

Starting with the seventh group of the periodic system, it was found that chlorine, bromine and iodine, all of the group that could be worked with, gave multiple spectra. The effect is particularly striking with bromine and iodine. Likewise all of the sixth group available gave multiple spectra, namely oxygen, sulphur, selenium and tellurium. In the fifth group nitrogen, phosphorus and arsenic exhibit

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<sup>a</sup> Plücker and Hittorf: "On the Spectra of Ignited Gases and Vapors, with Special Regard to the Different Spectra of the Same Elementary Gaseous Substance." Phil. Trans., 155, pp. 1-29; 1865.

multiple spectra while bismuth does not. With arsenic, however, the primary (anode) and secondary spectra have two prominent green lines in common. In the fourth group only the more metallic elements—tin and lead—could be used in a Plücker tube. The spectra of these was found not to change on connecting a condenser in shunt. In the third group aluminum, indium and thallium; and in the second group magnesium, zinc, cadmium, and mercury give only line spectra and these are not essentially altered by capacity in parallel. Hydrogen excepted, the first group shows only single spectra. Without taking up the much-discussed<sup>a</sup> question of the rôle of water vapor in gas conduction, from the point of view of multiple spectra the evidence is very strong that the fine-line spectrum of hydrogen is a true primary spectrum while the “four-line” is a true secondary. The latter is relatively enhanced under the same conditions that bring out the secondary spectra of other multiple-spectra elements. The primary and secondary spectra of other elements, notably chlorine and bromine, may be made to coexist in the same way, and the presence of water vapor certainly favors the preponderance of the secondary spectra of other elements as well as of hydrogen. Oxygen and sulphur are striking examples.

In the helium group there are wide differences. Helium itself shows but a single spectrum. Anode and cathode glows give identical spectra and both remain practically unchanged on the passage of the condenser discharge. Argon, however, shows at least four widely different spectra. The red primary (anode) and blue secondary spectra have been described by several observers.<sup>b</sup> But the cathode glow shows quite a different spectrum from the anode (capillary) glow. With low dispersion it appears nearly continuous in the yellow, green and blue, while the strong red line and violet group of the anode glow are lacking. When the blue condenser discharge is produced in the capillary there is a buff-colored glow in the bulbs at either end. The spectrum of this glow shows the blue group lacking, but the yellow group of the red primary anode spectrum present. This difference between bulb and capillary spectra can hardly be regarded as other than a current density effect, since this is the chief, if not the only, difference in the excitation. As we should expect, the bulb spectrum with condenser is intermediate between the capillary spectrum with

<sup>a</sup>Callendar: *Nat.* 56, p. 624; 1897. Trowbridge: *Phil. Mag.* [6], 2, p. 370; 1901. P. Lewis: *Phil. Mag.* [6], 3, p. 512; 1902. Parsons: *Astroph. J.*, 18, p. 112; 1903.

<sup>b</sup>Trowbridge and Richards: *Phil. Mag.* [5], 48, p. 77; 1897. Kayser: *Berlin Akad.*, 1896; Travers, *Expl. Study of Gases*, 1901, p. 312.

<sup>c</sup>Baly: *Chem. News*, 88, p. 26; 1903.

condenser and the red capillary spectrum without condenser. Of neon, krypton, and xenon only the spectra of the two latter are said to be seriously affected by the addition of a condenser.<sup>c</sup> No differences between anode and cathode capillary and bulb spectra appear to have been recorded.

In the accompanying Mendeleef table the elements showing multiple spectra are indicated by heavy-faced type. Elements showing distinct anode and cathode spectra are bracketed.

| O  | I         |    | II |    | III      |    | IV       |    | V   | VI       |     | VII      |
|----|-----------|----|----|----|----------|----|----------|----|-----|----------|-----|----------|
|    | <b>H</b>  | Li | Be |    | <b>B</b> |    | <b>C</b> |    | (N) | <b>S</b> | (O) | <b>F</b> |
| Ne | He        | Na | Mg | Al | Si       | Ti | P        | As | V   | Se       | Cr  | (Cl) —   |
| —  | (A)       | K  | Ca | Ga | Ge       | Zr | Nb       | Sb | Mo  | —        | —   | —        |
| —  | <b>Kr</b> | Rb | Sr | In | Sn       | Ce | Bi       | —  | —   | —        | —   | —        |
| —  | <b>Xe</b> | Ag | Cd | Ba | La       | Pb | —        | —  | —   | —        | —   | —        |
| —  |           | Cs | Hg | Tl |          |    |          |    |     |          |     |          |
| —  |           | Au |    |    |          |    |          |    |     |          |     |          |

Excepting then the anomalous helium group and hydrogen, we may say that in general, so far as observations go, *the acid-forming elements have multiple spectra*, while metallic elements do not.

Most metallic spectra are greatly altered by the addition of capacity, it is true, but the changes produced are by no means comparable with the change from primary to secondary, while metallic spectra, by every test known, are *already* secondary spectra before adding capacity. They are series spectra, show the Zeeman effect and the displacement effect due to pressure, and the new lines brought in by added capacity are of the same (secondary) character. That so many metallic vapors exhibit banded absorption spectra would seem to indicate that metals may have primary spectra as well as nonmetals, but if so they are to be sought in the electrodeless discharge and cathode luminescence where the excitation is very feeble, rather than in the condenser discharge. Nor would the oxyhydrogen flame, banded spectra of the metals studied by Hartley and Ramage be comparable with primary spectra. In these spectra the lines of the series spectra simply become the heads of the bands in the flame spectra, while the lines of a true secondary spectrum appear to have no relation to the bands of the corresponding primary. Another notable distinction between the spectra of electro-negative and electro-positive elements is that the former, both primary and secondary, are remarkably invariant, while it is difficult to obtain the spectrum of a metal twice alike.

Why should a condenser discharge produce a secondary spectrum? The oscillation frequency of a condenser is so much less than the frequency of collision of an electron in a gas at 1 mm pressure<sup>a</sup> that an electron must collide a great number of times before the impelling force is reversed. Hence the oscillatory nature of the discharge *per se* could have little effect on the spectrum of a gas. On the other hand, the current density during discharge is thousands of times as great as during the steady flow of the same current. In testing other gases for the effect of excessive current density, specially designed Plücker tubes were used. One form had a very fine (thermometer) capillary and hollow cylindrical electrodes; another had a third bulb interposed in the middle of the capillary. In this way the current density could be increased a hundred or thousand times in parts of the tubes. Bromine, iodine, and hydrogen easily show the current density effect within the range of current that a common glass tube will carry. The primary spectrum changes continuously over into the secondary spectrum as the current is increased. Nitrogen and the sulphur group failed to show the secondary spectrum with the largest steady currents that glass tubes could carry. Argon is so sensitive to changes in current density that, in conjunction with a pocket spectroscope, it might be used as a milliammeter.

It was further undertaken to determine how much capacity was necessary to just produce the secondary spectrum in different gases, how this critical capacity varies with the spectral wave length, with the density of the gas, the amount of inductance and resistance in circuit, distance apart of electrodes, and sectional area of the discharge. In this work photographic methods were employed throughout. Spectra obtained under varied conditions were photographed side by side on the same plate so that the minutest changes could be observed and followed. For this purpose a large model Fuess' quartz spectrograph was used. This was provided with a large flint glass prism giving a spectrum about 15 cm long from 300 to 600  $\mu$ . Ten or more spectra could be recorded on the same plate. A large glass condenser was used, composed of 20 plates well separated and provided with mercury cups, so that the capacity could be varied by a plate at a time. Current was supplied by transformers of 1,000, 2,000 and 5,000 volts, and by a set of generators giving 5,000 volts continuous current. For inductance, a Seibt tuning solenoid of 120 turns 20 cm in diameter was used. The greater part of the work was done with ordinary short, stout Plücker tubes made by Boehm, of Chicago. These had electrodes about 4 cm apart, and capillary por-

<sup>a</sup> Davis: Phys. Rev., 17, p. 501; Dec., 1903.



tions 2 mm in diameter and 12 mm long. Some of the work was done with tubes without any central constriction. To keep the pressure of the inclosed gas more nearly constant, a half liter bulb was sealed to each tube while in use.

With a tube of air at 13 mm pressure spectra were photographed with capacities of 0.12, 0.09, 0.06, 0.03 microfarad and with no capacity in parallel. A sudden change from secondary to primary spectrum was found to occur at a capacity of about 0.05 microfarad, equivalent to that of about 14 one-gallon Leyden jars. Adding capacity indefinitely above 0.06 microfarad produced little, if any, effect, nor do secondary lines usually appear in the primary spectrum until the capacity is nearly 0.03 microfarad.

*Critical capacity and wave length.*—Drawing a line separating primary and secondary spectra on the photographic plate (see Pl. I) the ordinates of the curve represent roughly critical capacity, abscissas wave length. The curve drops off very steeply on the short wave length end in spite of the greater dispersion, indicating that for waves perhaps not shorter than  $300\mu\mu$  the critical capacity becomes infinite. Critical capacity expressed as a function of wave length appears to be of the form

$$C = a e^{\frac{b}{\lambda - \lambda_0}}$$

Approximate numerical results for air are given in the table:

| Pressure.   | Wave length in $\mu\mu$ . |                    |                    |
|-------------|---------------------------|--------------------|--------------------|
|             | 350.                      | 450.               | 550.               |
|             | <i>Microfarad.</i>        | <i>Microfarad.</i> | <i>Microfarad.</i> |
| 760 mm..... | 0.02                      | 0.01               | < 0.006            |
| 20 mm.....  | 0.07                      | 0.03               | 0.02               |
| 1 mm.....   | > 0.15                    | 0.06               | 0.03               |

*Critical capacity and pressure.*—The critical capacity increases slightly as the pressure decreases down to about 2 mm air pressure, when it suddenly becomes infinite, i. e., no amount of capacity (without an external spark gap in series) will cause the secondary spectrum to appear. With spark gap, secondary spectra may be obtained at pressures of but a few tenths of a millimeter, nearly to the pressure at which cathode-ray fluorescence appears. This critical minimum pressure at which the disruptive discharge becomes possible is considerably

lower for hydrogen (about 1 mm), but depends on the condition of the surface of the electrodes and the presence of impurities. At high pressures the critical capacity continues to gradually decrease. With a spark in open air, either the primary or secondary nitrogen spectrum may be obtained according to the electrical conditions in circuit. Using a special tube the distance apart of whose electrodes could be varied to suit the potential (5,000 volts) employed, the critical capacity of air and of hydrogen was observed at various pressures up to atmospheric pressure. At atmospheric pressure it was found to be about one-fourth what it was at 10 mm pressure. The critical capacity appears to be roughly inversely proportional to the cube root of the pressure, or directly proportional to the mean distance apart of the molecules.

*Critical capacity and nature of the gas.*—Hydrogen, sulphur, nitrogen, oxygen, bromine, and iodine were tested and the critical capacity found to be practically the same for all for the same pressure and wave length. Critical capacity is more sharply marked in sulphur, nitrogen, and iodine. With hydrogen the lines of the secondary (four-line) spectrum invariably appear in the primary (many-line) spectrum, the capacity of the wires leading to the tube being a considerable factor in their dominance. All the substances show the same great increase in critical capacity for the very short wave lengths and decrease with increasing pressure.

Critical capacity appears to be nearly or quite independent of the voltage employed (up to 5,000 volts) and of the distance apart of the electrodes. Tests were made with the electrodes at various distances down to 3 mm apart. At much shorter distances the metallic lines from the electrodes become prominent at high pressure. Critical capacity increases with rise in temperature of the electrodes. No measurements were made, but care was taken during other tests to have the temperature of the electrodes fairly constant.

In the capillary of a Plücker tube the critical capacity is less than in the bulb or less than in a tube without a central constriction. This is confirmatory to a view expressed in a previous part of this paper that the production of a secondary spectrum was not so much the effect of capacity *per se* as of increased current density.

*Critical capacity and inductance.*—The effect of introducing inductance is always to relatively weaken the secondary and enhance the primary spectrum. But introducing a certain inductance is by no means equivalent in its effect to removing a definite corresponding capacity. Inductance was added in steps of 0.008 millihenry. The first inductance added, though very small, weakened the secondary

spectrum very markedly and introduced primary lines, and this whether the capacity used was just above the critical capacity or five times that amount. Adding more and more inductance produces less and less effect. Apparently no amount of inductance, however great, will *completely* annul the effect of any capacity, however small. Capacity and inductance effects are shown graphically in the figure. Ordinates represent the change from primary to secondary spectra.

*Critical capacity and resistance.*—The effect of resistance is as pronounced as that of inductance in changing the secondary spectrum back to the primary. Even as little as 20 ohms (non inductive) resistance brings in primary lines, while 100 ohms gives a nearly pure primary. The resistance effect curve has very nearly the same form as the inductance effect curve, as shown in the figure. The effect of an external spark gap is always to favor the production of a secondary

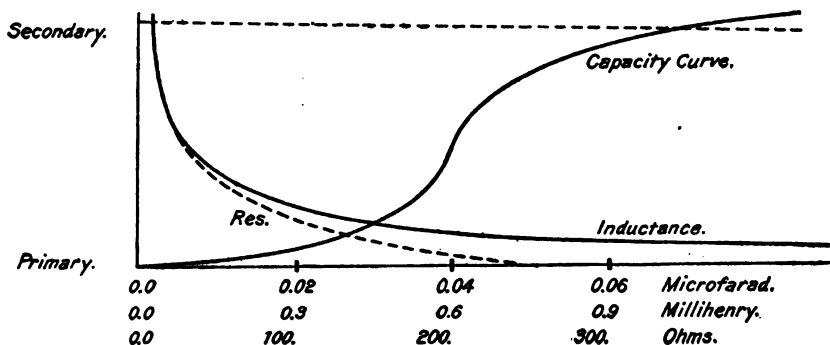


FIG. 1.

spectrum. Secondary spectra may be obtained with much less capacity and at much lower pressures with spark gap than without. But inductance appears to have the same effect with spark gap as without, while the ohmic resistance of the gap must be less than 10 ohms, judging by its effect on the discharge.

*Critical capacity of mixtures.*—Mixtures of hydrogen and nitrogen, sulphur and hydrogen, iodine and nitrogen, iodine and hydrogen, nitrogen and sulphur, hydrogen and oxygen, and mercury and nitrogen were tested, and each component was found to have its own critical capacity independently of the presence of the others. In a previous paper (The Spectra of Mixed Gases, this bulletin) it was shown that the relative intensities of two primary spectra (of a mixture of gases) depended on the relative atomic weight of the component gases, while the relative intensities of the secondary spectra depend only on the relative numbers of the different kinds of atoms present. For example,

it is easy to prepare a tube of mixed hydrogen and nitrogen that shows a ("four-line" secondary) hydrogen spectrum with capacity but a nitrogen (primary) spectrum without.

To obtain a pure *primary* spectrum, then, it is necessary to avoid introducing capacity of any kind (particularly long lead wires) into the tube circuit. It is useless to attempt to neutralize the effect of even a small capacity by introducing inductance or resistance. To obtain a pure *secondary* spectrum; inductance and resistance are to be avoided and at least 0.05 microfarad capacity must be added. With more than 10 ohms resistance or 0.01 millihenry inductance in the tube circuit, a pure secondary can not be obtained by merely adding an excess of capacity.

The names *primary* and *secondary* were given by Plücker and Hittorf in their classic monograph on multiple spectra. In recent years the terms have fallen into disuse, but the writer is strongly in favor of their retention. Some specific distinctive names are very desirable. The names *band* and *line* spectra are far from being distinctive, while the distinguishing term *series* spectrum applied by Kayser and Runge to secondary spectra is not sufficiently specific. When more than a single primary spectrum exists (as with nitrogen), it is easy to specify *anode* primary or *cathode* primary. And the terms primary and secondary are not likely to be confused with *first order* and *second order* applied to grating spectra.

Before discussing the results it may be well to summarize them, together with the results of other workers:

Many elementary gases and vapors possess two entirely different spectra (Plücker and Hittorf). These elements are in number about one-fourth of those known and are largely electro-negative.

The lines of primary spectra do not show the Zeeman effect, while those of secondary do (Runge and Paschen).

The transition from primary to secondary is in most cases very abrupt. Capacity appears to be essential to the production of a pure secondary spectrum. Great current density and a spark gap in series favor, while inductance, resistance, and heated electrodes hinder, the production of secondary spectra. The first inductance added produces the greatest effect per given increment of inductance, while the transition from secondary to primary spectra occurs at about 0.1 millihenry for a great range of capacity or pressure, with or without spark gap in circuit.

Critical capacity is different for different portions of the spectrum, being least in the red and being much greater and rapidly increasing

at the violet end. It appears to approach a constant value, possibly zero, in the infra red, and the value infinity in the ultra violet perhaps not farther out than  $300\mu$ . Critical capacity increases steadily as the density of a gas decreases, becoming suddenly infinite at about one thousandth of an atmosphere.

The electrical conditions concerned with the production of secondary spectra may be stated with some certainty. The *steepness of the wave front* of the discharge appears to be the principal factor; that is, the rate of variation of the electromagnetic field in the gas or of the number of advancing electrons per unit area of cross section, according to the view taken of the nature of the discharge. Putting in capacity vastly increases the steepness and amplitude of the current wave, while inductance tends to smooth it out again; hence capacity favors the production of secondary spectra, while inductance favors the primary. An external spark gap increases the suddenness of the discharge, while resistance cuts down its amplitude; hence a spark gap helps the production of secondary spectra, while resistance favors the primary. Merely increasing the frequency of a simple alternating (say 2,000 volt) current through a gas should change its spectrum from primary to secondary. But high frequency alone (without amplitude) is not sufficient to produce secondary spectra, for the electrodeless "ring" discharge gives a primary spectrum. In this case the frequency may be as great as  $10^8$ . On the other hand, an intense current (even a continuous one) does tend to change the primary spectrum over to the secondary.

Several theories of the mechanism of radiation have been advanced to account for double spectra. Plücker and Hittorf thought that the secondary spectrum of nitrogen indicated either an allotropic state of the gas or else a chemical decomposition under the violence of the discharge, and were more favorable to the former view. Deslandres, who has done a great deal of work on the primary spectrum of nitrogen, supposes the primary spectrum to be given off by molecules excited by the impact of electrons, while the secondary spectrum is given off by the negative electrons alone. Runge has called attention to the fact that the absence of the Zeeman effect indicates that the primary spectrum comes from uncharged particles or from particles whose masses are large relatively to their charges. Hence, primary spectra might be due to positive ions, secondary to negative ions. But band and line spectra do not exist together, so that it would be necessary to consider the positive electrons taking up all the energy until it became of great intensity and then the negative taking it all. Deslandres has questioned whether positive and negative electrons

revolving about one another can have other than identical periods. Hemsalech considers the primary nitrogen spectrum due to impacts between positive and negative ions, the secondary to impacts between electrons and positive ions, without, however, discussing a possible mechanism of radiation in either case.

We have to consider a transformation from electrical energy to radiant energy, a change from the energy of convection to energy of wave motion in an extended medium. On the one hand, work on gas conduction has shown beyond question that an electric current in a gas (probably in *every* medium) consists in a convection of positively and negatively charged particles. The masses, charges carried, numbers and velocities of these charged particles under various conditions, have been determined within narrow limits. On the other hand, Larmor, Lorentz, and others have shown that at least all optical radiation must be due to the motions of charged particles moving with variable velocities. An uncharged particle, if it could exist, would be unable to produce a wave motion in the ether, whatever its motion, while a neutral aggregate of charged particles could lose energy by radiation only when the equilibrium of its internal motion is disturbed by external forces. Here again the magnitude of the charges as well as the acceleration necessary to produce a given radiation has been quite definitely determined.

During the process of disturbance from equilibrium of motion such a neutral aggregate absorbs energy which it radiates in recovering equilibrium. An aggregate not electrically neutral would gradually lose its energy of internal motion until it either broke up and became neutral or until its internal motion ceased. Now the equilibrium of motion of a neutral aggregate could be disturbed by the impact of either an electro-magnetic wave or pulse or of another particle or aggregate. Or the (static) electric field might be so intense as to separate positively and negatively charged particles against their attraction for one another. Undoubtedly all three forms of excitation are effective in producing radiation under various conditions. But disruption appears to occur whenever luminosity is produced, even in the electrodeless "ring" discharge; hence we must always consider the presence of free charged particles and a disruption of part of the neutral aggregate as well as a general disturbance of motion without disruption. And radiation, although not concerned with the steady motion of neutral aggregates, must yet be caused by motions approaching the steady state.

Two forms of atomic structure appear to be capable of steady motion and of radiating observed spectra when this steady motion is

disturbed, namely, systems in which the satellites (negative electrons) are arranged in rings and those in which the satellites move independently. Under severe stresses a ring system might break down into a system having independent satellites. Considering the ring system as discussed by Thompson as the general type of atom, metallic as well as nonmetallic, we find nearly all the phenomena related to secondary spectra capable of simple interpretation. We should have primary spectra given off by the lateral oscillations of the rings when their steady motion was disturbed, secondary spectra by the independent satellites after the rings had been broken up by violent excitation. The ring systems of the atoms of acid-forming elements are not easily broken up, while those of the metals are already broken up before sufficiently excited to become luminous. The Zeeman effect would be shown only by secondary spectra and the wave lengths of the lines in the primary bands would fall under quite a different spectral series from those of the secondary. Lines of a secondary spectrum would not be related in position to those of the corresponding primary. A nitrogen atom would have as many rings as there are bands in its primary spectrum. Critical capacity would vary with the wave length and become infinite for short waves, because the larger, slower rings would be the first to break up, while the innermost rings could not perhaps be broken up at all. The abrupt and complete transition from primary to secondary spectrum shown by nitrogen, sulphur, iodine, and bromine would indicate that when the discharge exceeds a certain intensity the rings on *all* the atoms are broken up at once and remain so, while in hydrogen only a portion of the rings would appear to be disrupted at one time, probably on account of very rapid recombination.

## EXPLANATION OF PLATES.

PLATE I. The upper series of spectra are of nitrogen at 1.5 mm pressure, showing the transition from primary to secondary by gradually adding capacity in steps of 0.03 m. f. The first spectrum of the series (c) is of the white cathode glow. The second spectrum was taken with the same capacity (0.06 m. f.) as the fifth, but taken in the bulb instead of capillary. The bulb spectrum is seen to be a nearly pure primary, while the capillary is a secondary in the green and yellow, illustrating very well the effect of current density on the production of a secondary. The third spectrum (p) is a pure (anode) primary, while those beneath were taken with the capacity indicated. Drawing a line roughly separating primary and secondary, this curve represents critical capacity as a function of wave length.

The lower series shows the same capacity effect in hydrogen. The capacity used is indicated, the bottom spectrum being a nearly pure primary.

PLATE II. The upper series shows the same capacity effect in sulphur. The first two spectra are both primary, but the upper of the two was taken when the vapor density was greater.

The lower series of spectra shows the current density effect in hydrogen. The spectrum at the bottom was taken with a very feeble current and long exposure, the second from the bottom with a greater current, the third with the largest current (0.06 amp.) the tube would carry. The two upper spectra were taken with the capacities indicated.  $H_{\beta}$  appears even with a feeble steady current, but  $H_{\gamma}$ ,  $H_{\delta}$ , and  $H_{\epsilon}$  appear only with heavy current or capacity.

PLATE III. This plate shows the effect of inductance in hydrogen and nitrogen in bringing the secondary back to the primary. In each case the secondary at the bottom was taken with a large capacity (0.1 m. f.). The spectrum just above was taken with the same capacity with a small inductance (0.1 millihenry). The next spectrum was taken with the same capacity and a large inductance (0.8 millihenry). The topmost spectrum is a pure primary, in each case taken with neither capacity nor inductance.



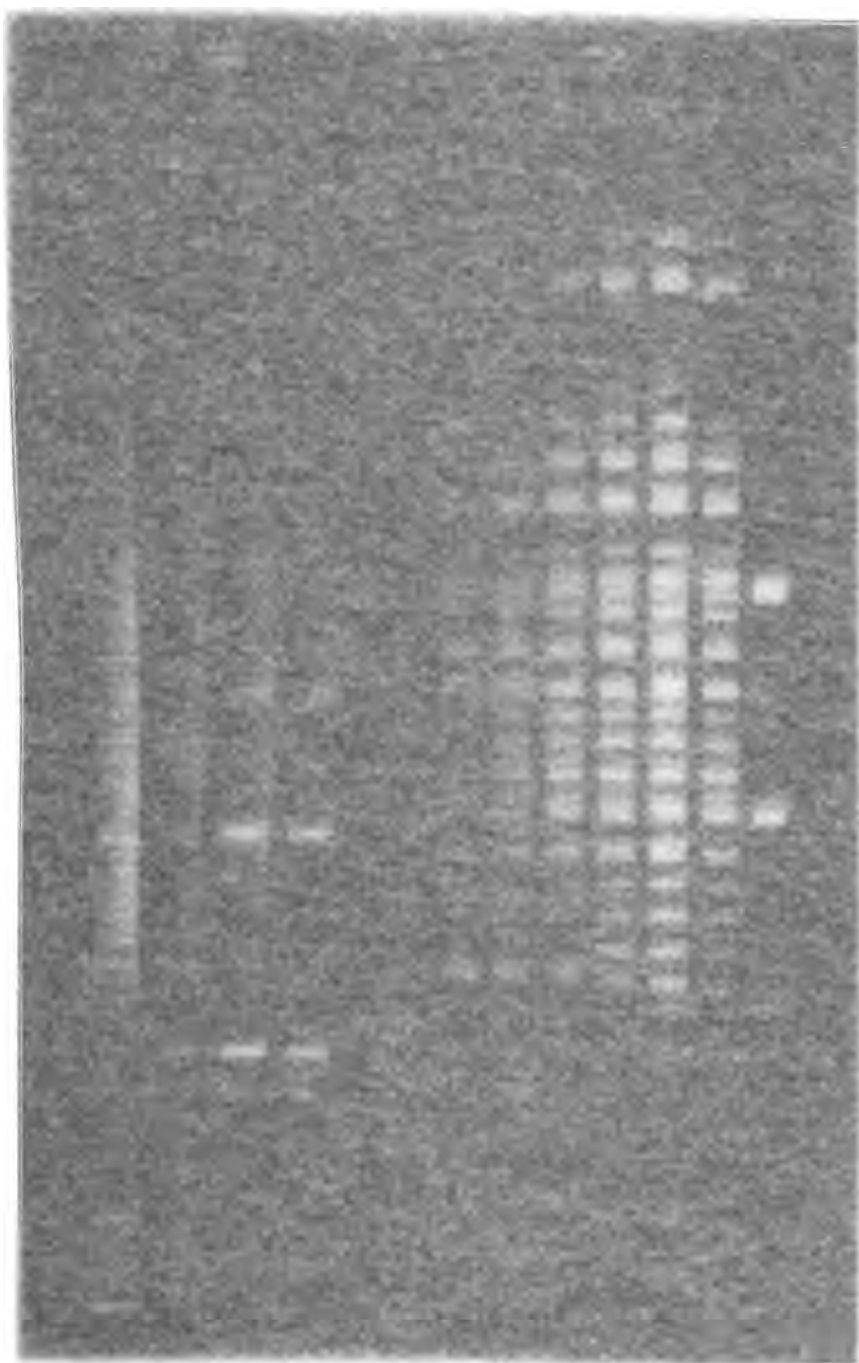
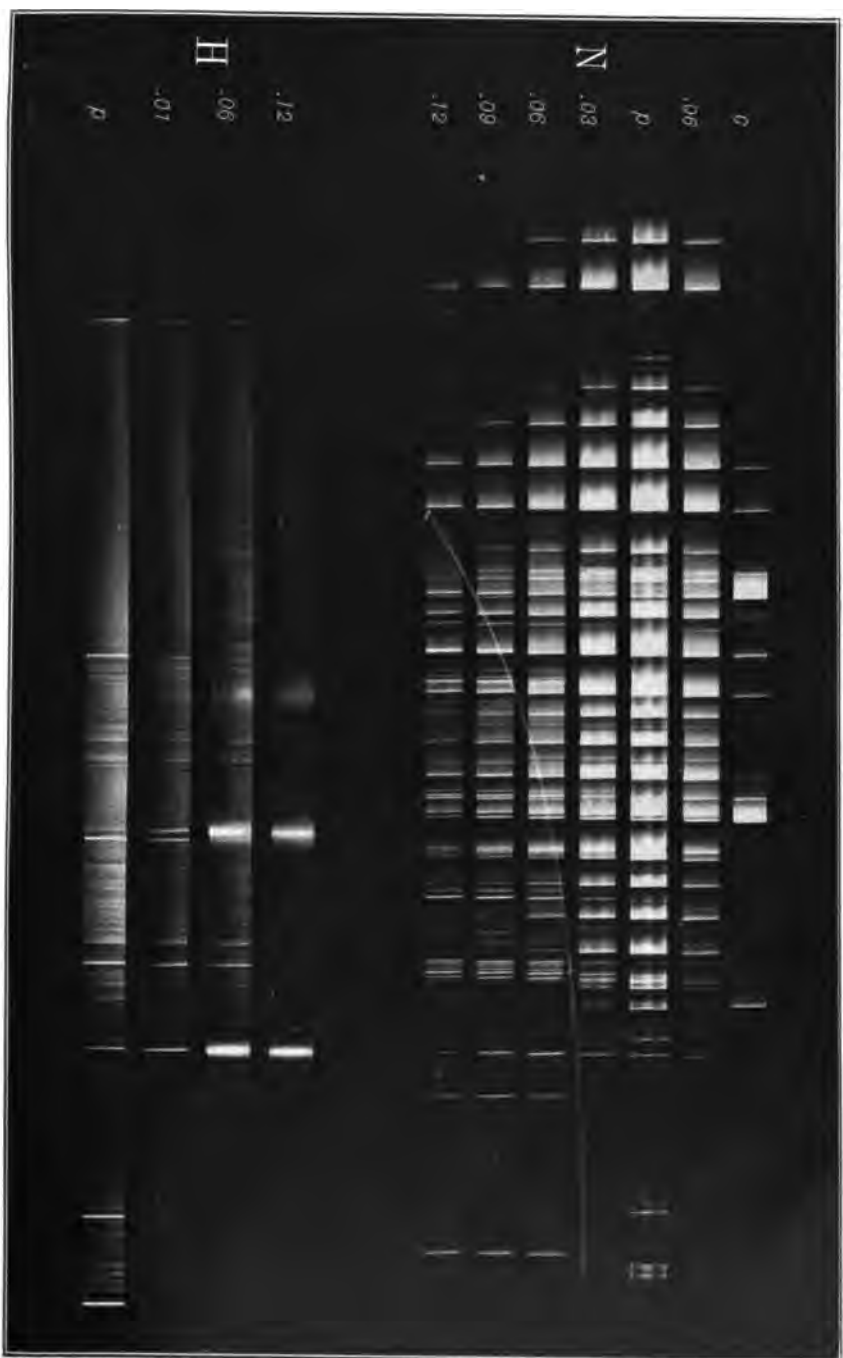




PLATE 1.





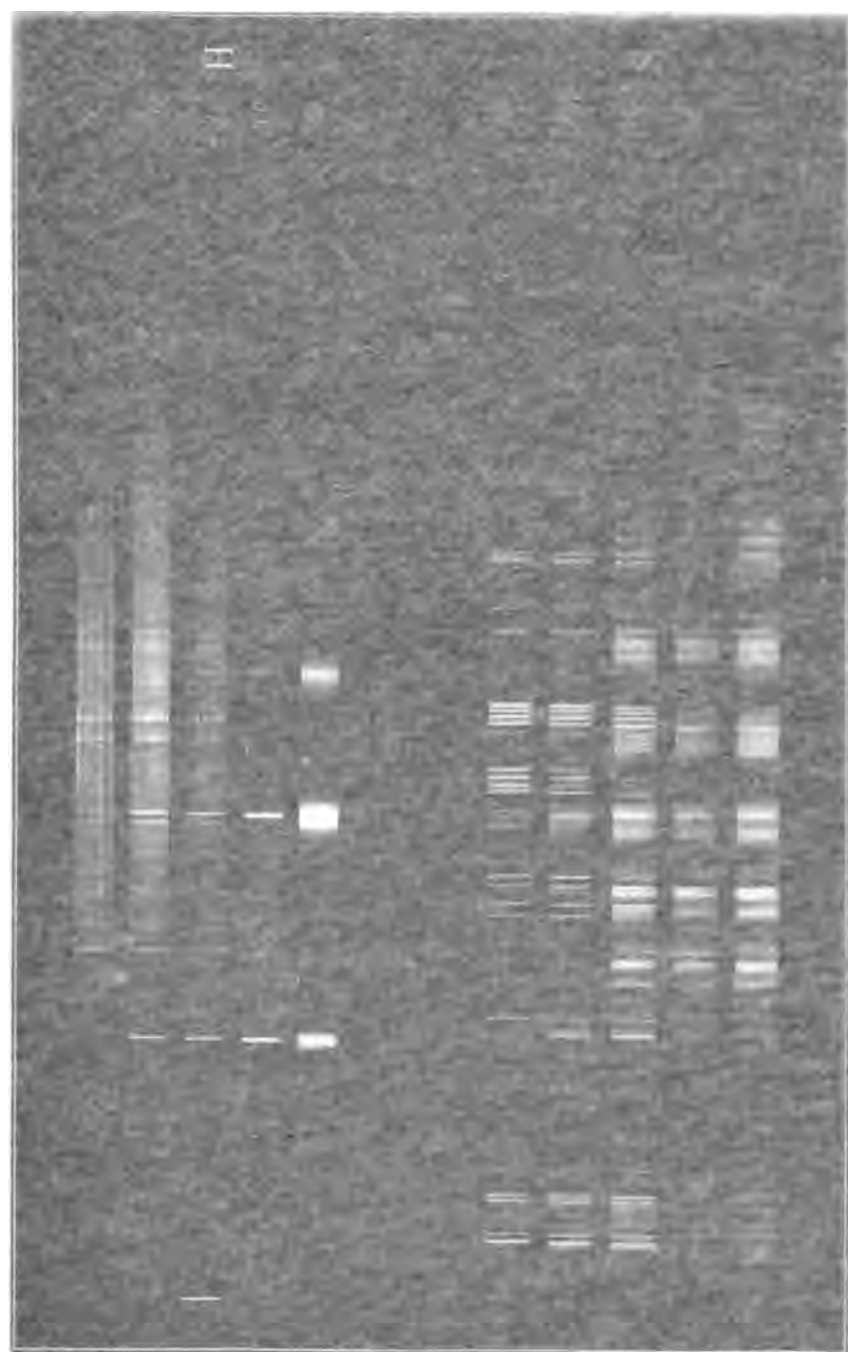
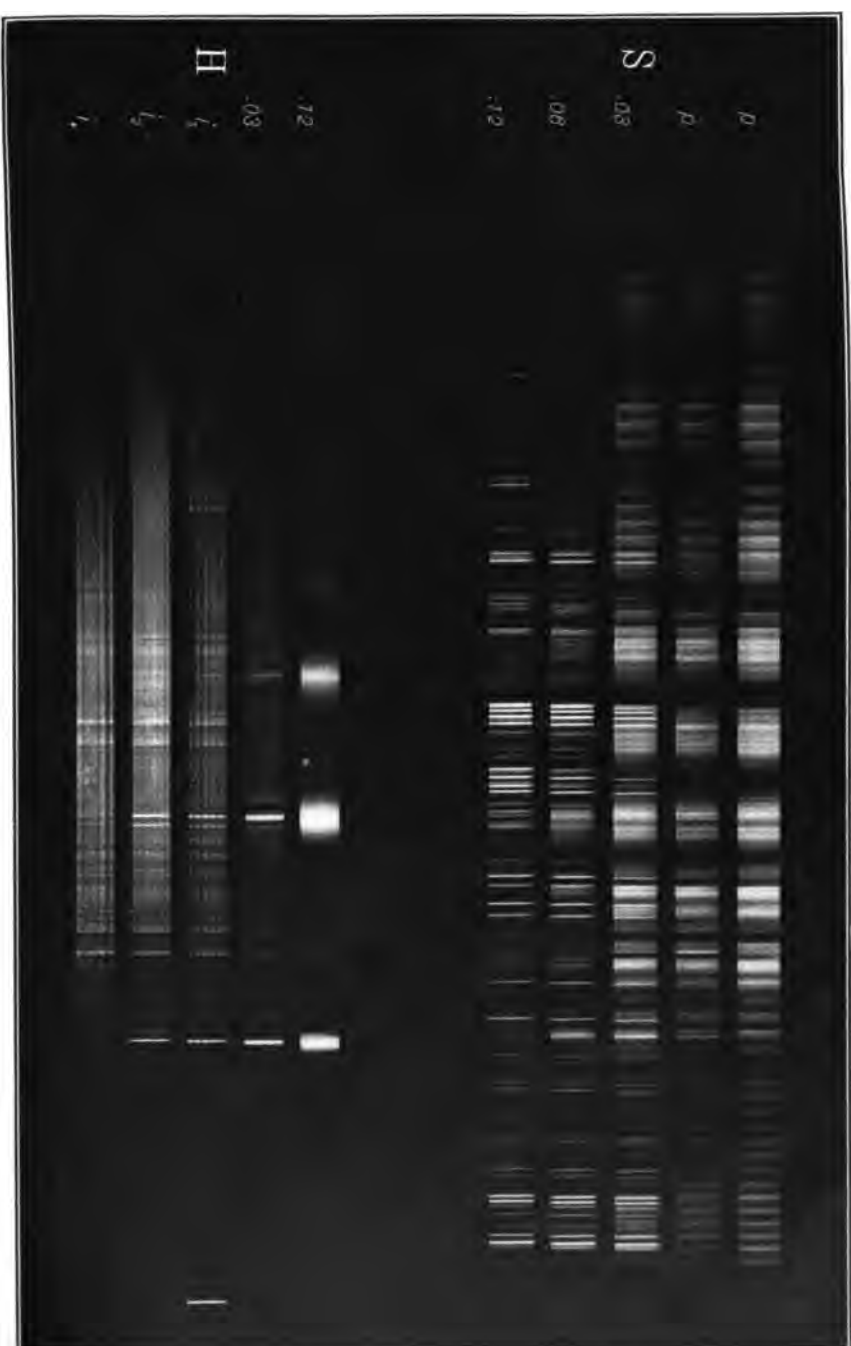




PLATE II.







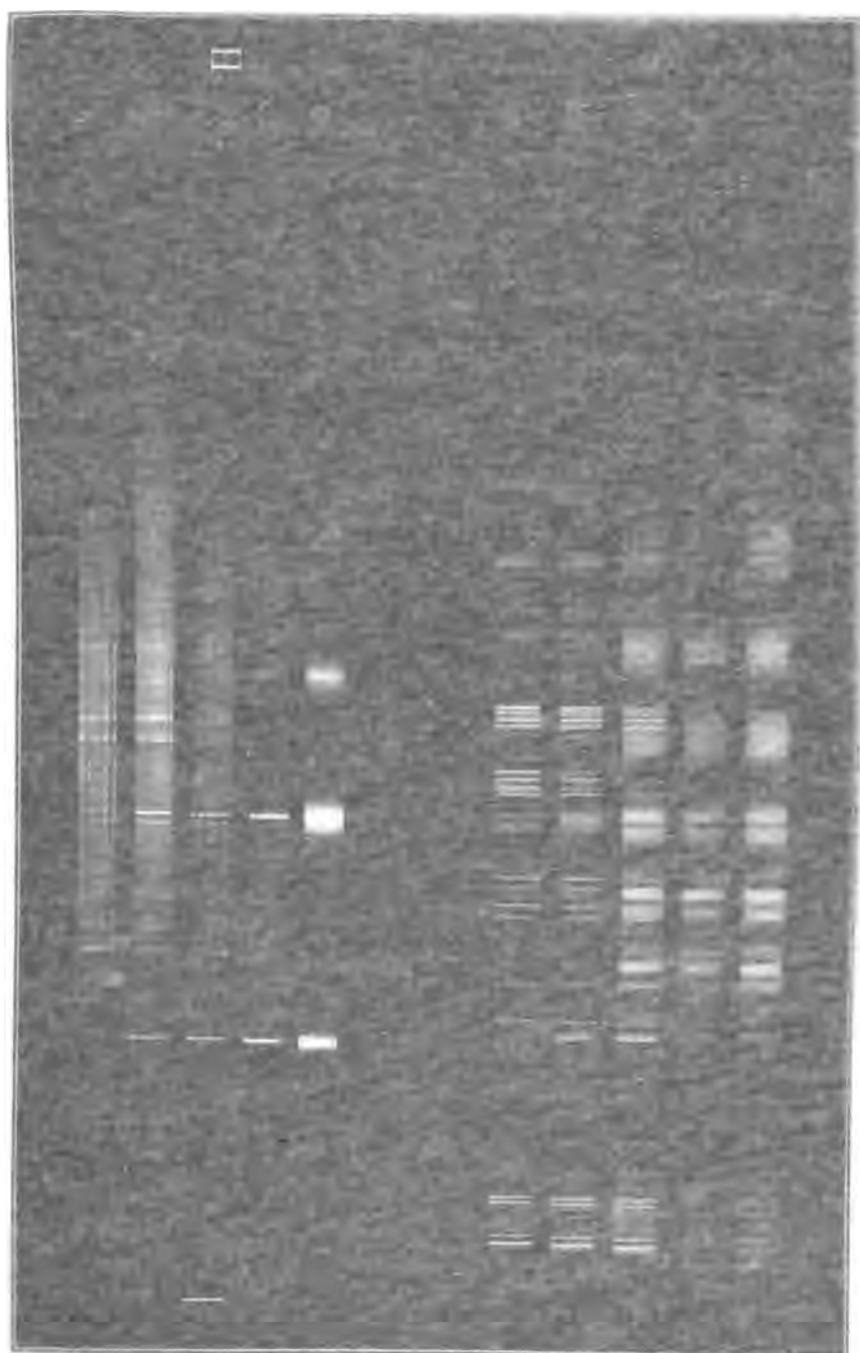
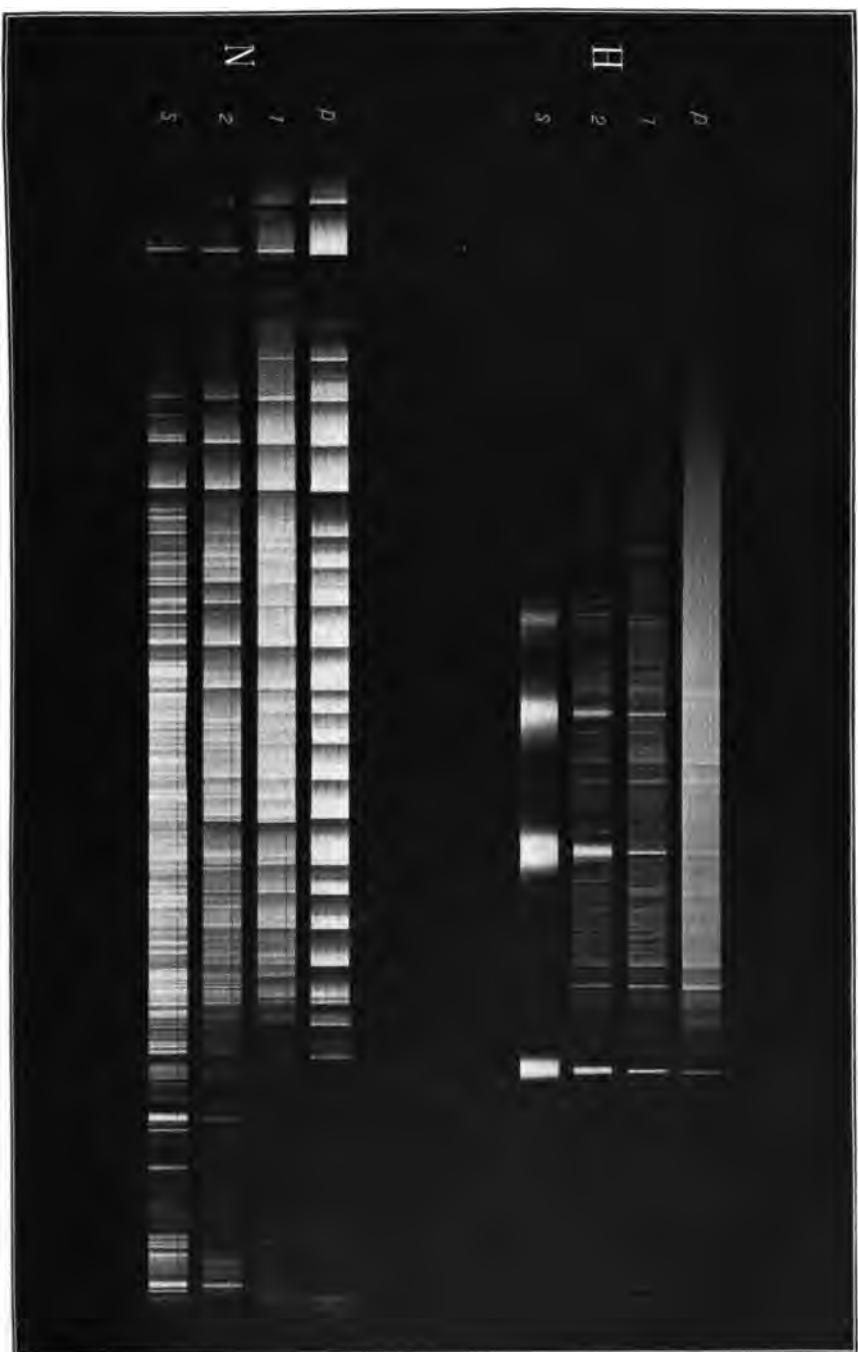




PLATE III.





## SOME NEW RECTIFYING EFFECTS IN CONDUCTING GASES.

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By P. G. NUTTING.

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The passage of electricity from a metal to a gas, or from a gas to a metal, occurs in a manner not yet well understood. The present investigation was undertaken to determine what conditions of the metal or gas favor the transfer of electricity from one to the other, and, if possible, to throw some light on the mechanism of surface conduction. While the results here presented are crude, they bear directly on the theory of metal-gas conduction, and the method, though very simple, is believed to be new. The work suggested itself during some spectroscopic work with Plücker tubes.

If we use a direct current in studying the electrode drop in potential, we are confronted by the the great difficulty in keeping the condition of the surface, temperature, pressure, etc., constant, and in reproducing these conditions in successive tests. But if we use an alternating current of a maximum voltage just a little above the maximum total drop in potential between electrodes, conditions can not appreciably change between successive alternations, and the excess of current in one direction may be taken as a rough measure of the relative surface conductivity at the two electrodes. It is hardly conceivable that there can be any rectifying effect due to the gas itself away from the electrodes, for this would require not only molecular or atomic dissymmetry, but a fixed orientation of these particles. Hence we must regard rectification as due to conditions existing at the metal-gas surface and in the immediately adjacent metal and gas. The general form assumed by a single-phase alternating current wave when a conducting gas is included in the circuit is discussed later on in relation to the dissymmetry of the wave caused by partial rectification.

The greater part of the work here described was done with the current from a transformer of 100 : 2,000 volts and 600 watts capacity. Occasionally some of the work was repeated on smaller 100 : 1,000 volt transformers of 100 or 300 watts capacity. To control the current in the secondary a variable noninductive resistance of about 100 ohms was placed in the primary of the transformer. The current in

the secondary varied from 10 to 40 milliamperes, according to the density of the conducting gas. Most of the work was done with ordinary Plücker tubes slightly altered according to the conditions desired. They were of the short, stout form now preferred and had either rod or disk aluminum electrodes. The tubes were connected with rubber pressure tubing to a Geryk pump and hydrogen generator by a three-way cock, so that they could be quickly exhausted and filled or washed

with hydrogen. A direct-current milliammeter in series with the tube gave the amount of current rectified, while an alternating current voltmeter used in series as an ammeter, gave the total current. In this way rectification as small as half of one per cent could be easily measured and a tenth as much detected. It is difficult to prepare a tube that will not show as much as one per cent rectification. The effects here recorded are as large as 5 to 40 per cent and decidedly reversible.

The principal rectifying effects obtained are indicated diagrammatically in the accompanying figures. The number at the left corresponds with the number of the descriptive paragraph in the context. The percentages at the right indicate the maximum percentage of the current rectified, while the intermediate arrow indicates its direction. Except the first, the effects showed but little variation in percentage of rectification with the density of the conducting gas, as the pressure on this was varied from 6 to 0.1 mm.

1. The *difference in pressure* of the gas about the two electrodes was accom-

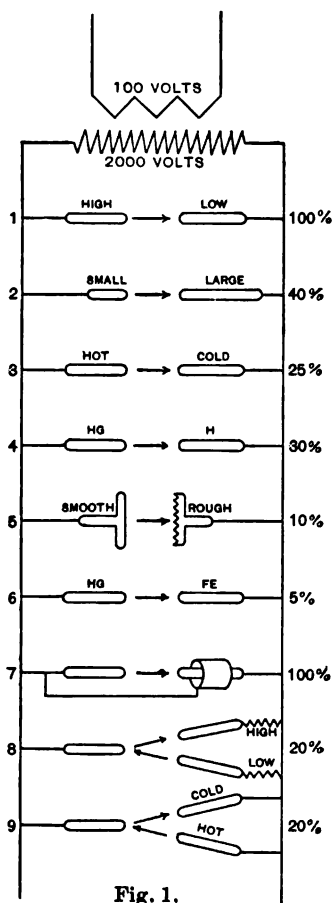


Fig. 1.

plished by constricting the tube to a fine capillary between the electrodes and rapidly exhausting. If the exhaustion is sufficiently rapid, complete rectification occurs, i. e., half of the current wave fails to pass at all. Judging by the width of the cathode dark space, the corresponding pressures in the two bulbs are about 4 mm. and 0.3 mm. Sudden admission of gas into an exhausted tube reverses the rectification.

We can hardly attribute this effect to any differences within the electrodes nor to the ease of escape of electrons from metal to gas. But if we consider the current carried by the negative electrons alone, then the current will be able to start much more easily from the electrode surrounded by the more rarefied gas, for here the electrons will have a greater free path and will be more easily able to attain ionizing velocity. Hence the (negative) current will travel most easily from low to high pressure as observed. A cruder and less tenable explanation is possible if we consider the current carried by both positive and negative electrons. The strong current of gas through the capillary from high to low pressure would tend to sweep the larger positive electrons with it rather than the much smaller negative electrons. Hence the (positive) current would flow from high to low pressure as observed.

2. The *relative size of the electrodes* is an important factor in producing rectification. The excess of current flows from the small to the large electrode. When the large electrode was three times the area of the smaller, a rectification of 12 per cent was observed; when it was ten times the area, 35 per cent was observed. The tests were made with aluminum electrodes in Plücker tubes. When the pressure is so great and the current so small that the glow covers only a portion of the larger electrode the effect is null or reversed. After the current has been passing for some time the heating effect next described (3) may mask the effect of relative size. There appears to be little if any difference in size of the cathode dark space at the large and small electrodes.

The current density being less at the larger electrode, any competition between the electrons carrying the current will be less at this electrode. At the larger electrode negative electrons starting out would be less interfered with by the pressure of positive electrons. The effect can hardly be traced to the difference in capacity of the large and small electrodes, for connecting one electrode to a large capacity or even to earth produces but a slight rectifying effect.

3. A *difference in temperature* of the electrodes gives a partly rectified current from the hot to the cold electrode. When one bulb of a Plücker tube was heated to about  $250^{\circ}$  and the other bulb kept cold by a wet cloth and protected by an asbestos screen, a rectification as large as 25 per cent was observed on starting the current. This decreased rapidly as the current heated the cold electrode. Using tubes with disk electrodes the cathode dark space was at least 50 per cent larger at the hot electrode than at the cold. At the same time the canal ray glow within the dark space was scarcely perceptible while at the cold electrode it nearly filled the cathode dark space.

A continuous current heats the cathode much more energetically than the anode. Hence when an alternating current passes between a hot and a cold electrode we should expect the rectified portion to pass in such a direction as to heat the cold electrode more than the hot, and so produce equilibrium. This is what actually occurs. But the electron theory would lead us to expect a current from cold to hot. At the hot electrode negative electrons escape from the metal most easily. They have a longer free path to get started and produce ionization, and this movement is less obstructed by the presence of positive electrons. Hence we should expect a negative current from hot to cold, the reverse of that observed.

4. *Gas about the electrode.*—When one bulb of a Plücker tube is filled with mercury vapor and the other with air or hydrogen there is a decided rectifying effect from the mercury vapor toward the hydrogen. In spectroscopic work I have frequently observed complete rectification under similar conditions.

In some previous work on the spectra of mixed gases (this bulletin) it was shown that in a mixture, other things being equal, gases or vapors of great atomic weight appear to give always greater spectral energy than those of lower. If we take this to mean that they carry current more readily or are more readily ionized, we have a simple explanation for the above rectifying effect. The current is more easily started in the bulb containing heavier vapor so that half the alternating current wave is reduced in area according to the diagram discussed later.

5. The condition of the *surface of the electrodes* appears to determine a slight but decided rectifying effect. Aluminum disk electrodes were used, one polished and the other cut in ridges, like a file, with a sharp knife. An excess of current passed from the smooth to the rough electrode. Whether this effect was anything more, than that due to a greater exposed surface could not be determined.

6. With one electrode mercury and the other iron of the same size, but a slight rectifying effect from mercury to iron was observed on starting the current. After the current had been running some time\* mercury vapor over the mercury electrode tended by (4) to increase the rectification, while the much hotter iron electrode by (3) tended to decrease it. It would appear that the action of the Cooper Hewitt rectifier on a three-phase current is largely due to the much greater surface exposed by the mercury electrode and the greater density of mercury vapor in its immediate neighborhood. If cathode drop of potential played an important rôle, we should have obtained greater rectification with the tube just described and a single-phase current.



When aluminum was used against iron in atmospheres of air and hydrogen, a rectification of 20 per cent was again observed. When hydrogen was used, the rectification was just perceptibly less than in air.

7. Placing a metallic band about one electrode and connecting this band with the other electrode, a strong, often complete, rectification is produced, the current passing toward the electrode having an external band. It has frequently been noticed that placing the band near a discharge tube steadies the discharge. An earthed metallic band about the tube near one electrode was much more effective in strengthening

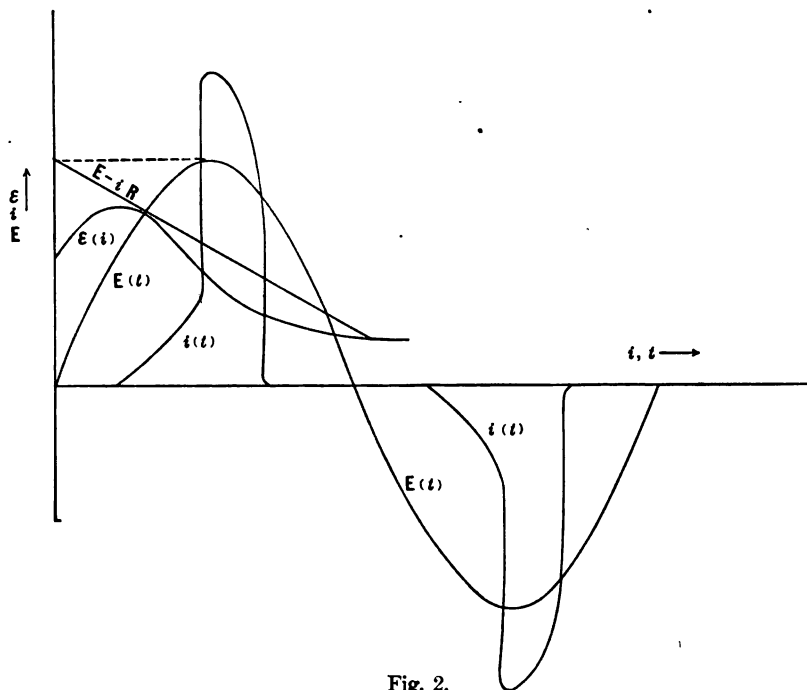


Fig. 2.

the discharge. Finally, this band was connected with the opposite electrode and complete rectification obtained. A similar device has been used to facilitate starting in mercury vapor lamps, a metallic covering being placed about the cathode and connected to the anode.

Throwing an intense beam of sunlight on the front face of one of the (disk) electrodes by means of a lens produced no perceptible rectification. The effect of exposing one electrode or the adjacent gas to the action of X rays was not tested, but from (4) and (6) we should expect the side exposed to become anode.

Tests 8 and 9 were made with tubes provided with three mercury electrodes. One of these was connected to one 2,000-volt terminal, and the other two, through different resistances, to the other. Current was found to enter through the low resistance and leave by the high. Such an effect by (3) would be produced by the low-resistance electrode being more heated by the greater current. When connections were made with equal resistance and one electrode heated the current passed from hot to cold in accordance with (3) and (4).

We may get a better idea of what actually occurs in the process of partial rectification from the accompanying figure (fig. 2). Let the curve  $\epsilon(i)$  be the characteristic gas-conduction curve for the tube used—i. e., the fall of potential between electrodes as a function of the current. Impress the alternating voltage  $E(t)$  on the tube having such an external (ohmic) resistance in circuit that the line  $E-iR$  will have the slope indicated, then  $i(t)$ , here plotted directly, will represent the form of the corresponding curve of current through the tube as a function of time. Anyone who has ever plotted these current curves knows how entirely their form changes with even a slight change in the form of the gas curve  $\epsilon(i)$ , and these  $\epsilon(i)$  curves change in form greatly as conditions in the tube are varied. For instance, at very low pressures the  $\epsilon(i)$  curve is elevated and flattened. At high pressures it may rise continually, particularly if the electrodes be small or pointed. If the tube be illuminated by X rays the maximum of the  $\epsilon(i)$  curve will be sharpened, lowered, and displaced toward the left. If then the electrodes are under different conditions, the  $\epsilon(i)$  curve will vary in form with the direction of the current through the tube, and therefore the current curves  $i(t)$  will no longer be symmetrical about the horizontal axis. The reversed (lower) part of the curve will be more flattened or rounded say than the upper, and the effect will be shown as partial rectification.

A tube having one internal and one external electrode showed no trace of rectification. This might have been expected, since in this case the  $\epsilon(i)$  curve rises steeply and continuously and the current curve  $i(t)$  is a low, broad sine curve which would show little, if any, dissymmetry.

## ON FIBERS RESEMBLING FUSED QUARTZ IN THEIR ELASTIC PROPERTIES.

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By K. E. GUTHE.

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The advantages of quartz fibers as suspensions, principally their small elastic fatigue, great strength, and the possibility of drawing very fine threads, have led to their use in a great many important investigations where fine suspensions and a steady zero point are required.

In my work, suspensions were needed, able to carry a load of 500 grams and more and at the same time having as small an elastic fatigue as possible. Naturally fused quartz was selected as the substance promising the best results. Such fibers must be rather thick, from 0.1 to 0.2 mm in diameter. Since it was desired to draw them at least 1 meter in length, it required the melting of a rather large bead free from air bubbles. The process of building up quartz rods is known to be rather tedious. Shenstone's method<sup>a</sup> is simpler, and consists of heating the crystals to about 1,000° C. and suddenly quenching them in cold water. The crystals after such treatment are not shattered, and do not break when brought into the oxy-hydrogen flame. A stick of the proper dimensions is easily formed, containing, however, a large number of air bubbles. To remove these is a very tedious task and particularly exasperating, because quartz at the high temperature of the blowpipe flame is quite volatile, about one-half of the mass evaporating during the process.

Whichever method is employed it means a considerable loss of time, if a great many fibers of the dimensions necessary for this work have to be drawn.

Boys, in his first paper on this subject,<sup>b</sup> mentions his experiments on a great many minerals, of which only a few, however, could be drawn into fibers, and these he says were far inferior to those made from

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<sup>a</sup> Shenstone: *Nature*, 64, p. 65; 1901.

<sup>b</sup> Boys: *Phil. Mag.*, 23, p. 489; 1887.

fused quartz. I have also tried a great number of substances, all silicates of magnesium: Enstatite, Olivine, Serpentine, and Meerschaum, without success. Only the Amphibol-Asbestos,  $\text{Mg}_3\text{CaSi}_4\text{O}_{12}$ , in the ordinary mineral form, as well as when specially prepared for chemical purposes, and the Steatite or Soapstone,  $\text{Mg}_3\text{H}_2\text{Si}_4\text{O}_{12}$ , give clear beads before the oxy-hydrogen blowpipe. These, under proper precautions, can be drawn easily to fibers of the desired dimensions. The soapstone is especially easily worked and gives fibers of practically the same elastic properties as those of quartz. Since the method of making them is so simple and requires little time, a short description may be of general interest. The soapstone when heated to a high temperature becomes exceedingly hard, and is used commercially under the name of "lava" for making apparatus intended to be able to stand high temperatures—for example, tips of gas-burners. While it can not be melted before an ordinary blowpipe, it does so before an illuminating gas-oxygen jet and forms a clear bead, usually of a greenish tint, due to the presence of a trace of iron. The original soapstone or the "lava" will do equally well, the most convenient form being small cylindrical sticks. (I obtained such cylinders 3 mm in diameter and 7 cm long, giving colorless transparent beads about 5 mm in diameter, from the Chattanooga Sunlight Lava Manufacturing Company.) If the flame of the blowpipe is too long or the mixture of the gases not well adjusted, the substance will boil violently, while with a small, quiet flame it melts without boiling, a slight development of gases being noticeable only at the upper, cooler part of the bead. After taking the pearl out of the flame the thread is drawn, its thickness depending upon the temperature and rapidity with which it is drawn. Very fine fibers can thus be secured. The fibers should not be heated after being drawn. In a Bunsen burner, for instance, they will immediately become white and break to pieces.

*Elastic fatigue.*—It is very well known that fine quartz fibers show hardly any elastic fatigue,<sup>a</sup> but with thicker fibers of 0.1 to 0.2 mm diameter the effect of elastic fatigue became quite apparent. Careful experiments showed that small twists, or twists of small amplitude, do not affect the zero point, while larger twists, being continued for several minutes, would displace the zero point. The apparatus used allowed a turning of the torsion head, while the lower end of the fiber was held in its original position. After release the zero could be redetermined in a few seconds and the slow disappearance of the effect of elastic fatigue observed. With fibers 70 cm long and 0.1 mm diameter

<sup>a</sup> Boys: Phil. Mag., 23, p. 496, 1887; Threlfall: Phil. Mag., 30, p. 113; 1890.

the displacement, after release from a torsion of  $360^\circ$  lasting five minutes, amounted to as much as 2 in 3,000. Fibers made of steatite showed about the same effect, which is, however, very much smaller than any other substance experimented with, and about one-half to one-third of that shown by steel or phosphor bronze.

For these experiments the quartz and steatite fibers had been silvered at their ends, then coppered, and finally soldered into large brass pins clamped securely in the supports. Thus the elastic fatigue observed can not be attributed to an effect of the material with which fibers were fastened to the other parts of the instrument.

*Tensile strength.*—The tensile strength of very thin quartz fibers is quite large,  $10 \times 10^9$  dynes per  $\text{cm}^2$ , but it decreases appreciably with increasing thickness.<sup>a</sup> My experiments show the same results. From the following table it is seen that the tensile strength of the steatite fibers is at least as large as that of quartz. For comparison the values given by Boys (B) are added. The decrease of tensile strength for the thicker fibers is very striking. For the thickest fibers it is, however, still as large as that of brass.

TABLE I.

| Quartz.     |                                    | Lava.     |                                    |
|-------------|------------------------------------|-----------|------------------------------------|
| Diameter.   | Tensile strength.                  | Diameter  | Tensile strength.                  |
| <i>Cm</i>   | $\frac{\text{Dynes}}{\text{cm}^2}$ | <i>Cm</i> | $\frac{\text{Dynes}}{\text{cm}^2}$ |
| (B) 0.00048 | $11.5 \times 10^9$                 |           |                                    |
| (B) .00175  | 8.0                                |           |                                    |
| .0075       | 6.21                               | 0.006     | $13 \times 10^9$                   |
| .0095       | 6.91                               | .0095     | 9.96                               |
| .010        | 6.16                               | .010      | 9.11                               |
| .015        | 4.5                                | .011      | 7.19                               |
| .0175       | 3.91                               | .0135     | 6.92                               |
| .018        | 4.08                               |           |                                    |

For thick rods the tensile strength is considerably smaller, as shown recently by Schulze,<sup>b</sup> who found a value of  $0.6 \times 10^9$  dynes/ $\text{cm}^2$  for a rod having a cross section of  $0.272 \text{ cm}^2$ .

<sup>a</sup> Boys: Phil. Mag., **30**, p. 116; 1890.

<sup>b</sup> Schulze: Ann. d. Phys., **14**, p. 384; 1904.

*Coefficient of simple rigidity.*—The torsional coefficient was determined by the well-known method of vibration. The diameter and length of the fibers were carefully measured, a cylinder made of Tobin bronze was suspended by the fibers, and the period of the torsional vibrations determined by means of a chronometer. Then

$$n = \frac{8\pi IL}{l^4 T^2}$$

Since it became apparent that different fibers of the same substance gave slightly different values, and since I measured the diameter of the fibers only with an accurate micrometer, I did not take the trouble to correct for the coefficient of expansion.<sup>a</sup>

The mean of a number of experiments gave for quartz  $n = 3.4 \times 10^{11}$ ; for the steatite fiber a value from  $2.6$  to  $4.2 \times 10^{11}$ . Also in this respect the new fiber almost exactly equals fused quartz. The temperature coefficient of the torsional constant of quartz is given by Threlfall as  $+0.000133$ ; by Barnett as  $+0.000115$ ;<sup>b</sup> I obtained  $+0.000149$ . Steatite does not show the remarkable property of becoming more rigid at higher temperatures. Its temperature coefficient is negative,  $\alpha = -0.000193$ . The results obtained are given in Table II and plotted in fig. 1.

TABLE II.

| Quartz.      |         |                         | Lava.        |         |                         |
|--------------|---------|-------------------------|--------------|---------|-------------------------|
| Temperature. | Period. | $n$                     | Temperature. | Period. | $n$                     |
| °            |         |                         | °            |         |                         |
| 8.5          | 8.3083  | $3.4044 \times 10^{11}$ | 11.45        | 8.7507  | $3.9830 \times 10^{11}$ |
| 19.0         | 8.3020  | 3.4096                  | 17.10        | 8.7556  | 3.9777                  |
| 22.45        | 8.2997  | 3.4115                  | 22.05        | 8.7590  | 3.9755                  |
| 26.2         | 8.2973  | 3.4134                  | 26.75        | 8.7625  | 3.9723                  |

<sup>a</sup> See also Threlfall; loc. cit., p. 108.

<sup>b</sup> Barnett: Phys. Rev., 6, p. 119; 1898.

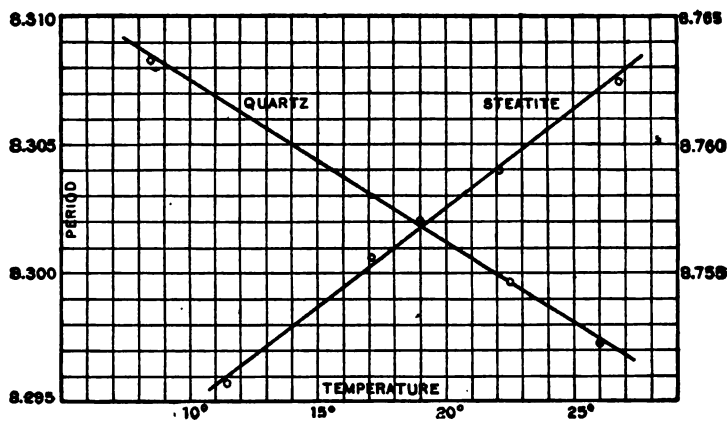


FIG. 1.

*Coefficient of expansion of fused steatite.*—The following experiments were made by Mr. L. G. Hoxton, of the Bureau of Standards, using a rather thick fiber of fused steatite.

The length between two fairly well defined marks 110.4 mm apart was compared at different temperatures with that of the standard nickel-steel decimeter No. 43, belonging to the Bureau and furnished with a certificate from the Bureau International des Poids et Mesures, which gives its linear coefficient as  $\alpha = 0.96 \times 10^{-6}$ . In the following table the differences between the length of the fiber and the standard are marked F-NS. In the third column is given the linear coefficient of expansion, counting from the lowest temperature, and in the last the number of observations made for each temperature.

TABLE III.

| Temperature. | F-NS. | $\alpha$ | Number. |
|--------------|-------|----------|---------|
| °            | $\mu$ |          |         |
| +19.31       | +10.3 | 0.0317   | 5       |
| + 3.56       | 6.4   | 0.058    | 7       |
| - 2.08       | 2.8   | -----    | 5       |
| +21.8        | 12.4  | 0.0365   | 4       |

The average relative coefficient is found from the normal equation

$$1059.4x = 410.1 \times \frac{1}{110400} \\ x = 0.00000350$$

The absolute linear coefficient of expansion is therefore

$$\alpha = 0.0000045.$$

The comparisons were made on a Zeiss horizontal comparator, which allows a direct reading of  $1.0\mu$ .

An attempt to obtain a fiber with a zero temperature coefficient by melting quartz and steatite together was unsuccessful. As soon as too much quartz is present the bead loses its transparency. If beads made of soapstone should not be clear, it is generally due to an excess of quartz in the substance. In such a case the addition of a small amount of magnesia or magnesium carbonate will clear up the bead.

*Index of refraction of fused steatite.*—Two small prisms were ground from steatite beads, and their index of refraction for sodium light determined by the method of minimum deviation. The instrument used was a spectrometer reading to half minutes. Table IV contains the results.

TABLE IV.

|              | Angle of prisms. | Minimum deviation. | $\mu$ |
|--------------|------------------|--------------------|-------|
| Prism 1..... | 60° 3'           | 44° 1'             | 1.576 |
| Prism 2..... | 60° 4'           | 44° 17'            | 1.578 |

Also for these determinations I am indebted to Mr. L. G. Hoxton.

From the above comparison we conclude that we have found in fused steatite a substance which shows all the characteristic properties of fused quartz. The method of drawing fibers from it is very simple, and since the substance is easily secured (an old jet of a gas burner will furnish a large number of fibers), these fibers may, with advantage, be used where economy of time is an important question. One more advantage lies in the easy handling of the fiber. While thick quartz threads break easily when bent, those of steatite may be bent considerably more without breaking.

Barnett<sup>a</sup> working with fibers from 0.005 to 0.007 cm in diameter did not detect any time effect upon the period. But I found a distinct effect with either quartz or steatite fiber. The quartz fibers became

<sup>a</sup> Barnett: loc. cit., p. 116.



more rigid in course of time, a behavior resembling that of metals.<sup>a</sup> Professor Carhart and the writer<sup>b</sup> have found, however, a decrease of rigidity in the case of phosphor bronze. Continued vibrations, especially of very large amplitude, seem to have a distinct influence upon this "settling down" to a steady state, if such can be obtained.

Tests have been made concerning this time change in order to determine whether different specimens of quartz will differ in their behavior—mine contained quite a large amount of strontium—or whether the difficulties are to be attributed simply to the size of the fibers. Fibers were drawn from a stick of quartz glass (Heraeus). These show no time effect if the load is very small in comparison with the tensile strength, but the time effect becomes more and more pronounced the larger the load. After a fiber 0.02 cm in diameter had been vibrated for about a week with a load of 400 grams, the latter was removed and the fiber allowed to rest for a couple of days. It showed a partial return to the original state. It seems that the thickness of quartz fibers can not be increased beyond 0.002 or 0.005 cm without the loss of the desirable properties which make them so well adapted for very fine suspensions.

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<sup>a</sup> Nichols and Franklin: Elements of Physics, I, p. 99.

<sup>b</sup> Carhart and Guthe: Phys. Rev., 9, p. 292; 1899.



## ON THE TEMPERATURE OF THE ARC.

By C. W. WAIDNER and G. K. BURGESS.

The comparison of the methods of extrapolation that must be resorted to in the estimation of extremely high temperatures is of growing importance in establishing a satisfactory tentative scale of temperature which is already required in many scientific and industrial operations.

In a study of the possibilities of the application of optical and radiation methods of pyrometry to the estimation of extremely high temperatures we have been led to compare a number of carefully calibrated optical pyrometers at the "temperature of the arc."

The early attempts to estimate the so-called temperature of the arc, or more precisely, the temperature of the hottest portion of the positive crater, were based on the extrapolation of empirical relations connecting radiation and temperature (Newton, Dulong and Petit, Rosetti, etc.), that were only applicable through very narrow ranges of temperature, and the results to which they have led are now only of historical interest.

The first important measurement was that of Le Chatelier,<sup>a</sup> who determined for a number of bodies the relation between the photometric intensity of the red light emitted and the temperature. The photometric measurements were made with his optical pyrometer, and the temperature measurements with the now well-known Le Chatelier thermocouple (platinum, platinum-rhodium 10 per cent). This empirical relation is based on experiments extending over the range 700° C. to nearly 1800° C. Le Chatelier found by the extrapolation of this relation ( $I = 10^{6.7} T^{-\frac{3210}{T}}$ ) for the temperature of the arc 4400° abs. The red light was obtained by passing the radiation through red glass, which probably lets through the shorter wave lengths at high temperatures, so that the measured intensity would increase more rapidly than the formula would indicate. This would act in the direction of making the result come out too high.

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<sup>a</sup>Le Chatelier: C. R., 114, p. 737; 1892; J. de Phys. (3), 1, p. 185; 1892.

Violle<sup>a</sup> made an estimate of the temperature of the arc by a calorimetric method. A small removable button of carbon on the end of the positive crater was dropped into the calorimeter. The specific heat of carbon was measured and found to obey a linear relation for a considerable range above 1000°, and the relation thus established was assumed to hold as far as the temperature of the arc. The final value found by Violle was given as 3875° abs. Among the great experimental difficulties of this method are those due to loss of heat by the button in falling to the calorimeter and to nonuniformity of heating. Violle investigated these by varying the height of fall and size of the button. In general criticism of this method it should be noted that the specific heat of most substances increases more rapidly as the temperature corresponding to change of state is approached than at low temperatures. Although the difficulties of this method are very great—as admitted by Violle—the result is nevertheless interesting as a determination by a method widely different from those usually employed, based on the extrapolation of relations connecting radiation and temperature.

Wilson and Gray<sup>b</sup> made an estimate of the temperature of the arc by a method in which the radiation from the positive crater falling on one junction of a differential radiomicrometer was balanced by the radiation on the other junction from a known area of incandescent polished platinum strip, whose temperature was measured by its expansion after the principle of the Joly meldometer. The relation between the radiation of polished platinum and of platinum covered with copper oxide was then determined and the assumption made that the radiation from the carbon obeyed the same law as from the copper oxide, which is very nearly the case. Knowing then the apparent areas of the two sources of radiation, and knowing the relation connecting radiation and temperature, they could at once find the temperature to which the platinum strip would have to be raised to balance the arc if the apparent areas were equal. The result to which these experimenters were led was 3600° abs. In discussing this experiment which was admirably carried out it must be remembered that it was done at a time when the laws of radiation were not so well understood as they were a few years later. The method of measuring the temperature made use of in these experiments is capable of considerable precision. Their temperature scale is probably about 20° low at 1000°, inasmuch as the melting point of gold was taken as

<sup>a</sup> Violle: C. R., 95, p. 1273; J. de Phys. (3), 2, p. 545; 1893; C. R., 120, p. 868; 1895.

<sup>b</sup> Wilson and Gray: Proc. Roy. Soc., 58, p. 24; 1895; Phil. Trans. A., 185, p. 361, 1894, for details of apparatus, same as used in estimation of temperature of the sun.

1041° in the calibration of the platinum strip. The radiation of the bare platinum was found to increase approximately as the 4th power of the absolute temperature and of the blackened platinum about as the 3.4th power. This is not in agreement with subsequent researches of Paschen and of Lummer, Pringsheim, and Kurlbaum. The bare platinum was inclosed in a gilt case, which made it approximate more or less to a black body. The strongest criticism of this method is undoubtedly that it is based on the extrapolation of an empirical relation which was studied only through a comparatively narrow range of temperature (650° to 1150°), and the results can not therefore be given so much weight as those obtained by the extrapolation of Planck's law, which has been found to satisfy the results of experiments throughout the range from -200° C. to +1500° C.

Wanner<sup>a</sup> studied experimentally the relation connecting the temperature and the photometric intensity of monochromatic light of different wave lengths emitted by several black bodies of different construction, up to about 1400° absolute, and found that the results were in agreement with Wien's relation for the spectral distribution

of energy for a black body,  $J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$ , as was shown by the linear

relation between  $\log J$  and  $\frac{1}{T}$ . Wanner then applied this method to an estimation of the temperature of the arc, by comparing with a spectrophotometer the intensity of the red ( $\lambda = 0.6563\mu$ ) and green ( $\lambda = 0.5461\mu$ ) radiation with the intensity of the same radiation emitted by a black body (indirectly for convenience through the intermediary of an amylacetate flame whose radiation for these wave lengths had been compared with black body radiation). Using red light, Wanner finds for the temperature of the hottest portion of the positive crater 3720°, and using green light 3700° abs. when cored carbons (Dochtkohle) are used; using retort carbons he finds for measurements with the red and green light 3875° and 3895° abs., respectively. If it can be assumed that the Wien equation continues to hold for such extremely high temperatures, which will be referred to again, this determination of Wanner must be given considerable weight, as the method is capable of precision. It must be noted that inasmuch as this determination is based on the extrapolation of the Wien equation which applies to a black body, the temperature thus found is the "black body temperature," i. e., the temperature that a black body would have to emit light of the same intensity. Barring the presence

<sup>a</sup> Wanner: Ann. d. Phys., 2, p. 141; 1900.

of luminescence, this method therefore gives the lower limit of temperature, so that the true temperature of the positive crater must at least be higher, by an amount depending on how much its radiation differs from black body radiation.

Lummer and Pringsheim<sup>a</sup> made use of the relation  $\lambda_m T = \text{const.}$ , connecting the wave length  $\lambda_m$  having maximum energy and the absolute temperature  $T$  of the radiating body, to estimate the limiting temperatures of a number of incandescent bodies, such as the Nernst filament, the Wellsbach mantle, argand gas flame, and the electric arc. The value of the constant for a black body is 2940, according to their experiments, and 2921 by those of Paschen,<sup>b</sup> a most satisfactory agreement. For the radiation from polished platinum Lummer and Pringsheim have found for the constant 2630. Assuming, then, that the "displacement law" continues to hold, and that the radiation from the crater of the arc is of the same character as from the black body and platinum, and is intermediate between these, which they have shown is very probable, these investigators concluded that the temperature of the positive crater must be between

$$T_{\min.} = \frac{2630}{0.7} = 3750^\circ \text{ and } T_{\max.} = \frac{2940}{0.7} = 4200^\circ \text{ abs.}$$

From the flattened form of the energy curve and the effect of atmospheric absorption, it is difficult to locate the position of the maximum with precision. It must be said that the "displacement law" on which this method is based is one of the best established laws of radiation, both from the theoretical and experimental side.

By an examination of a spectral energy curve of the positive pole of an electric arc, in a paper by Abney and Festing,<sup>c</sup> F. W. Very<sup>d</sup> was led to fix the maximum at about  $0.73 \mu$ . This gives for the upper and lower limits between which the temperature of the positive carbon must lie

$$T_{\max} = \frac{2940}{0.73} = 4025^\circ \text{ abs.}$$

$$T_{\min} = \frac{2630}{0.73} = 3600^\circ \text{ abs.}$$

<sup>a</sup> Lummer and Pringsheim: *Verh. d. Deutsch. Phys. Ges.*, 1, p. 235; 1899; *Verh. d. Deutsch. Phys. Ges.*, 3, p. 36; 1901.

<sup>b</sup> Paschen: *Ann. d. Phys.*, 4, p. 277; 1901.

<sup>c</sup> Abney and Festing: *Proc. Roy. Soc.*, 35, p. 334, Diagram II; 1883.

<sup>d</sup> F. W. Very: *Astrophysical Journal*, 10, p. 208; 1899.

Petavel<sup>a</sup> has found that his observations on platinum radiation satisfy the formula

$$(t-400)=889.6\sqrt[6.9]{b}$$

where  $t$  is degrees centigrade and  $b$  the intrinsic brilliancy per square centimeter measured photometrically. For the crater of the arc he found  $b=11000$  candles per  $\text{cm}^2$ , which would give  $t=3830^\circ\text{C}$  (4100 abs.) assuming that carbon and platinum obey the same radiation law. This assumption gives too high a value for  $t$ . With the constants proper to carbon in the above formula, the method might give good results.

More recently Féry<sup>b</sup> has made estimates of this temperature by two different methods. In one the radiation from the positive carbon was focused by a fluorite lens on the blackened junction of a minute iron-constantan thermocouple which was joined in circuit with a galvanometer. A preliminary calibration of this thermo-electric telescope with the radiation from an electrically heated black body as far as  $1500^\circ\text{C}$  showed that the observed deflections of the galvanometer were in most satisfactory agreement with those calculated from the Stefan-Boltzmann law. By this method Féry was led to the value  $3763^\circ\text{abs.}$  as the "black body temperature" of the arc. In this connection it is of interest to note that Lummer and Kurlbaum<sup>c</sup> have found that while the energy of total radiation from iron oxide, which is, very approximately, the same as from carbon, is only 30 per cent of that from a black body at  $654^\circ\text{abs.}$ , at  $1481^\circ\text{abs.}$  it has already grown to 60 per cent. This would indicate that the "black body temperature of the arc," as found by the energy of total radiation from carbon, would not differ very much from its true temperature, probably by less than  $200^\circ$ .

Using the photometric method and a modified form of the Le Chatelier optical pyrometer and assuming that Wien's law continues to hold, Féry finds  $4140^\circ\text{abs.}$  with red light and  $4170^\circ\text{abs.}$  using green light. As an explanation of this high value, want of monochromatism of the glass used at once suggests itself. The explanation certainly applies for the usual red glasses that are sent out by Pellin with the Le Chatelier optical pyrometer. Moreover, the "center of light" transmission of this glass is nearer to  $0.631\mu$  than  $0.659\mu$ , the value usually assumed from Le Chatelier's early determinations, probably made with a different kind of glass.

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<sup>a</sup> Petavel: Phil. Trans. Roy. Soc., A 191, p. 515; 1898.

<sup>b</sup> Féry: C. R., 134, pp. 977, 1201; 1902.

<sup>c</sup> Lummer and Kurlbaum: Verh. Phys. Ges., Berlin, 17, p. 106; 1896.

## EXPERIMENTAL RESULTS.

The methods we have used are all dependent on the extrapolation of Wien's equation for the distribution of energy in the spectrum of a black body,

$$J=c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

The researches of Lummer and Pringsheim,<sup>a</sup> Paschen,<sup>b</sup> Rubens and Kurlbaum,<sup>c</sup> and Beckmann<sup>d</sup> have shown that this law does not hold for long wave lengths and high temperatures. The results of experiment are better represented by the equation deduced by Planck.<sup>e</sup>

$$J=c_1 \frac{\lambda^{-5}}{e^{\frac{c_2}{\lambda T}}-1}$$

The experiments of Rubens and Kurlbaum have shown that this equation is in most satisfactory agreement with the results of experiments throughout the widest range of measurable temperatures,  $-200^{\circ}$  C. to  $+1,500^{\circ}$  C., and for the longest waves of the infra red portion of the spectrum obtained by multiple reflection from fluorite ( $\lambda=24.0\mu$  and  $31.6\mu$ ) and rock salt ( $\lambda=51.2\mu$ ) surfaces.

However, for the wave lengths of the visible spectrum, and even for the region of the infra red in which the largest part of the total energy of the spectrum is found, Wien's law applies with sufficient precision. Lummer and Pringsheim have shown that for values of  $\lambda T$  less than 3000 Wien's equation represents the results of experiments to an order of accuracy of 1 per cent. For the photometric measurements in the visible spectrum where the wave length does not exceed  $0.65\mu$ , the use of Wien's equation is justified for extrapolation to temperatures exceeding  $4000^{\circ}$  C.

<sup>a</sup> Lummer and Pringsheim: *Verh. d. Deutsch. Phys. Ges.*, 1, pp. 23, 215; 1899; *ibid.* 2, p. 163; 1899; *ibid.* 3, p. 6; 1901. Lummer: *Sur le rayonnement des corps noirs*: *Int. Cong. Rep.*, Paris, 1900; p. 929.

<sup>b</sup> Paschen: *Ann. d. Phys.*, 4, p. 277; 1900.

<sup>c</sup> Rubens and Kurlbaum: *Ber. d. K. Akad. d. Wiss.*, Berlin, 41, p. 929; 1900; *Ann. d. Phys.*, 4, p. 649; 1900.

<sup>d</sup> H. Beckmann: *Inaug.-Dissert.*, Tübingen, 1898.

<sup>e</sup> Planck: *Verh. d. Deutsch. Phys. Ges.*, 2, pp. 202, 237; 1900; *Ann. d. Phys.*, 4, p. 553; 1901.



We have determined the temperature of the brightest portion of the positive crater of the electric arc by means of the three pyrometers devised, respectively, by Le Chatelier, Wanner, and Holborn and Kurlbaum. The three instruments are photometers. In the Wanner a narrow spectral band in the red is used as monochromatic source, while in the other two dependence has to be placed upon colored glasses for monochromatism. In the Holborn-Kurlbaum<sup>a</sup> pyrometer the tip of a very fine incandescent lamp filament is made to disappear against the bright background observed, by varying the current through the lamp. The current is then a function of the temperature which may be found by calibration against a thermocouple.

In the Wanner<sup>b</sup> instrument a polarizing device permits balancing photometrically the intensities of the same spectral hues coming from a standard light and the source observed. The angle of the analyzer may be converted into degrees of temperature by calibration. In the Le Chatelier<sup>c</sup> instrument the light from the source is cut down by a cat's-eye diaphragm until photometric equality is had with light from a comparison source, when from the opening of the cat's-eye the temperature may be calculated after the instrument has been calibrated.

Between such a bright source as the arc and any of these instruments it is necessary, in order to render photometric comparison possible, to interpose absorbing glasses whose absorption factors ( $K$ ) have to be determined for the different wave lengths used.

Wien's law gives:

$$(1) \quad \log_{10} K = \log \frac{J_1}{J_2} = \frac{c_2}{\lambda} \log_{10} e \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

where  $c_2 = 14500$  for a black body, where  $\lambda$  is expressed in  $\mu$ ,  $e$  is the base of Napierian logarithms,  $J_1$  and  $J_2$  are the light intensities expressed in any units corresponding to the temperatures  $T_1$  and  $T_2$ ,  $T_1$  being the absolute black-body temperature of the source and  $T_2$  its apparent temperature when the absorbing glasses are interposed. This formula may be used with any of the above pyrometers to determine the arc temperature when  $K$  is known and the reading of each instrument has been found for some source which can be viewed without the absorption glasses. As a convenient reference temperature, or source of constant intensity, that of the central area of a definite acetylene flame viewed normally was used. The apparent black body

<sup>a</sup> Holborn and Kurlbaum: Ann. d. Phys., 10, p. 225; 1902.

<sup>b</sup> Wanner: Phys. ZS., 3, p. 112; 1902.

<sup>c</sup> Le Chatelier: J. de Phys., (3), 1, p. 185; 1892.

temperature of this source was 1625 abs., the different instruments agreeing to within  $3^{\circ}$  in this determination. Each of the instruments was also calibrated over a considerable temperature range in terms of a thermocouple using a Lummer-Kurlbaum black body as source, thus obtaining a whole series of comparison points. The two methods gave concordant results.

The values of absorption factors ( $K$ ) vary with the wave lengths of the light used,  $K$  decreasing quite rapidly for shorter wave lengths. Thus a given absorbing glass gave  $K=1500$  for  $\lambda=0.651\mu$ , and only 360 for  $\lambda=0.550\mu$ . Also with these glasses the value of  $K$  for  $n$  glasses is not  $K^n$ , as is generally assumed, but something less than this quantity. Four glasses of the same quality and thickness gave, for  $\lambda=0.651\mu$ , the following results, as determined from a large number of observations, using all three pyrometers in this determination:

| Number of glasses..... | 1.   | 2.   | 3.    | 4.     |
|------------------------|------|------|-------|--------|
| Value of $K$ .....     | 11.9 | 137  | 1 500 | 15 500 |
| Increment of $K$ ..... | 11.9 | 11.5 | 10.9  | 10.3   |

The results of our determination of the arc temperature for a current of 15 amperes are given in the three following tables. The carbons were exceptionally homogeneous, prepared especially for optical projections and arcing very readily. The positive carbon (13 mm diameter) was mounted horizontally and the negative (11 mm diameter) vertically. The potential across the arc was about 65 volts. In order to prevent the formation of a deep crater on the under side of the positive carbon, the latter was occasionally trimmed down so that the hottest part of the crater could be readily viewed in a horizontal line.

#### DETERMINATIONS.

##### WITH WANNER PYROMETER.

| Date.        | Wave length used. | $K =$<br>Absorption factor. | Absolute temperature of arc. |
|--------------|-------------------|-----------------------------|------------------------------|
| 1904.        | $\mu$             |                             |                              |
| June 20..... | 0.656             | 1 500                       | 3 680                        |
| 20.....      | .656              | 1 500                       | 3 700                        |
| Mean .....   |                   |                             | 3 680                        |

## DETERMINATIONS—Continued.

## WITH HOLBORN-KURLBAUM PYROMETER.

| Date.        | No. of lamp<br>used in<br>H.-K. py-<br>rometer. | Wave length<br>used. | K=<br>absorption<br>factor. | Absolute<br>temperature<br>of arc. |
|--------------|-------------------------------------------------|----------------------|-----------------------------|------------------------------------|
| 1904.        |                                                 | $\mu$                |                             |                                    |
| June 14..... | 132                                             | 0.651                | 1 500                       | 3 670                              |
| 14.....      | 156                                             | .651                 | 1 500                       | 3 680                              |
| 14.....      | 145                                             | .651                 | 1 500                       | 3 680                              |
| 15.....      | 132                                             | .651                 | 1 500                       | 3 630                              |
| 15.....      | 156                                             | .651                 | 1 500                       | 3 690                              |
| 15.....      | 145                                             | .651                 | 1 500                       | 3 670                              |
| 20.....      | 132                                             | .630                 | 1 350                       | 3 750                              |
| 15.....      | 156                                             | .630                 | 1 350                       | 3 790                              |
| 16.....      | 132                                             | .550                 | 360                         | 3 640                              |
| Mean .....   | .....                                           | .....                | .....                       | 3 690                              |

## WITH LE CHATELIER OPTICAL PYROMETER.

| Date.        | Wave length<br>used. | K =<br>absorption<br>factor. | Absolute<br>temperature<br>of arc. |
|--------------|----------------------|------------------------------|------------------------------------|
| 1904.        | $\mu$                |                              |                                    |
| June 17..... | 0.651                | 15 500                       | 3 740                              |
| 17.....      | .651                 | 15 500                       | 3 750                              |
| 17.....      | .651                 | 15 500                       | 3 750                              |
| 17.....      | .651                 | 15 500                       | 3 690                              |
| 16.....      | .651                 | 1 500                        | 3 730                              |
| 18.....      | .630                 | 14 850                       | 3 680                              |
| 18.....      | .630                 | 1 350                        | 3 730                              |
| Mean .....   | .....                | .....                        | 3 720                              |

Although the three pyrometers give practically an identical value ( $3700^{\circ}$  abs.) for the arc temperature, yet we do not consider all the observations of equal weight. The red glass  $\lambda=0.630\mu$  is not sufficiently homogeneous for good photometric measurements; with the Holborn-Kurlbaum instrument, for instance, the filament of the lamp can not be made to disappear when using this glass. Glass  $\lambda\ 0.651\mu$ , on the other hand, leaves practically nothing to be desired as to its homogeneity, and the results obtained with it have the greatest certainty. The values of the equivalent wave lengths for the red glasses were verified by Dr. Nutting to an accuracy well within the limits of error from other sources.

The relative temperatures obtained with the various instruments are independent of the absorption factors, the latter being dependent upon the absorption glasses, which were the same for all the pyrometers, and upon the wave length used. The absolute values we have obtained for the arc temperatures would evidently be in error if the values of the absorption coefficients are incorrect. The values chosen are the best that could be deduced from several hundred observations made with all the pyrometers. The absorption factor for green light ( $\lambda=0.550\mu$ ) is, however, less well determined.

As to the pyrometers themselves and the reliance to be placed upon the results obtained with the different instruments, the greatest uncertainty seems to be with the Wanner, because it is more difficult with this instrument than with the others to be sure that one is sighting upon the desired spot, as the instrument is not a telescope; also, the extrapolation of the scale of this particular instrument is somewhat uncertain. As between the determinations with the Le Chatelier and Holborn-Kurlbaum instruments, the preference is in favor of the latter, as a smaller area is used in the photometry and there are more independent checks upon its calibration; thus, using three comparison lamps, identical results are obtained.

On the whole, a better agreement among different instruments employing such varied methods of measurement could hardly be expected at such a high temperature. Our experiments with pure graphite have shown that the value,  $3700^{\circ}$  abs., would not be increased by more than  $50^{\circ}$ . The effect of material of carbons and of current density will also be considered.

## VARIATIONS IN TEMPERATURE OF THE ARC.

(a) *With current.*—It is generally accepted, since Violle<sup>a</sup> so stated, that the temperature of the arc is independent of the current, and this temperature is assumed to be the boiling point of carbon. There are, however, at least two reasons for which we should expect this temperature to vary with the current. As the current is increased there will be a tendency to superheat the viscous carbon layer from which the vapor boils, even though this vapor does not have a higher temperature than the normal boiling point, and as it is this viscous layer which is observed, variations of current above a certain limit should be accompanied by changes in temperature. Again, with low current a smaller area is heated, so that it will be more cooled by conduction.

An examination of the observations from which it has been inferred that the arc temperature is independent of current seems to indicate that this conclusion is unwarranted, for no observer has published a series of results sufficiently concordant in themselves or of sufficient number to enable him to state with certainty whether or not the effect exists. This has been due largely to the tediousness or inadequacy of the methods employed.

The constancy of brightness was first announced as probable by Rossetti in 1878, and Violle was the first to state as a result of his own experiments that “the brightness of the positive carbon is rigorously independent of the electric power expended to produce the arc, changing from within the limits 10 amperes at 500 watts to 400 amperes at 34000 watts.” Carbons of 3.5 cm diameter were used for the high currents, but no details of observations are published nor the precision of his spectrophotometric method, which is certainly sensitive, though it would be expected that any effect for very high currents would be neutralized, in part at least, by the larger carbons used. Furthermore, results with carbons of varying size and quality are not comparable. Although Violle showed undoubtedly that with carbons of different sizes adapted to carrying currents of from 10 to 400 amperes, there is no considerable change in temperature, yet it would seem that he did not show conclusively that for a given sized carbon there is no variation of temperature with current for the brightest portion of the positive crater.

Wilson and Gray found that changing from 14 to 25 amperes “the temperature then appeared to be a little higher than with the smaller current. \* \* \* Later experiments \* \* \* indicate an exact

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<sup>a</sup> Violle: J. de Phys. (3), 2, p. 545; 1893.

equality of temperature." They do not give any data, but their published observations on the arc temperature indicate that it would be difficult to detect variations by their method.

Wanner states that "changing the current one and a half times (15 to 22 amperes) remains without influence." Wanner's published results for a given kind of carbon are few in number (four) and vary over  $90^{\circ}$  under the same conditions, so that any differences due to current changes might be masked.

Our experiments on the variation of arc temperature with current were first made with a Holborn-Kurlbaum pyrometer, which is peculiarly well adapted for these measurements, as it is readily sighted, and thus the wandering of the brightest spot on the crater can be easily followed. A very small photometric area is employed, and observations may be taken rapidly within intervals of a few seconds.

In order to secure a good-sized image of the crater, so as to facilitate the photometric measurements, the instrument, provided with auxiliary lenses and suitable absorption glasses, was placed as near as possible (12 to 25 cm) to the arc, which was mounted as previously described. The current from a 120-volt storage battery was varied without the observer being aware of the actual changes, and to render the settings of the pyrometer as unbiased as possible an additional rheostat in its lamp circuit was also worked independently of the observer. No observations were taken on a hissing or humming arc, nor until the arc burned normally after changing current, and the conditions of constant length of arc and constant *P. D.* across the arc were very closely maintained, the latter being measured with a Weston voltmeter; the value of the *P. D.* was about 65 volts. Two observers obtained practically identical results, which are here summarized.

| Date.               | Number of observations. | Current in arc. | Temperature of arc (absolute). |
|---------------------|-------------------------|-----------------|--------------------------------|
|                     |                         | <i>Amperes.</i> | $^{\circ}$                     |
| June 23, 1904 ..... | 51                      | 15              | 3690                           |
|                     | 48                      | 22              | 3727                           |
|                     | 49                      | 30              | 3782                           |
| June 28, 1904 ..... | 23                      | 15              | 3690                           |
|                     | 37                      | 22              | 3741                           |
|                     | 42                      | 30              | 3762                           |

A few observations were also taken with a current of 40 amperes, which indicated that the effect persists with increasing current density to the limit that the auxiliary apparatus would stand.

The precision with which this determination can be made on a single carbon with the Holborn-Kurlbaum instrument is shown by the observations of June 23, 1904, namely, that the probable error of the mean in any one of the three series on that date is less than  $3^{\circ}$ , and no photometric observation with a current of 15 amperes was as high as the mean value for 30 amperes. Whatever may be the best value for the arc temperature, this value is apparently a function of the current when the *P. D.* across the arc is a constant and the arc is burning normally.

If the current is kept constant and the *P. D.* varied, it is necessary to vary simultaneously the length of the arc; therefore, temperature results obtained by this method of varying the power are not directly comparable, as the length of the arc is a variable which is not easily eliminated quantitatively.

It does not seem that our finding a small temperature change with current can be explained on physiological grounds, due to contrast in light intensities entering the eye, since with the smallest current used the image of the hottest part of the crater was many times larger than the image of the filament of the comparison lamp. To further test this point we determined the difference in temperature with a Le Chatelier optical pyrometer provided with auxiliary lens and a very homogeneous red glass before the eyepiece. A number of consistent observations indicate that changing the current from 15 to 30 amperes causes a rise in temperature of about  $70^{\circ}$ . With this instrument the eye does not receive light except through a 1 mm<sup>2</sup> opening, so that the effects of the surrounding field can not influence the eye.

(b) *With the material of carbons.*—Wanner finds a difference of about  $170^{\circ}$  in changing from cored to retort carbon, the former giving a low value, and our experiments also show that the kind of carbon used will influence the apparent temperature of the arc. The homogeneous carbons described above gave us results  $40^{\circ}$  lower than with a very pure graphite furnished by the Acheson Company. This result was obtained both with the Holborn-Kurlbaum and Le Chatelier optical pyrometers.

In commercial carbons the presence of salts seems to facilitate the arcing, and with large currents a steadiness can be had which can not be approached with pure graphite carbons. It appears that the arc temperature, as determined with commercial carbons, is lower than the point obtained for pure carbon.

(c) *Effect on the arc as a standard of light.*—These questions are of interest in connection with the proposed use of the radiation from a definite area of the brightest part of the crater of the + carbon of the electric arc as a standard source of light. If the variations in temperature are as great as indicated in our experiments, a variation of current from 15 to 22 amperes (corresponding to a temperature variation of  $40^{\circ}$  C. in the temperature of the arc) would correspond to a variation in the photometric intensity of several per cent. Even with the same current and the same carbons our results appear to show relative time changes of photometric intensity that seem considerably greater than the experimental errors and which may be due to varying shape of the carbons and changes in current density, as well as variations in the composition of the material. While it would be possible to define the current, it would not be as easy to control the form of the crater and the current density in the region from which the light is to be taken.

Another difficulty in the above method of defining a light standard arises from the variations that may result from the use of different kinds of carbon, as shown above. The experiments of Wanner indicate an enormous change in the intensity of light due to this cause.

Blondel<sup>a</sup> recognized the objections to the use of a screened area of the positive crater as a primary standard of light and recommended its use as a secondary standard for arc photometry. He found the nature of the carbon to be important in determining the absolute brightness (of a square millimeter); thus for five different qualities of homogeneous carbon an average value of 158 candles was found, the extremes being 150 and 163. With cored carbons he obtained values lower than 130 candles for the maximum brightness.

#### CONCLUSION.

A *résumé* of the estimates of the arc temperature that have been discussed in the preceding pages is given in the table on the next page.

From this table it will be seen that the photometric methods based on the extrapolation of Wien's equation show that the "black body temperature of the arc" (pure graphite) is at least  $3750^{\circ}$  abs., so that its true temperature must be higher than this; it is not possible to state how much, in the absence of more definite knowledge concerning the departure of carbon from black body radiation. In the light of the best evidence that is at present available it would seem that the true

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<sup>a</sup> Blondel: Proc. Int. Elec. Congress, Chicago, 1893.



temperature of the hottest part of the positive carbon is between  $3900^{\circ}$  and  $4000^{\circ}$  absolute.

| Observer.              | Absolute temperature of the arc.   | Method.                                                                     |
|------------------------|------------------------------------|-----------------------------------------------------------------------------|
| Le Chatelier .....     | 4370                               | Photometric: Intensity of red light.                                        |
| Violle .....           | 3870                               | Calorimetric: Specific heat of carbon.                                      |
| Wilson and Gray ..     | 3600                               | Total radiation of copper oxide, empirical relation for.                    |
| Wanner .....           | <sup>a</sup> 3700-3900             | (Varying with carbons used). Photometric in terms of Wien's law.            |
| Very .....             | Bet. 3600 and 4000                 | Wave-length of maximum energy, Wien's displacement law: $\lambda_m T = C$ . |
| Lummer and Pringsheim. | Bet. 3750 and 4200                 | Do.                                                                         |
| Féry .....             | <sup>a</sup> 3760                  | Total radiation: Stefan-Boltzmann law.                                      |
|                        | <sup>a</sup> 4150                  | Photometric: Wien's law.                                                    |
| Waldner and Burgess.   | <sup>a</sup> 3690                  | Holborn-Kurlbaum pyrometer.                                                 |
|                        | ( <sup>b</sup> ) <sup>a</sup> 3680 | Wanner pyrometer.                                                           |
|                        | <sup>a</sup> 3720                  | Le Chatelier pyrometer.                                                     |

<sup>a</sup> Black body temperature.

<sup>b</sup> Pure graphite gives a temperature not over  $50^{\circ}$  higher.

If we are justified in assuming that the high values found by Le Chatelier and by Féry, using photometric methods, are satisfactorily explained by the reasons we have advanced, a comparison of the results contained in the above table will show, when we consider the experimental difficulties and uncertainties that may enter due to varying conditions of the arc (such as current strength, length of arc, quality of carbon, etc.), that the results obtained, dependent as they are on the extreme extrapolation of different laws, are in quite satisfactory agreement, and diminish somewhat the doubt which is always present when we must venture beyond the domain of experiment.

The value obtained by Féry for the "black body temperature" of the arc, by making use of the Stefan-Boltzmann law for the total radiation of a black body, is in most satisfactory agreement with the results obtained by Wanner, dependent on the extrapolation of the Wien-Planck relation, and is nearly identical with our own photometric determinations based on the same relation. It is interesting to note in

this connection that Lummer and Pringsheim<sup>a</sup> have found these laws in agreement at 2300° abs.

If we can assume that the Wien-Planck and the Stefan-Boltzmann relations continue to hold good for thermodynamic radiation at these high temperatures, it would seem, in view of the above fact, that the phenomena of luminescence plays no very important part in the radiation from the positive pole of the electric arc; for, if so, it might be expected to lead to higher values in the estimates based on the photometric methods than would be found by methods dependent on total radiation.

An examination of the available estimates of the temperature of the sun, made by methods based on the photometric intensity, seem to indicate that these methods lead to higher values than those based on the energy of total radiation. Thus, using the former method, Le Chatelier<sup>b</sup> finds 7900°<sup>c</sup> abs. The estimates,<sup>d</sup> based on total radiation and on the position of the wave length of maximum energy in the solar spectrum, lead to values in the neighborhood of 5500° to 6000° absolute. If this difference in the methods really exists, it would at least suggest the presence of luminous radiation. It would also be of interest in this connection to use the photometric method with green and blue, in addition to red radiation, to see if the former led to higher values, as might be expected if luminescence played an important rôle.

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<sup>a</sup> Lummer and Pringsheim: *Verh. Deutschen Phys. Ges.* (5) I, p. 3; 1903.

<sup>b</sup> Le Chatelier: *C. R.*, 114, p. 737; 1892.

<sup>c</sup> As Le Chatelier's determination of the arc temperature is probably several hundred degrees high, his value of the effective temperature of the sun should be lowered.

<sup>d</sup> Day and Van Ostrand: *Astrophys. J.* 19, p. 39; 1904.









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S. W. STRATTON, Director

Issued February 1, 1905

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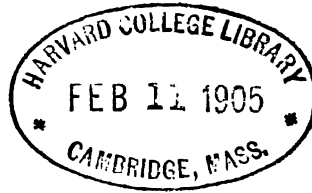
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*The Bureau.*

## THE ABSOLUTE MEASUREMENT OF INDUCTANCE.<sup>a</sup>

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By EDWARD B. ROSA and FREDERICK W. GROVER.

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### 1. METHODS OF MEASURING INDUCTANCE.

Self inductance may be determined in absolute measure (that is, in terms of resistance and time) by the methods of Maxwell, Wien, or Rowland. The first named is complicated and scarcely capable of giving results of high accuracy. The other two methods are probably capable of yielding results of satisfactory accuracy, but so far as we know few results by these methods have been published, and none of a degree of accuracy equal to the results which have been obtained in the absolute measurement of a capacity.

The most obvious method of directly determining the inductance of a coil, originally proposed by Joubert, consists in first determining the impedance of the coil and then calculating the inductance, after having found the ohmic resistance of the wire and the frequency of the current employed.

Brew<sup>b</sup> has given some determinations of inductance by this method, using a Cardew voltmeter, first in series with the inductive coil and second with the coil cut out. Knowing the resistance of the coil and of the instrument, and the frequency of the current, the inductance is calculated. The results on a single coil are given; they show considerable variations, as would be expected. Nothing is said of the wave form, although the formula employed presupposes a sine wave.

Several variations of this method are described by Gray<sup>c</sup> and Fleming.<sup>d</sup> According to Gray, a noninductive resistance is placed in series with the coil whose inductance is to be measured, and an alternating current passed through both. By means of an electrometer,

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<sup>a</sup> A paper presented at the International Electrical Congress, St. Louis, 1904.

<sup>b</sup> *Electrician*: 25, p. 206; 1890.

<sup>c</sup> Absolute measurements: II, pt. II, p. 488.

<sup>d</sup> *Handbook for the Electrical Laboratory*: Vol II, p. 205.

the differences of potential at the terminals of the noninductive resistance  $R_2$ , and of the inductive coil (resistance  $R_1$  and inductance  $L$ ) are measured. The inductance is then given by the expression

$$L = \frac{R_1}{p} \left( \frac{R_2^2 V_1^2}{R_1^2 V_2^2} - 1 \right)^{\frac{1}{2}}$$

$p$  being  $2\pi$  times the frequency of the current employed, which is to be as nearly simply harmonic as possible.

According to Fleming, we "first send through the coil a continuous current and observe the potential difference of the ends of the coil with an electrostatic voltmeter, and measure the current flowing through it. Then repeat the experiment, using the alternating e. m. f. The ammeter should be a Kelvin balance, or dynamometer, or hot wire ammeter, suitable for both continuous and alternating currents. Adjust the voltage so that the current is the same in both cases. Then if  $A$  is this current, and if  $V$  is the volt-fall down the coil with continuous current, and  $V'$  that with the alternating current, and if  $R$  is the resistance and  $L$  the inductance of the coil, we have

$$A = \frac{V}{R}$$

$$A' = \frac{V'}{\sqrt{R^2 + p^2 L^2}}$$

where  $p = 2\pi$  times the frequency of the alternating current."

Therefore,

$$L = \frac{R}{p} \sqrt{\frac{V'^2 - V^2}{V^2}}$$

or the volt-drop may be kept constant and the current measured in each case. Then

$$L = \frac{R}{p} \sqrt{\frac{A^2 - A'^2}{A'^2}}$$

If the current is not of sine wave form, a correction must be applied.

## 2. THE METHOD OF THIS PAPER.

It occurred to us that a modification of the method quoted above from Gray would be as well adapted to precision measurements as any other proposed. Instead of using the electrometer to measure the difference of potential at the terminals of the inductive coil and of a fixed resistance  $R$ , we vary the resistance  $R$  until the difference of potential at its terminals is equal to that at the terminals of the

inductive coil, as shown by an electrometer. Then, since the alternating current  $I$  is the same in both, and a sine wave form is assumed,

$$IR = I\sqrt{r^2 + p^2 L^2},$$

$r$  being the ohmic resistance of the inductive coil. Then

$$L = \frac{1}{p} \sqrt{R^2 - r^2}. \quad (\text{See fig. 1.})$$

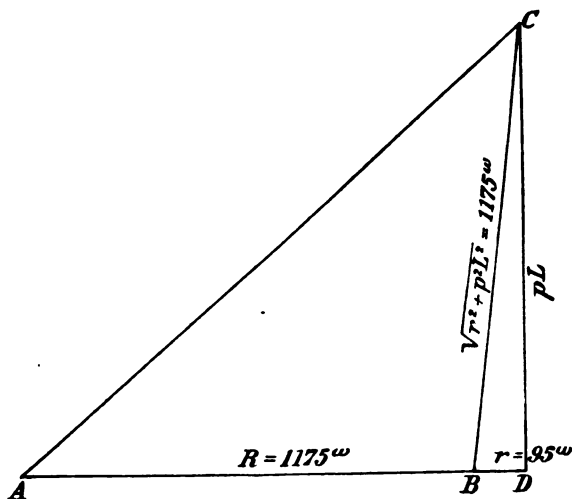


FIG. 1.— $AB$ =noninductive resistance;  $BCD$ =impedance triangle for inductive coil.

This is an extremely simple formula, in which only two quantities, the resistance and the frequency, have to be determined accurately. The resistance  $r$  is usually so small that an approximate value for it is sufficient. In the simplicity and directness of the method and the small number of quantities to be determined lie the advantages of this over other methods.

The chief objection to this method is that it is necessary to have a perfect sine current, or to know the exact form of the current wave in order to calculate the correction due to any harmonics that may be present. So far as we know no accurate determination of inductance by this method has ever been published, and probably because of this requirement. Most alternating current generators yield currents having harmonics of considerable magnitude, and the wave form of course varies according to the load. It is necessary, therefore, to determine the wave form of the particular current used in the experiment in order to obtain the proper correction factor.

The Bureau of Standards possesses an alternating generator designed especially for testing purposes, which has a smooth core armature and pole pieces so shaped as to give a nearly sine wave. The correction to be applied to the measured inductance due to a small departure from a sine form is correspondingly small, and we therefore believed that by using this machine it would be possible to measure inductance in this manner with a high order of accuracy, provided, of course, that small harmonics in the current were carefully determined and allowed for.

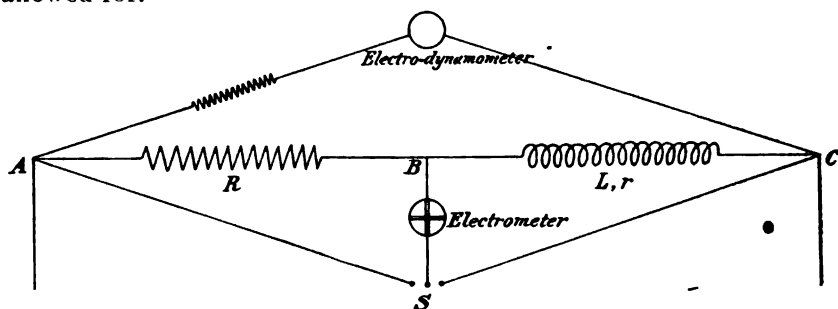


FIG. 2.—Diagram of connections of apparatus.

The method is illustrated in fig. 2. The noninductive resistance  $R$  (being an oil-immersed resistance of manganin of relatively large carrying capacity) was placed in series with an inductive coil having an inductance of about 1 henry and a resistance of about 95 ohms. An alternating current from the generator passes from  $A$  to  $C$  through these resistances, in parallel with which is an electro-dynamometer  $D$ , which serves as a very sensitive voltmeter. An electrometer is joined to the point  $B$  and to a switch  $S$ , by means of which it can be connected to  $A B$  and to  $B C$  successively. By means of the rheostats  $R_2$  and  $R_3$  (fig. 7), the first in series with the main current and the second in the field circuit of the alternator, the electromotive force at the terminals  $A C$  is kept constant. The resistance  $R$  is now varied until the deflection of the electrometer is the same on  $A B$  as on  $B C$ . Then

$$R = \sqrt{r^2 + p^2 L^2}$$

The frequency of the current is determined by means of a chronometer and chronograph. An electric contact being made for every 50 revolutions of the dynamo, a record is secured on the chronograph along with the second beats of the chronometer. Thus the frequency may be determined to a very high order of accuracy, provided the speed is maintained sufficiently constant. To do this a rotating commutator is directly connected to the alternator, and a Wheatstone bridge

and condenser are joined up exactly as in the absolute measurement of capacity. When the bridge has been balanced at the desired speed an assistant maintains the speed constant by means of the carbon rheostat  $R_3$ , the criterion of constant speed being that the galvanometer continues to give zero deflection. That this condition of constant speed is sufficiently met will appear in the following pages.

The requirement of constant voltage is satisfied by having the alternator directly coupled to its driving motor, running the latter from a storage battery having a very constant electromotive force, keeping the bearings and brushes in good order, and regulating the exciting current of the alternator by means of the manual adjustment of a carbon rheostat, so that the deflection of the volt-dynamometer  $D$  is maintained constant. The latter instrument, which gave a deflection proportional to the square of the voltage, was so arranged that a variation of 1 volt in 140 gave a change in the reading of about 25 mm. The readings of the electrometer were only made when the dynamometer deflection was within 0.2 mm. of the selected mark, hence variations in the electromotive force on  $A C$  were less than one part in ten thousand for all the separate readings. A differential electrometer might be used, and so obviate the necessity of maintaining so constant a voltage. We have designed such an instrument and it has been constructed, but we have not yet had an opportunity to use it in this work.

### 3. POSSIBLE SOURCES OF ERROR.

The current flowing into the electrometer would cause an error in the result if the electrometer had a sufficiently large capacity, since it shunts first  $A B$  and then  $B C$ . This effect would be very small when the electrometer shunts  $A B$ , since its current would differ in phase by 90° from that in  $A B$ ; but when in shunt with  $B C$  the electrometer current is nearly opposite in phase to the main current. The capacity of the electrometer is, however, so small that the current flowing into it from  $B$  is wholly inappreciable in comparison with the main current through  $A B C$ , which was usually nearly 0.1 ampere.

Any small inductance or capacity in the resistance  $R$  will produce no error, for its effect if present would be to slightly alter the phase of the electromotive force, but could not alter the impedance to be measured to an appreciable extent. Thus if  $R$  is 1,175 ohms, and we suppose that there is sufficient inductance present to make the impedance a hundredth of one per cent greater, or 1,175.12, this would require at a frequency of 180 cycles an inductance of 15 millihenrys, or a capacity of about 0.01 microfarad. A careful measurement shows

that the capacity effect of this resistance exceeds the inductance effect, but that the resultant effect is equivalent to a capacity of about 0.001 microfarad, and hence the error produced is wholly inappreciable.

The heating of the inductive coil causes a change in its resistance; in fact, its resistance serves as an excellent indication of its mean temperature. Any such change in the resistance, however, produces a very much smaller change in the impedance of the coil. Thus, if  $r$  changes from 95 to 95.5 ohms, the impedance will change only from, say 1,175 to 1,175.04. As the resistance  $r$  is determined at frequent intervals during a series of measurements, the uncertainty of the impedance due to uncertainty in this resistance need never be as much as one part in fifty thousand. The change in the inductance due directly to changing temperature is, however, appreciable and needs to be taken into account very carefully. Foucault currents in the wire of the coil may cause the inductance to vary appreciably with the frequency, when the wire is relatively coarse and the frequency is relatively high. To avoid any error due to this cause, the wire should be fine, or stranded if a low resistance is desired, and high frequencies avoided in the measurements.

The electrostatic capacity of the coil is also a source of error when the frequency is relatively high. Dolazalek<sup>a</sup> has shown that at a frequency of 2,500 per second the measured value of the inductance of a coil may be 3 or 4 per cent greater than its true value, due to this cause. The error is, however, proportional to the square of the frequency, so that at a frequency of 180 per second it amounts to only one or two parts in ten thousand, and, by properly designing the coil, we believe that this correction could be reduced to perhaps five parts in a hundred thousand and its value determined experimentally with a fair degree of accuracy. This experimentally determined correction would include also any effect due to eddy currents, which effect is of opposite sign to the effect due to capacity. Thus no correction need be applied to the result derived from the simple formula except those due to the electrostatic capacity of the coil and the wave form of the current. The former we have not attempted to determine experimentally; the latter we now proceed to ascertain.

#### 4. CORRECTION FOR WAVE FORM.

The correction factor to be applied when using a single coil of negligible resistance has been given by H. F. Weber.<sup>b</sup> Calling the har-

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<sup>a</sup>Dolazalek: *Ann. der Phys.* 1903, p. 1142.

<sup>b</sup>Wied. *Ann.*, 63, p. 366; 1897.



monic components of the electromotive force at the terminals of the coil  $E_1$ ,  $E_3$ ,  $E_5$ , etc., the correction factor for this case is

$$f = \sqrt{\frac{E_1^2 + \frac{1}{9} E_3^2 + \frac{1}{25} E_5^2 + \text{etc.}}{E_1^2 + E_3^2 + E_5^2 + \text{etc.}}}$$

This expression is, however, not applicable to the present case.

To find the correction due to the wave form where the resistance  $r$  is not negligible and where a resistance  $R$  is joined in series with the inductive coil, we remember that the square of the effective value of an alternating current is given by the following expression:

$$I^2 = I_1^2 + I_3^2 + I_5^2 + \text{etc.}, \text{ where } I_1, I_3, I_5, \text{ etc.},$$

are the values of the components of the current of which the relative frequencies are 1, 3, 5, etc.

The electromotive force  $E_a$  on  $A B$  is made equal in the experiment to the electromotive force  $E_b$  on  $B C$ . Therefore,

$$E_a^2 = E_b^2 \quad (1)$$

$$E_a^2 = R^2 (I_1^2 + I_3^2 + I_5^2 + \text{etc.}) \quad (2)$$

$$E_b^2 = E_1^2 + E_3^2 + E_5^2 + \text{etc.} = I_1^2(r^2 + p^2 L^2) + I_3^2(r^2 + 9 p^2 L^2) + I_5^2(r^2 + 25 p^2 L^2) + \text{etc.}$$

$$= r^2(I_1^2 + I_3^2 + I_5^2 + \text{etc.}) + p^2 L^2(I_1^2 + 9 I_3^2 + 25 I_5^2 + \text{etc.})$$

where  $p = 2\pi n$ , and  $n$  is the frequency of the fundamental.

Therefore, from (1),

$$(R^2 - r^2)(I_1^2 + I_3^2 + I_5^2 + \text{etc.}) = p^2 L^2(I_1^2 + 9 I_3^2 + 25 I_5^2 + \text{etc.})$$

Therefore,

$$p^2 L^2 = (R^2 - r^2) \left\{ \frac{I_1^2 + I_3^2 + I_5^2 + \text{etc.}}{I_1^2 + 9 I_3^2 + 25 I_5^2 + \text{etc.}} \right\}$$

$$\text{or, } L = \frac{1}{p} \sqrt{R^2 - r^2} \sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \text{etc.}}{I_1^2 + 9 I_3^2 + 25 I_5^2 + \text{etc.}}}$$

$$\text{or, } L = \frac{f}{p} \sqrt{R^2 - r^2}$$

$$\text{where } f = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \text{etc.}}{I_1^2 + 9 I_3^2 + 25 I_5^2 + \text{etc.}}}$$

and is the correction factor sought.

It will be seen from this expression for  $f$  that the presence of higher harmonics in the current causes the correction factor to depart from unity much more rapidly than the lower harmonics.

For example, suppose that the equation of the current is

$$I = I_1 \sin (pt - \phi_1) + I_3 \sin (3pt - \phi_3) + I_5 \sin (5pt - \phi_5) \\ + I_7 \sin (7pt - \phi_7) + I_9 \sin (9pt - \phi_9),$$

and that  $I_1 = 100$

$$I_3 = 2$$

$$I_5 = 2$$

$$I_7 = 1$$

$$I_9 = 1$$

$$\text{Then } f = \sqrt{\frac{100^2 + 2^2 + 2^2 + 1^2 + 1^2}{100^2 + 9 \times 2^2 + 25 \times 2^2 + 49 \times 1^2 + 81 \times 1^2}} = \sqrt{\frac{10010}{10266}} = .9875.$$

Thus the correction amounts to 1.25 per cent.

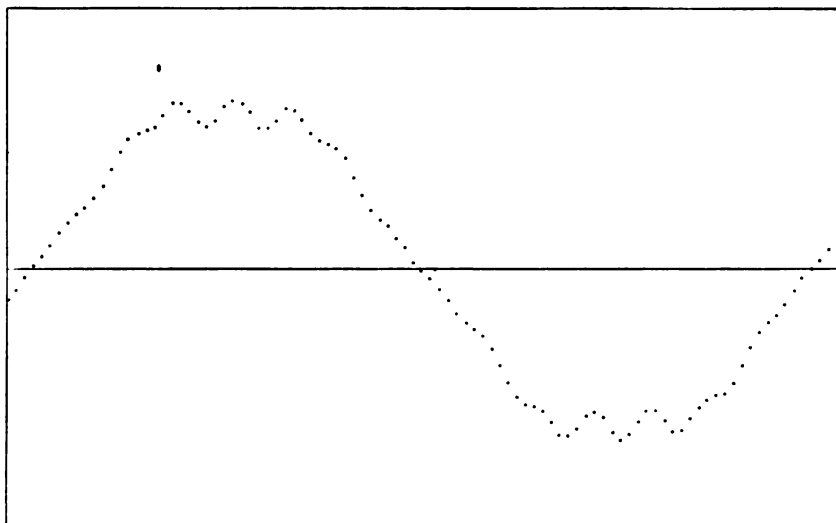


FIG. 3.—Curve of electromotive force of a Westinghouse alternator.

If the harmonics had been more pronounced, and  $I_7$  and  $I_9$  were each 5 instead of 1, then would

$$f = \sqrt{\frac{100^2 + 2^2 + 2^2 + 5^2 + 5^2}{100^2 + 9 \times 2^2 + 25 \times 2^2 + 49 \times 5^2 + 81 \times 5^2}} = \sqrt{\frac{10058}{13386}} = .8668.$$

The correction thus amounts to 13.32 per cent.

Professor Weber found the correction for a certain Ganz alternator with a greatly distorted wave to amount to 6.8 per cent.

The electromotive force shown in fig. 3, due to a small Westinghouse alternator with slotted armature, has the following equation:

$$E = 30.66 \sin(x - 0^\circ 56') + .80 \sin(3x - 6^\circ 17') - 1.65 \sin(5x - 14^\circ 22') + .40 \sin(9x + 7^\circ 50') + 1.11 \sin(13x - 30^\circ 45') - 1.18 \sin(15x - 38^\circ 40').$$

The third, seventh, ninth, and eleventh harmonics are very small; the fifth, thirteenth, and fifteenth are relatively large. The correction factor for this curve is .9933, differing from unity by only .0067, and yet this is a larger correction than is desirable for precision measurements of inductance.

#### 5. THE CURVES OF ELECTROMOTIVE FORCE AND CURRENT.

The electromotive force given by the alternator in our experiments and the current flowing through the circuit *A B C* under this impressed electromotive force are shown in the curves of fig. 5. These curves were drawn under the same conditions as those of the experiment. The third harmonic of the electromotive force curve is in such phase as to slightly flatten the curve; this component being smaller in the current curve, the latter is less flattened and more nearly a sine curve.

Although these curves were drawn with great care and are reasonably accurate, the analysis of the current curve did not give as good results as were desired. This was because the harmonics are so small that very slight errors in the curves produce relatively large errors in the harmonics.

All the odd harmonics were looked for up to the fifteenth, and small values found in every case; above the seventh their average value was not more than a thousandth part of the fundamental, and yet the values obtained from different sets of ordinates of the same curve varied considerably. This suggested that these harmonics were really absent in the curve, and that the inevitable errors in drawing the curves gave rise to small residuals. The actual values of the small upper harmonics in question was on the average only one-twentieth of a millimeter measured on the plotted curve. Nevertheless, they made an appreciable error in the value of the correction factor sought, and hence it was desirable to eliminate all such residual errors.

The harmonics of the current are readily computed from those of the electromotive force which produces the current, and since in an inductive circuit the latter are larger than the former, the errors of the curve and of the analysis are divided down in the calculation and a more accurate result may therefore be obtained.

Thus, for the third harmonic the impedance is

$$\sqrt{(R+r)^2 + 9p^2L^2} \text{ instead of } \sqrt{(R+r)^2 + p^2L^2}$$

and, in a particular case, its numerical value is as follows:

$$\begin{aligned} r &= 97.2 \\ R &= 1176.1 \\ L &= 1.0017 \\ p &= 2\pi n = 2\pi \times 186.23 = 1172.1. \end{aligned}$$

Therefore,

$$Imp_1 = \sqrt{(1273.3)^2 + (1172.1)^2} = 1730.6.$$

$$Imp_3 = \sqrt{(1273.3)^2 + 9(1172.1)^2} = 3739.7.$$

$$Imp_5 = \sqrt{(1273.3)^2 + 25(1172.1)^2} = 5997.2.$$

$$Imp_7 = \sqrt{(1273.3)^2 + 49(1172.1)^2} = 8302.8.$$

$$\text{Hence, } \frac{Imp_3}{Imp_1} = 2.161. \quad \frac{Imp_5}{Imp_1} = 3.465. \quad \frac{Imp_7}{Imp_1} = 4.798.$$

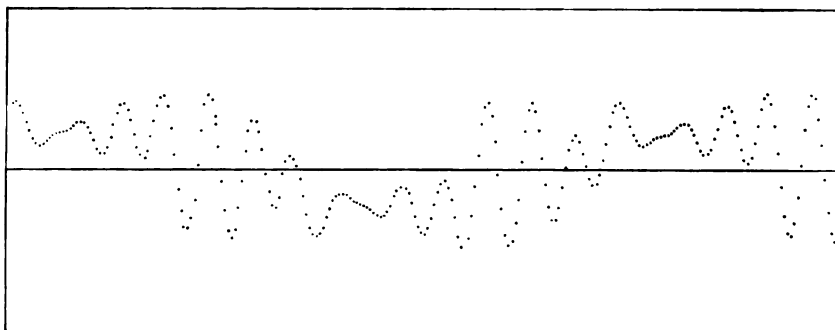


FIG. 4.—Current in condenser due to electromotive force of fig. 3.

These quotients show how much smaller relatively the third, fifth, and seventh harmonics are in the inductive current curve than in the electromotive force curve. Therefore the third, fifth, and seventh harmonics in the current curve may be found by dividing the corresponding harmonics in the electromotive force curve by the numbers 2.16, 3.46, 4.80, respectively. Small errors due to the curve itself or to the analysis are thus divided by these factors, and the values of the harmonics found are therefore more accurate and more consistent than if determined directly by analyzing the current curve.

This process may indeed be carried another step by placing a condenser in parallel with the circuit  $ABC$ , so that the alternating current flowing into it will be due to the same electromotive force that causes

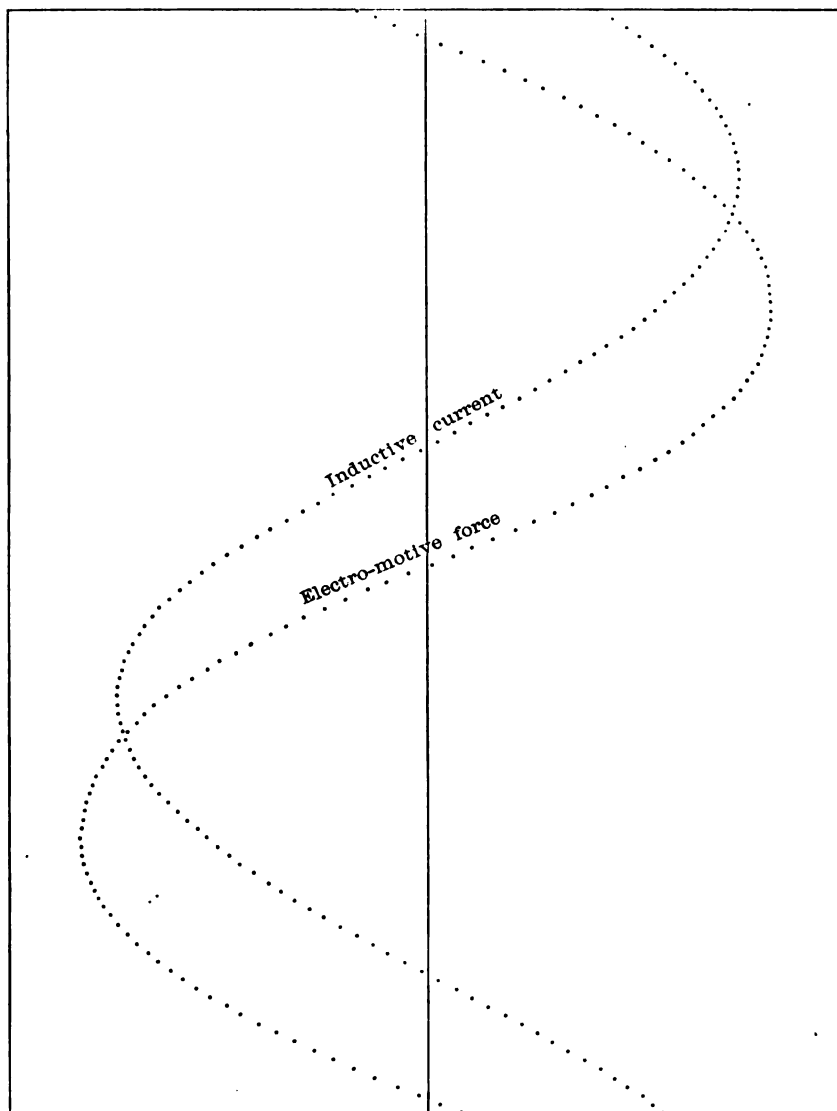


FIG. 5.

the current in the circuit  $ABC$ . In the condenser current, however, the harmonics are magnified in proportion to their order, the seventh harmonic in the condenser current being seven times as great, in pro-

portion to the fundamental, as in the electromotive force, and hence  $7 \times 4.798$  or 33.58 times as great as in the current through  $A B C$ .

Fig. 4 shows the current in a condenser due to such an electromotive

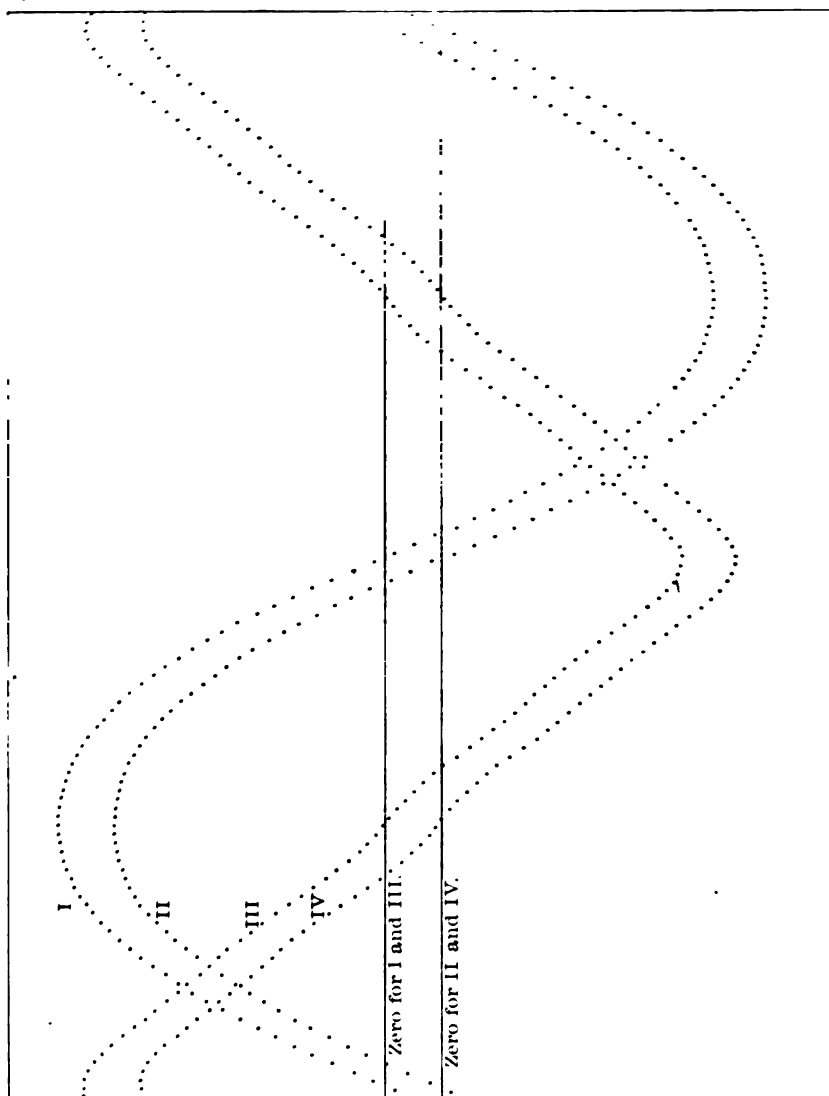


FIG. 6.

force as that of fig. 3, showing the prominence of the higher harmonics, especially those of thirteen and fifteen times the fundamental. In a similar manner we should expect that if there are any small harmonics

of these frequencies in the current through the circuit  $A B C$ , they would be multiplied by 114 and 153, respectively, in such a condenser current in parallel with  $A B C$ . On the other hand, if no such harmonics are found in the condenser current we may be sure that the small values found by analysis in the inductive current were, as we supposed, residual errors.

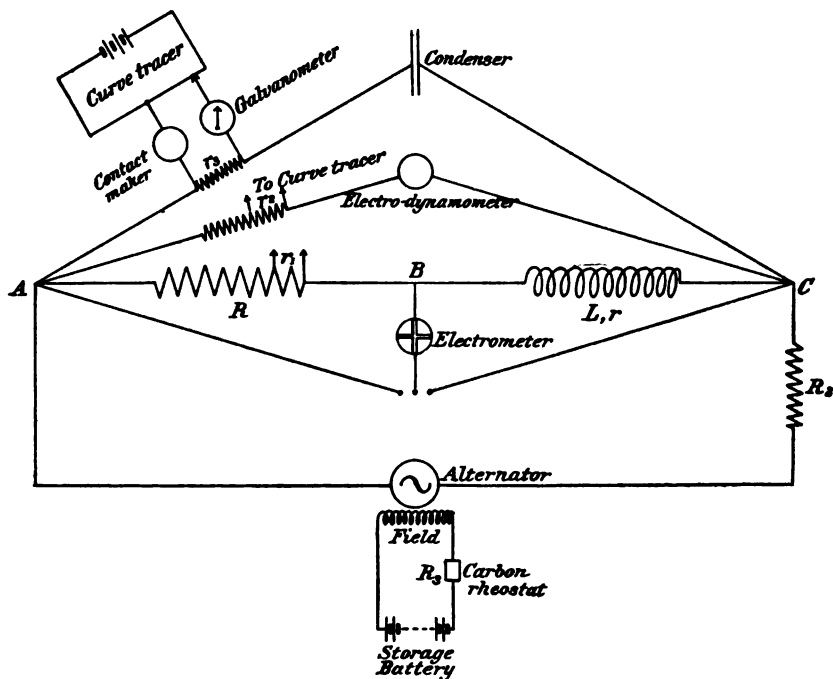


FIG. 7.—Diagram of connections for drawing curves of current and electromotive force.

Fig. 6 gives two curves  $I$  and  $II$  of e. m. f., and two curves  $III$  and  $IV$  of the condenser current. They were drawn under the same circumstances as prevailed during the measurement of the inductance, to be described later. The method of drawing the curves is illustrated in fig. 7. The curve tracer is connected to the terminals of the small noninductive resistances  $r_1, r_2, r_3$ , successively, these terminals being joined to the potentiometer of the curve tracer through the galvanometer on the one side and the contact maker on the other. Evidently a curve drawn with the connecting wires at the terminals of  $r_1$  will represent the inductive current through  $A B C$ ; similarly joining to  $r_2$  gives the e. m. f. on  $A C$ , and joining to  $r_3$  gives the

condenser current. The curves of fig. 6, which are far from being sine curves, but which evidently have no appreciable high harmonics, show the magnification of the harmonics in the condenser current. Since these condenser currents can be drawn with the same precision as the electromotive force and current through the inductive circuit, it is evident that greater accuracy will be secured by analyzing the condenser current curves and calculating the correction factor  $f$  therefrom than by using the curves of the inductive current. However, both pairs of curves, the e. m. f. and condenser current, have been analyzed and the correction factor  $f$  computed from both.

#### 6. THE ANALYSIS OF THE CURVES.

The curves were drawn on the Rosa curve tracer, which is essentially an alternating current potentiometer arranged with printing cylinder for automatically recording the curve. The spiral wire forming the potentiometer is not quite uniform in resistance; the cross-section paper on which the curves were printed was not ruled with as great accuracy as is necessary for the most precise work, and slight variations in the printed record were due to the backlash of the traveling carriage on the potentiometer. To eliminate all these errors at once the paper was replaced on the record cylinder of the curve tracer and the printing point was re-set in the dots (one at a time) of the record. At the same time an ordinary direct current potentiometer was joined to the center and traveling contact of the potentiometer wire of the curve tracer, a constant e. m. f. being maintained at the ends of the latter. Thus the values of the ordinates of the curves were measured, not from the paper record directly, but electrically by the direct current potentiometer, and the most important errors in the curves were thereby eliminated. These errors were, of course, all relatively small, but important enough to be avoided in the present work.

The curves of fig. 6 were drawn after those of fig. 5, and in virtue of a better adjustment of the brushes of the alternator the current was somewhat steadier and the curves a little more accurate. Hence the results of the analysis of the curves of fig. 6 will be given. The alternator has 12 poles, thus giving 6 complete periods in one revolution. The contact maker has 3 contact points  $120^\circ$  apart, and thus makes a contact on every other wave. The first curve of each pair of fig. 4 was drawn while the contact maker turned, step by step, through  $60^\circ$ , and the second curve of each pair was drawn while the contact maker turned through another  $60^\circ$ . Thus one curve may be regarded as a composite made up of every other wave of the train superposed,



TABLE I.—*Showing the values of the three sets of ordinates of fifteen each (at angular intervals of 12°) for each of the four curves analyzed.*

[The values of the ordinates are expressed in arbitrary units.]

| Curve I. Electromotive force. |          |          | Curve II. Electromotive force. |          |          |
|-------------------------------|----------|----------|--------------------------------|----------|----------|
| <i>a</i>                      | <i>b</i> | <i>c</i> | <i>a</i>                       | <i>b</i> | <i>c</i> |
| — 4.72                        | — 1.90   | + 0.96   | + 1.30                         | + 4.28   | + 7.05   |
| + 6.61                        | + 9.51   | 12.17    | 12.41                          | 14.96    | 17.56    |
| 17.05                         | 19.23    | 21.76    | 22.12                          | 24.16    | 26.12    |
| 26.13                         | 27.95    | 29.96    | 29.90                          | 31.63    | 33.28    |
| 33.38                         | 34.82    | 36.43    | 36.32                          | 37.63    | 38.86    |
| 39.02                         | 40.09    | 41.07    | 40.96                          | 41.82    | 42.57    |
| 42.64                         | 43.36    | 44.01    | 43.70                          | 44.18    | 44.44    |
| 44.50                         | 44.77    | 44.92    | 44.70                          | 44.74    | 44.72    |
| 44.96                         | 44.94    | 44.76    | 44.56                          | 44.22    | 43.88    |
| 44.12                         | 43.70    | 43.09    | 42.92                          | 42.43    | 41.55    |
| 41.86                         | 41.02    | 39.99    | 39.53                          | 38.57    | 37.49    |
| 37.75                         | 36.46    | 35.08    | 34.79                          | 33.06    | 31.50    |
| 31.98                         | 30.18    | 28.36    | 28.10                          | 26.13    | 23.92    |
| 24.60                         | 22.56    | 20.45    | 19.87                          | 17.47    | 14.94    |
| 15.27                         | 12.90    | 10.58    | 9.88                           | 7.12     | 4.52     |

| Curve III. Condenser current. |          |          | Curve IV. Condenser current. |          |          |
|-------------------------------|----------|----------|------------------------------|----------|----------|
| <i>a</i>                      | <i>b</i> | <i>c</i> | <i>a</i>                     | <i>b</i> | <i>c</i> |
| +49.26                        | +49.49   | +49.11   | +49.06                       | +49.26   | +48.87   |
| 46.62                         | 45.03    | 43.46    | 46.60                        | 45.15    | 43.41    |
| 39.63                         | 37.91    | 36.21    | 39.85                        | 37.70    | 36.04    |
| 32.46                         | 30.96    | 29.20    | 32.61                        | 31.00    | 29.14    |
| 25.88                         | 24.11    | 22.02    | 25.86                        | 24.22    | 22.42    |
| 17.80                         | 16.10    | 13.64    | 17.96                        | 16.06    | 13.86    |
| 9.46                          | 7.37     | 5.54     | 9.69                         | 7.78     | 5.98     |
| + 2.54                        | + 1.21   | + 0.11   | + 2.98                       | + 1.75   | + 0.67   |
| — 2.24                        | — 3.44   | — 5.08   | — 1.76                       | — 3.03   | — 4.76   |
| — 8.49                        | —10.31   | —12.54   | — 8.08                       | — 9.83   | —11.63   |
| —16.49                        | —18.30   | —20.18   | —16.10                       | —17.99   | —19.73   |
| —23.68                        | —25.28   | —27.08   | —23.33                       | —24.92   | —26.67   |
| —30.46                        | —32.24   | —33.88   | —30.16                       | —32.20   | —33.95   |
| —38.01                        | —39.86   | —41.87   | —37.50                       | —39.53   | —41.33   |
| —45.70                        | —46.94   | —48.25   | —45.08                       | —46.64   | —47.68   |

TABLE II.—*Results of analyses of the two e. m. f. curves (I and II) of fig. 6, and calculation of components of current in the inductive circuit.*[The values of  $E$  and  $I$  are expressed in arbitrary units.]

| Components of electromotive force found by analysis.                                                       |            |       |       |       |          |          |
|------------------------------------------------------------------------------------------------------------|------------|-------|-------|-------|----------|----------|
| Fundamental= $E_1$                                                                                         | Harmonics. |       |       |       |          |          |
|                                                                                                            | $E_3$      | $E_5$ | $E_7$ | $E_9$ | $E_{11}$ | $E_{13}$ |
| Curve I $\left\{ \begin{array}{l} a=46.880 \dots \\ b=46.838 \dots \\ c=46.882 \dots \end{array} \right.$  | 1.898      | 0.122 | 0.127 | 0.012 | 0.021    | 0.041    |
|                                                                                                            | 1.840      | 0.186 | 0.200 | 0.051 | 0.024    | 0.075    |
|                                                                                                            | 1.939      | 0.242 | 0.222 | 0.082 | 0.062    | 0.046    |
| Curve II $\left\{ \begin{array}{l} a=46.620 \dots \\ b=46.608 \dots \\ c=46.601 \dots \end{array} \right.$ | 1.889      | 0.223 | 0.154 | 0.066 | 0.022    | 0.055    |
|                                                                                                            | 1.833      | 0.206 | 0.220 | 0.036 | 0.031    | 0.031    |
|                                                                                                            | 1.833      | 0.185 | 0.241 | 0.075 | 0.041    | 0.009    |
| Divisors=1                                                                                                 | 2.16       | 3.46  | 4.80  | 6.14  | 7.48     | 8.84     |
| Components of inductive current calculated from above values.                                              |            |       |       |       |          |          |
| Fundamental= $I_1$                                                                                         | Harmonics. |       |       |       |          |          |
|                                                                                                            | $I_3$      | $I_5$ | $I_7$ | $I_9$ | $I_{11}$ | $I_{13}$ |
| Curve I $\left\{ \begin{array}{l} a=46.880 \dots \\ b=46.838 \dots \\ c=46.882 \dots \end{array} \right.$  | 0.878      | 0.035 | 0.027 | 0.002 | 0.003    | 0.005    |
|                                                                                                            | 0.851      | 0.054 | 0.042 | 0.008 | 0.003    | 0.008    |
|                                                                                                            | 0.897      | 0.070 | 0.046 | 0.013 | 0.008    | 0.005    |
| Curve II $\left\{ \begin{array}{l} a=46.620 \dots \\ b=46.608 \dots \\ c=46.601 \dots \end{array} \right.$ | 0.874      | 0.064 | 0.032 | 0.011 | 0.003    | 0.006    |
|                                                                                                            | 0.848      | 0.059 | 0.046 | 0.006 | 0.004    | 0.003    |
|                                                                                                            | 0.848      | 0.053 | 0.050 | 0.012 | 0.005    | 0.001    |

and the second curve similarly consists of the alternate waves (not included in the first) superposed. We have analyzed both pairs of curves.

In each wave of current and electromotive force 120 points are determined and printed at distances of  $3^\circ$  each in the wave, or  $30'$  each in the generator, since one revolution gives 6 complete waves. We have used 15 points in each half wave in each analysis for the fundamental and harmonics, the mean value of the ordinates of the positive and negative half waves being taken in each case. Three

TABLE III.—*Results of analyses of two condenser currents, and calculation of components of current in the inductive circuit.*

[The values of  $I$  are expressed in arbitrary units.]

| Components of condenser current found by analysis. |              |            |        |        |        |          |          |
|----------------------------------------------------|--------------|------------|--------|--------|--------|----------|----------|
| Fundamental= $I_1$                                 |              | Harmonics. |        |        |        |          |          |
|                                                    |              | $I_3$      | $I_5$  | $I_7$  | $I_9$  | $I_{11}$ | $I_{13}$ |
| Curve III                                          | 42. 374..... | 5. 119     | 0. 854 | 1. 095 | 0. 101 | 0. 052   | 0. 090   |
|                                                    | 42. 427..... | 5. 170     | 0. 900 | 1. 138 | 0. 069 | 0. 107   | 0. 053   |
|                                                    | 42. 393..... | 5. 110     | 0. 847 | 1. 172 | 0. 108 | 0. 150   | 0. 032   |
| Curve IV                                           | 42. 162..... | 5. 117     | 0. 809 | 1. 067 | 0. 088 | 0. 066   | 0. 058   |
|                                                    | 42. 160..... | 5. 132     | 0. 804 | 1. 101 | 0. 135 | 0. 083   | 0. 135   |
|                                                    | 42. 218..... | 5. 116     | 0. 809 | 1. 145 | 0. 119 | 0. 125   | 0. 060   |
| Divisors=1 .....                                   |              | 6. 48      | 17. 33 | 33. 58 | 55. 24 | 82. 22   | 114. 7   |

| Components of inductive current calculated from above values. |              |            |        |        |        |          |          |
|---------------------------------------------------------------|--------------|------------|--------|--------|--------|----------|----------|
| Fundamental= $I_1$                                            |              | Harmonics. |        |        |        |          |          |
|                                                               |              | $I_3$      | $I_5$  | $I_7$  | $I_9$  | $I_{11}$ | $I_{13}$ |
| Curve III                                                     | 42. 374..... | 0. 790     | 0. 050 | 0. 031 | 0. 002 | 0. 001   | 0. 001   |
|                                                               | 42. 427..... | 0. 798     | 0. 052 | 0. 034 | 0. 001 | 0. 001   | 0. 000   |
|                                                               | 42. 393..... | 0. 788     | 0. 049 | 0. 035 | 0. 002 | 0. 002   | 0. 000   |
| Curve IV                                                      | 42. 162..... | 0. 790     | 0. 047 | 0. 032 | 0. 002 | 0. 001   | 0. 001   |
|                                                               | 42. 160..... | 0. 792     | 0. 046 | 0. 033 | 0. 002 | 0. 000   | 0. 001   |
|                                                               | 42. 218..... | 0. 789     | 0. 047 | 0. 034 | 0. 002 | 0. 001   | 0. 001   |

separate sets of ordinates were in this way prepared for each curve, and the analysis carried through for each set to the thirteenth harmonic—that is, the harmonic having a frequency of thirteen times the fundamental. The ordinates used in these analyses are given in Table I.

The results of these analyses are given in Tables II and III, where in the first part of each table the six values of the fundamental and of each harmonic are given in the proper columns, the first three as found for one curve, and the second group of three for the other.

TABLE IV.—*Calculation of correction factor  $f$  from electromotive force curves.*

|                            | Curve I.  |           |           | Curve II.                   |           |           |
|----------------------------|-----------|-----------|-----------|-----------------------------|-----------|-----------|
|                            | $a$       | $b$       | $c$       | $a$                         | $b$       | $c$       |
| $I_1^2 =$                  | 2197.6846 | 2193.4475 | 2197.8827 | 2173.3750                   | 2172.3367 | 2171.6776 |
| $I_2^2 =$                  | 0.7714    | 0.7250    | 0.8051    | 0.7640                      | 0.7195    | 0.7195    |
| $I_3^2 =$                  | 0.0012    | 0.0029    | 0.0049    | 0.0041                      | 0.0035    | 0.0028    |
| $I_7^2 =$                  | 0.0007    | 0.0017    | 0.0021    | 0.0010                      | 0.0022    | 0.0025    |
| Sum ..                     | 2198.4579 | 2194.1771 | 2198.6948 | 2174.1441                   | 2173.0619 | 2172.4024 |
| $I_1^2 =$                  | 2197.6846 | 2193.4475 | 2197.8827 | 2173.3750                   | 2172.3367 | 2171.6776 |
| 9 $I_2^2 =$                | 6.9426    | 6.5250    | 7.2459    | 6.8760                      | 6.4755    | 6.4755    |
| 25 $I_3^2 =$               | 0.0310    | 0.0720    | 0.1220    | 0.1025                      | 0.0862    | 0.0712    |
| 49 $I_7^2 =$               | 0.0343    | 0.0853    | 0.1049    | 0.0490                      | 0.1054    | 0.1235    |
| Sum ..                     | 2204.6925 | 2200.1298 | 2205.3555 | 2180.4025                   | 2179.0038 | 2178.3478 |
| $f$                        | 0.99859   | 0.99865   | 0.99849   | 0.99856                     | 0.99863   | 0.99863   |
| Mean from curve I=0.998577 |           |           |           | Mean from curve II=0.998607 |           |           |

TABLE V.—*Calculation of correction factor  $f$  from condenser current curves.*

|                              | Curve III. |           |           | Curve IV.                   |           |           |
|------------------------------|------------|-----------|-----------|-----------------------------|-----------|-----------|
|                              | $a$        | $b$       | $c$       | $a$                         | $b$       | $c$       |
| $I_1^2 =$                    | 1795.6048  | 1800.0458 | 1797.1532 | 1777.6218                   | 1777.4734 | 1782.3466 |
| $I_2^2 =$                    | 0.6240     | 0.6360    | 0.6213    | 0.6230                      | 0.6267    | 0.6227    |
| $I_3^2 =$                    | 0.0024     | 0.0027    | 0.0024    | 0.0022                      | 0.0022    | 0.0022    |
| $I_7^2 =$                    | 0.0011     | 0.0012    | 0.0012    | 0.0010                      | 0.0011    | 0.0012    |
| Sum ..                       | 1796.2323  | 1800.6857 | 1797.7781 | 1778.2480                   | 1778.1034 | 1782.9727 |
| $I_1^2 =$                    | 1795.6048  | 1800.0458 | 1797.1532 | 1777.6218                   | 1777.4734 | 1782.3466 |
| 9 $I_2^2 =$                  | 5.6160     | 5.7240    | 5.5917    | 5.6070                      | 5.6403    | 5.6043    |
| 25 $I_3^2 =$                 | 0.0607     | 0.0675    | 0.0598    | 0.0550                      | 0.0540    | 0.0545    |
| 49 $I_7^2 =$                 | 0.0529     | 0.0564    | 0.0598    | 0.0490                      | 0.0524    | 0.0568    |
| Sum ..                       | 1801.3344  | 1805.8937 | 1802.8645 | 1783.3328                   | 1783.2201 | 1788.0622 |
| $f$                          | 0.99858    | 0.99856   | 0.99859   | 0.99857                     | 0.99856   | 0.99857   |
| Mean from curve III=0.998577 |            |           |           | Mean from curve IV=0.998567 |           |           |

In the middle of the table are given the divisors by which we determine any harmonic of the current in the inductive circuit  $A B C$  from the harmonic of corresponding order in the e. m. f. or condenser circuit in parallel with it. The calculation of these divisors has already been given (p. 134). The second part of the tables gives the values of the harmonics of the current curves so calculated.

There is a very noticeable drop in the values of the harmonics after the seventh; these small quantities are without doubt residual errors arising in drawing the curve and getting the values of the ordinates. When they are divided by the corresponding divisors to get the harmonics of the current they become insignificant, amounting on the average to about one part in 8,000 of the fundamental for the e. m. f. curves and one part in 40,000 of the fundamental for the condenser current, and thus having no appreciable effect whatever in the correction factor to be determined.

#### 7. THE CALCULATION OF THE CORRECTION FACTOR $f$ .

In the first part of Tables IV and V are given the values of the squares of the harmonics as found from the analyses of the twelve sets of ordinates of the four curves, and in the second part of the tables the same squares multiplied by 1, 9, 25, 49, the coefficients occurring in the expression for the correction factor  $f$ . In the last line the values of  $f$  are given.

The mean value of  $f$  found from curve I differs from that found from curve II by only 3 parts in 100,000, whereas the mean value of  $f$  calculated from curve III differs from that of curve IV by only 1 part in 100,000.

If we had omitted the small fifth and seventh harmonics in the calculation of  $f$ , using only the fundamental and the third harmonic, the result would have differed by only 3 parts in 100,000. This illustrates how unimportant are all the harmonics above the third. But the residual errors in the current curve determined directly, instead of indirectly through the condenser current, are much greater. That the values obtained from different curves and from different sets of ordinates in the same curve agree so closely shows that the curve tracer gives a very accurate reproduction of the waves. It also proves that the six waves produced by one revolution of the twelve-pole alternator are very accurately the same, inasmuch as curves I and II were produced in different parts of the revolution, and III and IV also.

*Summary of values of  $f$ .*

|                                                                                                                                                               |                       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| Mean value from curve I of e. m. f .....                                                                                                                      | =0.998577             |
| Mean value from curve II of e. m. f .....                                                                                                                     | =0.998607             |
| Mean value from both curves.....                                                                                                                              | =0.998592( <i>a</i> ) |
| Mean value from curve III, condenser current .....                                                                                                            | =0.998577             |
| Mean value from curve IV, condenser current.....                                                                                                              | =0.998567             |
| Mean value from both curves .....                                                                                                                             | =0.998572( <i>b</i> ) |
| Weighted mean of ( <i>a</i> ) and ( <i>b</i> ), giving the result from the condenser<br>current curves three times the weight of that of the e. m. f. curve.. | =0.998577             |

In what follows we shall use **0.99858** as the most probable value.

Having now determined the correction factor to be applied to the values of  $L$  found in the use of these alternating currents, we may proceed to an account of the determination of frequency.

## 8. MEASUREMENT OF FREQUENCY.

As already stated, the speed of the alternator was maintained as nearly constant as possible by hand control, using a carbon rheostat in the main circuit of the driving motor, the criterion of constant speed being that the deflection of a galvanometer, observed with a telescope and scale, should be kept zero. The galvanometer was joined to a Wheatstone bridge in which a condenser and a rotating commutator form one arm, precisely as though the object of the experiment was to determine the capacity of the condenser. This arrangement we have found to be a sensitive and satisfactory method of controlling the speed. The galvanometer is quick and nearly dead beat, and instantly shows any tendency of the motor to change its speed. Such tendency can be quickly checked by a slight change in the pressure on the carbon rheostat. Slight variations in speed above or below the normal cause corresponding deflections of the galvanometer to the right or left, and the operator balances these small deflections as nearly as possible during the period of the run.

To illustrate, we give the determinations of frequency for the first four runs of the observations of May 28, which are fair samples of all the runs. Contact is made on the chronograph every 50 revolutions of the alternator, and as the latter is a 12-pole machine there are 6 waves in every revolution, or 300 in every period of 50 revolutions. Ten contacts are read off at the beginning of each run and ten at the end, and from these the mean interval for the whole number of periods is determined. The first two runs were divided into two parts each, and the frequency found for each, in order to see what the change was. After that the mean speed was found for the whole period of the run.

TABLE VI.—Run 1.

| Chronograph record—First part.         |             |                  | Chronograph record—Second part. |                                        |             |                  |                 |
|----------------------------------------|-------------|------------------|---------------------------------|----------------------------------------|-------------|------------------|-----------------|
| Beginning.                             |             | End.             | Interval.                       | Beginning.                             |             | End.             | Interval.       |
| <i>Min.</i>                            | <i>Sec.</i> | <i>Min. Sec.</i> | <i>Seconds.</i>                 | <i>Min.</i>                            | <i>Sec.</i> | <i>Min. Sec.</i> | <i>Seconds.</i> |
| 10                                     | 11.16       | 13 11.60         | 180.44                          | 13                                     | 31.65       | 15 20.27         | 108.62          |
|                                        | 12.82       | 13.26            | .44                             |                                        | 33.33       | 21.94            | .61             |
|                                        | 14.49       | 14.94            | .45                             |                                        | 35.03       | 23.60            | .57             |
|                                        | 16.15       | 16.61            | .46                             |                                        | 36.67       | 25.28            | .61             |
|                                        | 17.84       | 18.28            | .44                             |                                        | 38.33       | 26.95            | .62             |
|                                        | 19.51       | 19.94            | .43                             |                                        | 40.02       | 28.62            | .60             |
|                                        | 21.18       | 21.62            | .44                             |                                        | 41.68       | 30.29            | .61             |
|                                        | 22.85       | 23.30            | .45                             |                                        | 43.34       | 31.96            | .62             |
|                                        | 24.52       | 24.97            | .45                             |                                        | 45.02       | 33.63            | .61             |
|                                        | 26.20       | 26.63            | .43                             |                                        | 46.69       | 35.30            | .61             |
| 108 periods =                          |             |                  | 180.443                         | 65 periods =                           |             |                  | 108.608         |
| 108 × 300 = 32400 waves.               |             |                  |                                 | 65 × 300 = 19500 waves.                |             |                  |                 |
| $n = \frac{32400}{180.443} = 179.557.$ |             |                  |                                 | $n = \frac{19500}{108.608} = 179.546.$ |             |                  |                 |
| Mean value of $n = 179.552.$           |             |                  |                                 |                                        |             |                  |                 |

TABLE VII.—Run 2.

| Chronograph record—First part.        |                  |                 | Chronograph record—Second part.       |                  |                 |
|---------------------------------------|------------------|-----------------|---------------------------------------|------------------|-----------------|
| Beginning.                            | End.             | Interval.       | Beginning.                            | End.             | Interval.       |
| <i>Min. Sec.</i>                      | <i>Min. Sec.</i> | <i>Seconds.</i> | <i>Min. Sec.</i>                      | <i>Min. Sec.</i> | <i>Seconds.</i> |
| 18 2.29                               | 22 9.59          | 247.30          | 24 53.34                              | 29 59.09         | 305.75          |
| 3.97                                  | 11.26            | .29             | 55.02                                 | 30 0.77          | .75             |
| 5.64                                  | 12.93            | .29             | 56.66                                 | 2.43             | .77             |
| 7.32                                  | 14.61            | .29             | 58.35                                 | 4.06             | .71             |
| 8.99                                  | 16.27            | .28             | 25 0.00                               | 5.77             | .77             |
| 10.66                                 | 17.95            | .29             | 1.68                                  | 7.44             | .76             |
| 12.34                                 | 19.62            | .28             | 3.34                                  | 9.07             | .73             |
| 14.02                                 | 21.30            | .28             | 5.01                                  | 10.78            | .77             |
| 15.67                                 | 22.96            | .29             | 6.70                                  | 12.46            | .76             |
| 17.34                                 | 24.64            | .30             | 8.36                                  | 14.10            | .74             |
| 148 periods = 247.289                 |                  |                 | 183 periods = 305.751                 |                  |                 |
| 148 × 300 = 44400 waves.              |                  |                 | 183 × 300 = 54900 waves.              |                  |                 |
| $n = \frac{44400}{247.289} = 179.542$ |                  |                 | $n = \frac{54900}{305.751} = 179.558$ |                  |                 |
| Mean value of $n = 179.550$           |                  |                 |                                       |                  |                 |

TABLE VIII.

| Run 3.<br>Chronograph record. |             |                  | Interval.       | Run 4.<br>Chronograph record. |             |                  | Interval.       |
|-------------------------------|-------------|------------------|-----------------|-------------------------------|-------------|------------------|-----------------|
| Beginning.                    |             | End.             |                 | Beginning.                    |             | End.             |                 |
| <i>Min.</i>                   | <i>Sec.</i> | <i>Min. Sec.</i> | <i>Seconds.</i> | <i>Min.</i>                   | <i>Sec.</i> | <i>Min. Sec.</i> | <i>Seconds.</i> |
| 5                             | 51.69       | 12 59.11         | 427.42          | 14                            | 21.55       | 20 19.97         | 358.42          |
|                               | 53.37       | 13 0.81          | .44             |                               | 23.24       | 21.65            | .41             |
|                               | 55.06       | 2.50             | .44             |                               | 24.92       | 23.34            | .42             |
|                               | 56.75       | 4.18             | .43             |                               | 26.61       | 25.00            | .39             |
|                               | 58.43       | 5.87             | .44             |                               | 28.28       | 26.71            | .43             |
| 6                             | 0.10        | 7.53             | .43             |                               | 29.94       | 28.39            | .45             |
|                               | 1.78        | 9.23             | .45             |                               | 31.65       | 30.06            | .41             |
|                               | 3.47        | 10.90            | .43             |                               | 33.33       | 31.74            | .41             |
|                               | 5.16        | 12.60            | .44             |                               | 35.00       | 33.43            | .43             |
|                               | 6.85        | 14.28            | .43             |                               | 36.70       | 35.11            | .41             |
| 254 periods                   |             | =                | 427.435         | 213 periods                   |             | =                | 358.418         |
| 254 × 300                     |             | =                | 76200 waves.    | 213 × 300                     |             | =                | 63900 waves.    |
| $n = \frac{76200}{427.435}$   |             | =178.272.        |                 | $n = \frac{63900}{358.418}$   |             | =178.284.        |                 |

The examples given being the first runs of the first of the three days' work are not as good as the best, but show that the frequency can be maintained very nearly constant. The first two runs were made with the same resistances in the Wheatstone bridge, and the speed should therefore be the same. The mean frequencies found, 179.552 and 179.550, are practically identical. The resistance in the third arm of the bridge was now changed from 41,600 to 41,900, and the speed decreased until a balance was obtained. The frequencies for runs 3 and 4 were found to be, as shown in the table, 178.272 and 178.284. To show how nearly the mean speed remains constant, while the bridge resistance is unaltered, Table IX is given.

TABLE IX.—Frequencies of alternating current in ten runs of May 28.

[R=resistance in the third arm of the auxiliary Wheatstone bridge.]

|                |                    |                     |
|----------------|--------------------|---------------------|
| R=41,600 ohms. | Run 1, $n=179.552$ | Run 2, $n=179.550$  |
| R=41,900 ohms. | Run 3, $n=178.272$ | Run 4, $n=178.284$  |
| R=42,200 ohms. | Run 5, $n=177.009$ | Run 6, $n=177.009$  |
| R=42,250 ohms. | Run 7, $n=176.804$ | Run 8, $n=176.793$  |
| R=41,250 ohms. | Run 9, $n=181.083$ | Run 10, $n=181.081$ |



9. THE DETERMINATION OF  $R$ .

Instead of attempting to make the electromotive force on the non-inductive resistance  $AB$  exactly equal to that on  $BC$ , we adjusted  $AB$  to the nearest ohm and found the readings of the electrometer when the latter was joined first to  $AB$  and then to  $BC$ , three pairs of readings being taken. If the resistance  $AB$  was found to be too small it was then increased by one ohm and three pairs of readings of the electrometer again taken. By interpolation the value of the resistance was then found which would exactly balance the impedance of the inductive coil. As examples, the first three runs of June 2 are given.

TABLE X.—*Electrometer readings.*

|                | On $AB$ | On $BC$ | Difference $\times 100$ . | Mean.  | Interpolated value of $R$ |
|----------------|---------|---------|---------------------------|--------|---------------------------|
| RUN 1.         |         |         |                           |        |                           |
| $L_C$          |         |         |                           |        |                           |
| $R=1193$ ..... | 14.92   | 14.765  | -15.5                     |        |                           |
|                | .92     | .76     | -16.                      |        |                           |
|                | .925    | .77     | -15.5                     | -15.7  |                           |
| $R=1194$ ..... | 14.85   | 14.865  | + 1.5                     |        |                           |
|                | .85     | .86     | + 1.0                     | + 1.2  | 1193.92                   |
| RUN 2.         |         |         |                           |        |                           |
| $L_P$          |         |         |                           |        |                           |
| $R=1178$ ..... | 14.92   | 14.95   | + 3.                      |        |                           |
|                | .925    | .955    | + 3.                      |        |                           |
|                | .925    | .96     | + 3.5                     | + 3.17 |                           |
| $R=1177$ ..... | 14.975  | 14.87   | -10.5                     |        |                           |
|                | .975    | .86     | -11.5                     |        |                           |
|                | .975    | .87     | -10.5                     | -10.83 | 1177.77                   |
| RUN 3.         |         |         |                           |        |                           |
| $L_C$          |         |         |                           |        |                           |
| $R=1194$ ..... | 14.78   | 14.79   | + 1.                      |        |                           |
|                | .77     | .78     | + 1.                      | + 1.0  |                           |
| $R=1193$ ..... | 14.85   | 14.69   | -16.                      |        |                           |
|                | .85     | .71     | -14.                      |        |                           |
|                | .85     | .70     | -15.                      | -15.0  | 1193.94                   |

A summary of the results of three sets of measurements, taken on three separate days, is given in Tables XI, XII, and XIII. In each of these sets measurements are made successively on an inductance standard by Carpentier, of Paris, having a nominal value of one henry, and another by Franke & Co., of Hannover, of the same nominal value. In the tables the results for each coil are grouped together, the numbers of the runs indicating the order in which the measurements are made. The several columns give (1) the corrected values of the noninductive resistance  $R$ , (2) the ohmic resistance  $r$  of the inductive coil, (3) the frequency of the current, (4) the computed value of  $L$  from the formula (p. 127) uncorrected for wave form, (5) the mean of these latter values, and finally (6) the mean after correction for wave form, using  $f=0.99858$ . The corrected values of the resistances were found by means of a carefully calibrated bridge, and were redetermined each day.

TABLE XI.—Results of May 28.

|              | 1       | 2     | 3       | 4                                | 5                 | 6       |
|--------------|---------|-------|---------|----------------------------------|-------------------|---------|
|              | $R$     | $r$   | $n$     | $L_1$ uncorrected for wave form. | Mean of $L_1$     | $L$     |
| $L_c$        |         |       |         |                                  |                   |         |
| Run 1 .....  | 1149.98 | 97.6  | 179.551 | 1.01567                          | .....             | .....   |
| Run 4 .....  | 1141.90 | 97.75 | 178.284 | 1.01564                          | .....             | .....   |
| Run 5 .....  | 1133.93 | 97.9  | 177.009 | 1.01575                          | 1.01572           | 1.01428 |
| Run 8 .....  | 1132.69 | 98.05 | 176.793 | 1.01586                          | (D=7.2 in 100000) |         |
| Run 9 .....  | 1159.76 | 98.2  | 181.083 | 1.01566                          | .....             | .....   |
| $L_f$        |         |       |         |                                  |                   |         |
| Run 2 .....  | 1134.01 | 97.4  | 179.550 | 1.00144                          | .....             | .....   |
| Run 3 .....  | 1125.93 | 97.6  | 178.272 | 1.00140                          | .....             | .....   |
| Run 6 .....  | 1118.19 | 97.7  | 177.009 | 1.00156                          | 1.00144           | 1.00002 |
| Run 7 .....  | 1116.68 | 97.8  | 176.804 | 1.00135                          | (D=5.2 in 100000) |         |
| Run 10 ..... | 1143.62 | 97.9  | 181.081 | 1.00145                          | .....             | .....   |

$D$ =average deviation of a single determination.

TABLE XII.—Results of May 31.

|             | 1       | 2    | 3       | 4                                | 5                       | 6       |
|-------------|---------|------|---------|----------------------------------|-------------------------|---------|
|             | $R$     | $r$  | $n$     | $L_1$ uncorrected for wave form. | Mean of $L_1$           | $L$     |
| $L_C$       |         |      |         |                                  |                         |         |
| Run 1 ..... | 1194.42 | 97.0 | 186.550 | 1.01561                          | .....                   | .....   |
| Run 4 ..... | 1193.44 | 97.3 | 186.437 | 1.01540                          | .....                   | .....   |
| Run 5 ..... | 1192.36 | 97.5 | 186.230 | 1.01560                          | 1.01560                 | 1.01416 |
| Run 8 ..... | 1192.54 | 97.7 | 186.225 | 1.01577                          | ( $D = 9.5$ in 100000)  |         |
| $L_F$       |         |      |         |                                  |                         |         |
| Run 2 ..... | 1178.16 | 96.9 | 186.561 | 1.00168                          | .....                   | .....   |
| Run 3 ..... | 1177.10 | 97.1 | 186.440 | 1.00141                          | .....                   | .....   |
| Run 6 ..... | 1176.13 | 97.2 | 186.234 | 1.00166                          | 1.00160                 | 1.00018 |
| Run 7 ..... | 1176.16 | 97.3 | 186.237 | 1.00168                          | ( $D = 10.2$ in 100000) |         |

 $D$  = average deviation of a single determination.

## 10. DISCUSSION OF THE RESULTS.

The results of June 2 are somewhat more uniform than those of the two preceding days, due largely to the fact that the electrometer was adjusted to a greater sensibility and readings were consequently more accurate. The average deviations of the separate determinations from the mean is less than 4 parts in 100,000 in this set.

The two determinations of  $L$  at lower frequency on June 2 are subject to a correction for wave form which was not separately determined, and probably the capacity correction, which would be smaller for the lower frequency, accounts for part of the difference. A larger number of determinations at the lower frequency should be made to determine what difference is due to the different frequency. We mean to resume the experiments and make a longer series of measurements at other frequencies.

Table XIV shows a progressive change in the values of  $L_C$  and  $L_F$ , but in opposite directions; in both cases the change between May 28 and June 2 is about 3 parts in 10,000. Referring to Tables XI, XII, and XIII, in which the values of  $r$ , the resistances of the coils are given, it will be observed that the resistances happened to be almost exactly equal for the two coils, and that both were lower on May 31

TABLE XIII.—*Results of June 2.*

|                      | 1        | 2        | 3        | 4                                               | 5                             | 6        |
|----------------------|----------|----------|----------|-------------------------------------------------|-------------------------------|----------|
|                      | <i>R</i> | <i>r</i> | <i>n</i> | <i>L</i> <sub>1</sub> uncorrected for waveform. | Mean of <i>L</i> <sub>1</sub> | <i>L</i> |
| <b>L<sub>C</sub></b> |          |          |          |                                                 |                               |          |
| Run 1 .....          | 1194.32  | 96.9     | 186.579  | 1.01541                                         |                               |          |
| Run 3 .....          | 1194.35  | 96.9     | 186.584  | 1.01541                                         |                               |          |
| Run 5 .....          | 1194.67  | 96.9     | 186.621  | 1.01549                                         | 1.01542                       | 1.01398  |
| Run 8 .....          | 1182.92  | 96.9     | 184.791  | 1.01539                                         |                               |          |
| Run 10 .....         | 1182.96  | 96.9     | 184.793  | 1.01541                                         | (D=2.6 in 100000)             |          |
| Run 11 .....         | 687.53   | 96.9     | 106.704  | 1.01525                                         |                               |          |
| Run 13 .....         | 687.60   | 96.9     | 106.704  | 1.01535                                         | 1.01530                       |          |
| <b>L<sub>F</sub></b> |          |          |          |                                                 |                               |          |
| Run 2 .....          | 1178.18  | 96.7     | 186.578  | 1.00162                                         |                               |          |
| Run 4 .....          | 1178.56  | 96.7     | 186.619  | 1.00173                                         |                               |          |
| Run 6 .....          | 1178.56  | 96.7     | 186.621  | 1.00171                                         | 1.00173                       | 1.00031  |
| Run 7 .....          | 1167.11  | 96.7     | 184.778  | 1.00183                                         |                               |          |
| Run 9 .....          | 1167.07  | 96.7     | 184.784  | 1.00174                                         | (D=4.8 in 100000)             |          |
| Run 12 .....         | 678.25   | 96.7     | 106.693  | 1.00142                                         |                               |          |
| Run 14 .....         | 678.35   | 96.7     | 106.698  | 1.00152                                         | 1.00147                       |          |

*D*=average deviation of a single determination.TABLE XIV.—*Summary of results.*

|              | <i>L<sub>C</sub></i> | <i>L<sub>F</sub></i> | Ratio = $\frac{L_C}{L_F}$ |
|--------------|----------------------|----------------------|---------------------------|
| May 28 ..... | 1.01428              | 1.00002              | 1.01426                   |
| May 31 ..... | 1.01416              | 1.00018              | 1.01398                   |
| June 2 ..... | 1.01398              | 1.00031              | 1.01367                   |

than on May 28, and still lower June 2, the total difference being almost exactly one ohm for each coil. This corresponds to about  $2.5^{\circ}\text{C.}$ , and is due to the lower temperature of the laboratory on the later days. We were surprised to find evidence of a positive temperature coefficient in one coil and a negative coefficient in the other, and therefore made some direct comparisons of the two coils with each other with a view to testing this point.  $L_C$ , being maintained at a constant temperature of about  $21.5^{\circ}\text{C.}$ ,  $L_F$  was cooled about  $3^{\circ}$  by leaving it in a cooler room over night. The two coils being balanced against each other, with a variable inductance included with the smaller,  $L_F$  was warmed in an inclosed space and its inductance was observed to *decrease* about 3 parts in 10,000. On another day  $L_F$  was kept constant and  $L_C$  heated in a similar manner. The result was an *increase* in the value of  $L_C$ .  $L_F$  being again heated while  $L_C$  remained constant, its value *decreased* with respect to  $L_C$ . An exact measure of the change of temperature was not obtained, and hence no definite value of the temperature coefficient was found.

A possible explanation of the opposite sign of the temperature coefficients suggested itself when we removed the covering of  $L_F$ . This coil is wound on a spool of serpentine, and the wire is embedded in paraffin. The formula for the induction of such a coil is

$$L = 4\pi n^2 a \left( \log \frac{8a}{R} - 2 \right)$$

where  $a$  is the mean radius of the coil and  $R$  is the geometric mean distance of the wires in the cross section of the coil. When the paraffin (which has a temperature coefficient many times larger than copper) expands, it tends to increase the geometric mean distance of the wires and so decrease  $L$ , and this effect may be greater than the increase due to the expansion of the copper, which increases  $a$ . The other coil, however, is wound on a spool of mahogany with dry, silk-covered wire, and there is no such tendency to increase  $R$ . Whether part of the observed increase of  $L_C$  with increase of temperature is due to the spool itself we do not know.

It is evident that we must either keep these coils continuously at a constant temperature when measuring their inductances, or else get some new ones not subject to so large temperature coefficients. Whether this is possible we do not know, but hope soon to make some trials in this direction and also to study more carefully the magnitude of the temperature coefficients of these coils and their causes.

This method of measuring inductance is capable of yielding somewhat better results than those given above, when all possible refinements are introduced. It seems to us desirable to measure in this way some carefully constructed inductance standards whose values can be computed from their dimensions. The determination of such pairs of values of  $L$  would amount to an absolute determination of the international ohm.

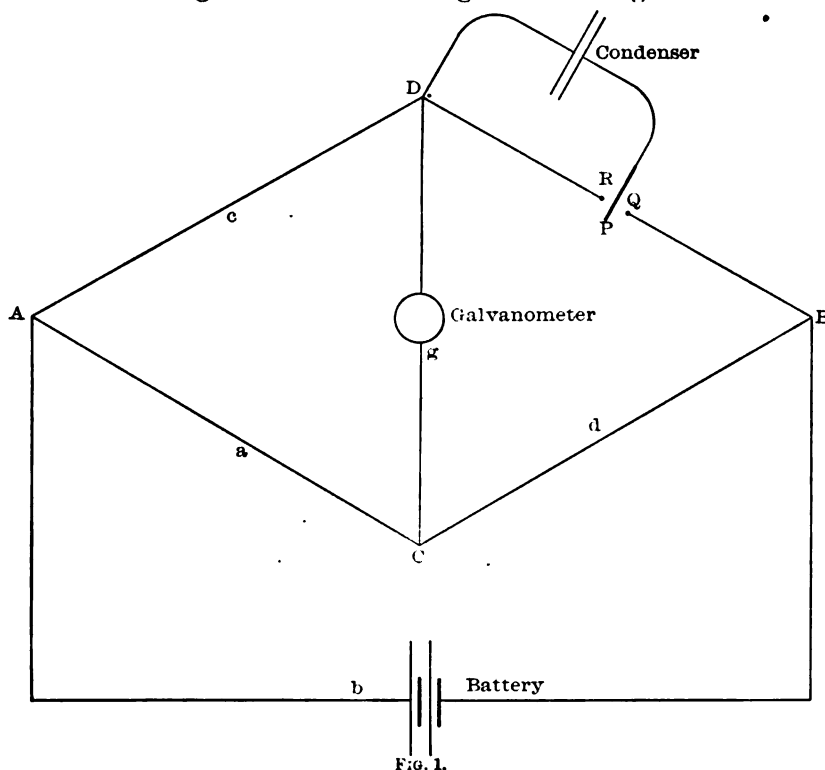
We are indebted to Mr. C. E. Reid for assistance in making some of the observations recorded in this paper and to Dr. N. E. Dorsey for assistance in analyzing the curves.

# THE ABSOLUTE MEASUREMENT OF CAPACITY.

By EDWARD B. ROSA and FREDERICK W. GROVER.

## 1. THE METHOD EMPLOYED.

The usual method of determining the capacity of a condenser in electromagnetic measure is Maxwell's bridge method, using a tuning fork or a rotating commutator to charge and discharge the condenser,



which is placed in the fourth arm of a Wheatstone bridge. This is a null method, is adapted to measuring large and small capacities equally well, and requires an accurate knowledge only of a resistance and the rate of the fork or commutator.

The formula for the capacity  $C$  of the condenser as given by J. J. Thomson,<sup>a</sup> is as follows:

$$C = \frac{a}{ncd} \cdot \frac{1 - \frac{a^2}{(a+b+d)(a+c+g)}}{\left(1 + \frac{ab}{c(a+b+d)}\right) \left(1 + \frac{ag}{d(a+c+g)}\right)}, \quad (1)$$

in which  $a$ ,  $c$ , and  $d$  are the resistances of three arms of a Wheatstone bridge,  $b$  and  $g$  are battery and galvanometer resistances, respectively, and  $n$  is the number of times the condenser is charged and discharged per second. When the vibrating arm  $P$  touches  $Q$  the condenser is

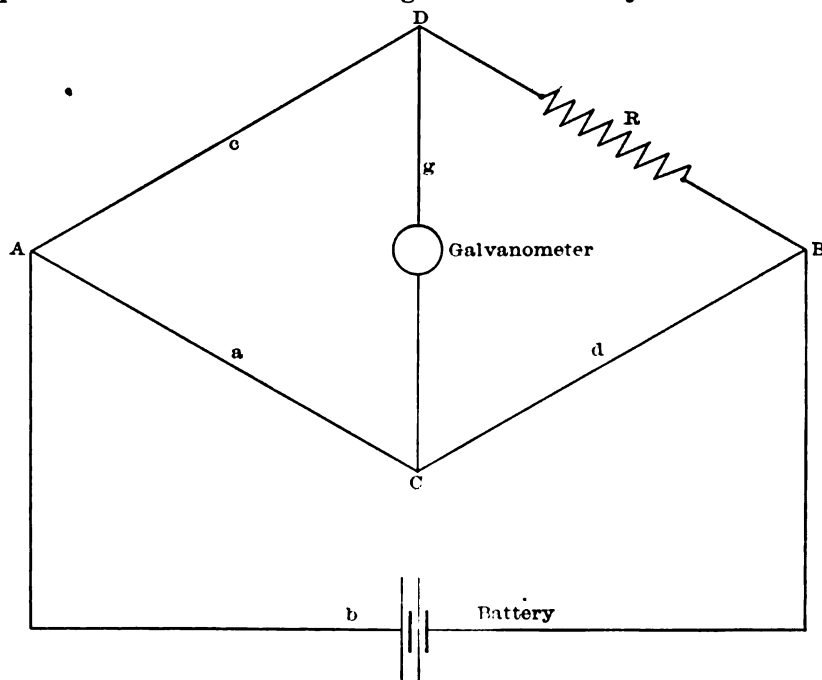


FIG. 2.

charged, and when it touches  $R$  it is short circuited and discharged. When  $P$  is not touching  $Q$  the arm  $BD$  of the bridge is interrupted and a current flows from  $D$  to  $C$  through the galvanometer; when  $P$  touches  $Q$  the condenser is charged by a current coming partly through  $c$  and partly through  $g$  from  $C$  to  $D$ . Thus the current through the galvanometer is alternately in opposite directions, and when these opposing currents balance each other there is no deflection of the galvanometer. This is accomplished by varying one of the resistances,  $a$ ,  $c$ , or  $d$ . The capacity may then be computed from the formula (1).

<sup>a</sup> Phil. Trans., 1883.



Under ordinary circumstances the capacity of the condenser is very nearly given by the first part of the formula  $\frac{a}{ncd}$ , the remainder constituting a correction factor which we may represent by  $F$ . Then

$$C = \frac{Fa}{ncd}. \quad (2)$$

If after the experiment we replace the condenser and commutator by an accurately calibrated resistance box, and vary its resistance until the bridge is balanced while  $a$ ,  $c$ , and  $d$  remain unchanged, then  $\frac{a}{cd} = \frac{1}{R}$  and hence

$$C = \frac{F}{nR}. \quad (3)$$

This is not practicable for very small capacities, as  $R$  would then be inconveniently large at ordinary values of  $n$ . But for capacities of 0.01 microfarad and larger this can usually be done. Of course, if  $a$ ,  $c$ , and  $d$  are all known with sufficient accuracy this operation may be omitted.

## 2. THE CORRECTION FACTOR $F$ .

The correction factor  $F$ , as we have called it, shown in equation (1), may be written as follows:

$$F = \frac{1 - \frac{1}{\left(1 + \frac{b}{a} + \frac{d}{a}\right)\left(1 + \frac{c}{a} + \frac{g}{a}\right)}}{\left(1 + \frac{c}{b\left(1 + \frac{b}{a} + \frac{d}{a}\right)}\right)\left(1 + \frac{d}{g\left(1 + \frac{c}{a} + \frac{g}{a}\right)}\right)} \quad (4)$$

It is, of course, desirable to have this factor as nearly equal to unity as possible, and therefore the quantities  $\left(1 + \frac{b}{a} + \frac{d}{a}\right)$  and  $\left(1 + \frac{c}{a} + \frac{g}{a}\right)$  should be as large as possible. To this end  $a$  should be small,  $c$  and  $d$  relatively large. Under these circumstances the values of  $b$  and  $g$  are not very important, although it is generally better to have them small. If  $b$  is nearly zero, the first parenthesis of the denominator disappears, as well as one term in the first parenthesis of the numerator, and the expression for  $F$  is somewhat simplified. Since  $c$  and  $d$  are opposite branches of the bridge, their product is constant, and therefore increasing  $c$  decreases  $d$ ; both should be as large as possible, in order to make  $F$  as near unity as possible, but there are other reasons to be given later which make it desirable to have  $d$  much larger than  $c$ .

To show how the value of  $F$  varies with the values of the several quantities  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $g$ , we have computed and plotted a number of values in the curves of figs. 3 and 4. In each case  $a=c=100$  ohms. In the curves of fig. 3 the galvanometer resistance is 20 ohms, while the battery resistance is 0, 6, and 12 ohms in the three curves, respectively.

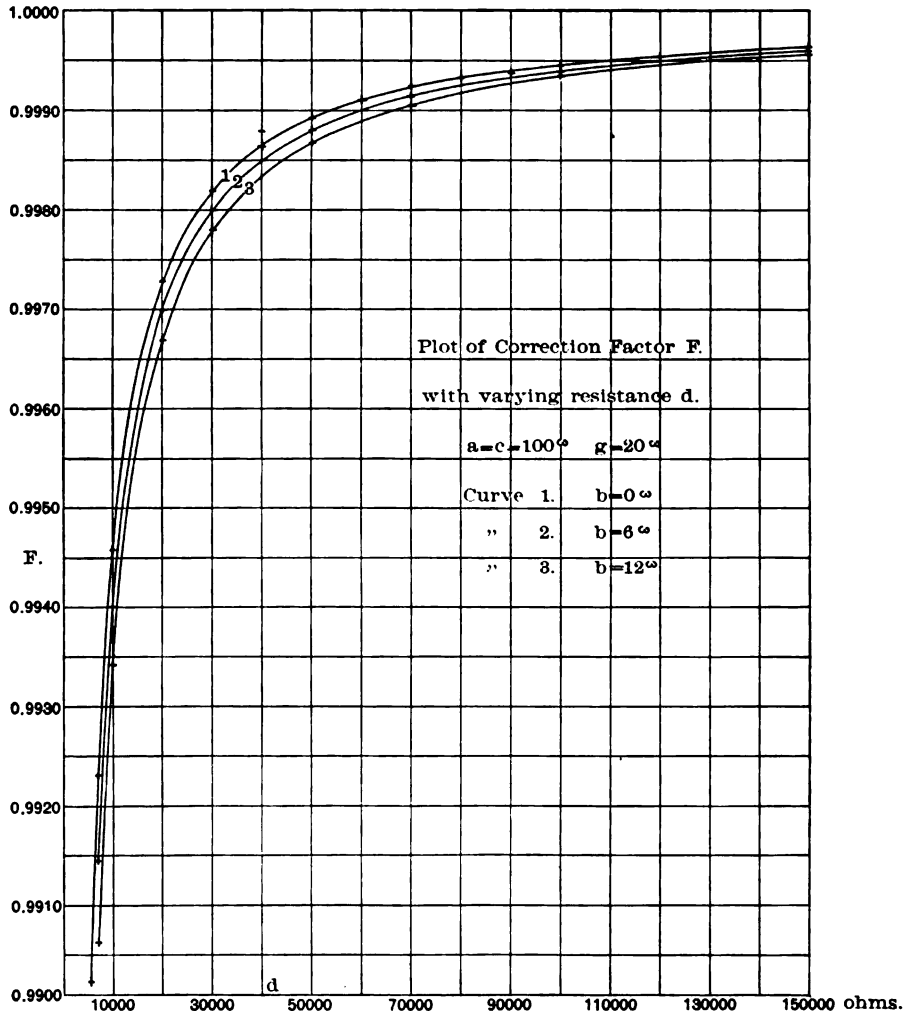


FIG. 3.

Ordinates show the values of  $F$  and abscissas increasing values of  $d$ . The value of  $F$  falls as low as 0.99 for  $d=6,000$  ohms, running up to 0.995 at about 12,000 ohms, 0.999 for 60,000 ohms, and 0.9996 for  $d=150,000$  ohms. If  $n=100$  per second, these values correspond to capacities of 1.67, 0.83, 0.167, and 0.056 microfarads, respectively.

Evidently  $F$  is more nearly unity as the frequency  $n$  is smaller and the capacity of the condenser is smaller.

The difference in  $F$  due to change in  $b$  from 0 to 6 ohms is only 0.0006 at  $d=10,000$  and 0.00006 at  $d=100,000$ . In our own experi-

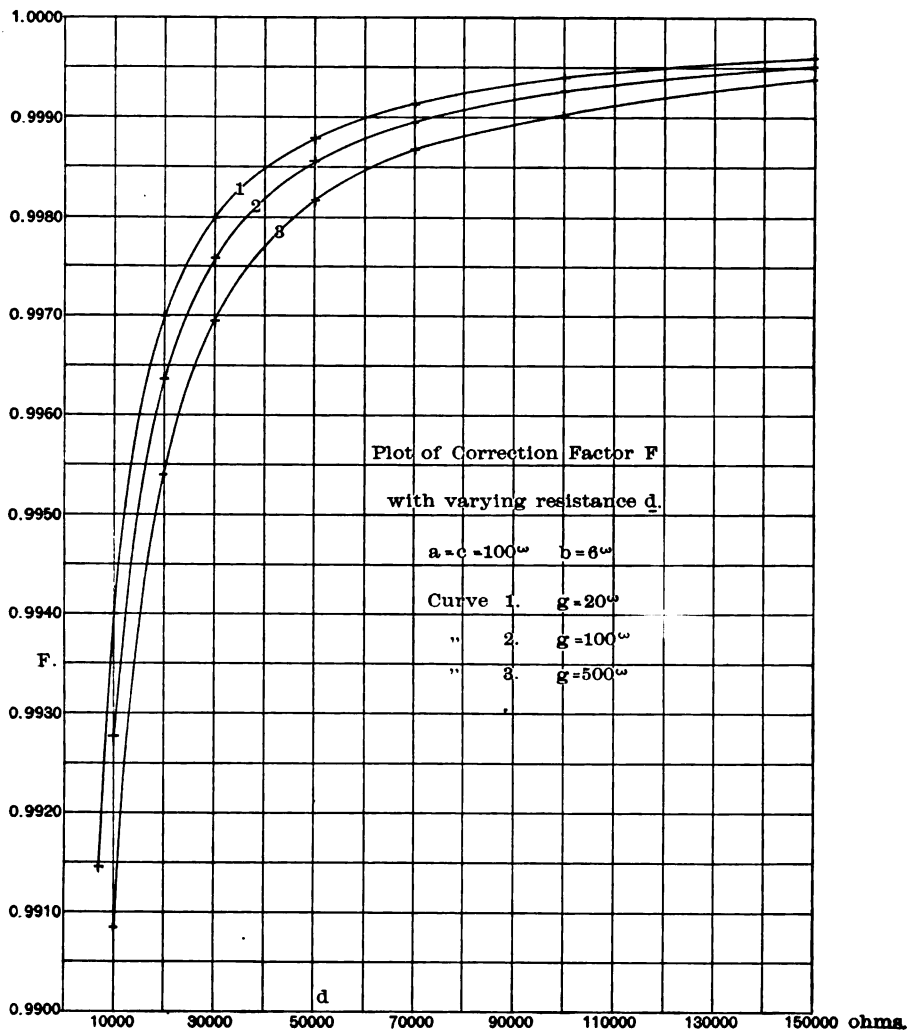


FIG. 4.

ments  $b$  was usually either 3 or 6 ohms, and hence the change in  $F$  due to changing the battery was very small, and a suitable correction easily made.

The effect of changing the galvanometer resistance is shown in the curves of fig. 4. Here  $a=c=100$  as before,  $b=6$  and  $g=20, 100$ , and

500 in the three curves, respectively. These represent actual values employed in our own experiments. The differences in  $F$  are greater than those due to changes in the battery, amounting to 0.0016 at  $d=20,000$  ohms and 0.00037 at 100,000 ohms. Obviously, in careful measurements, it is important to ascertain the value of  $F$  for each case, and such curves as are here given are convenient for the purpose.

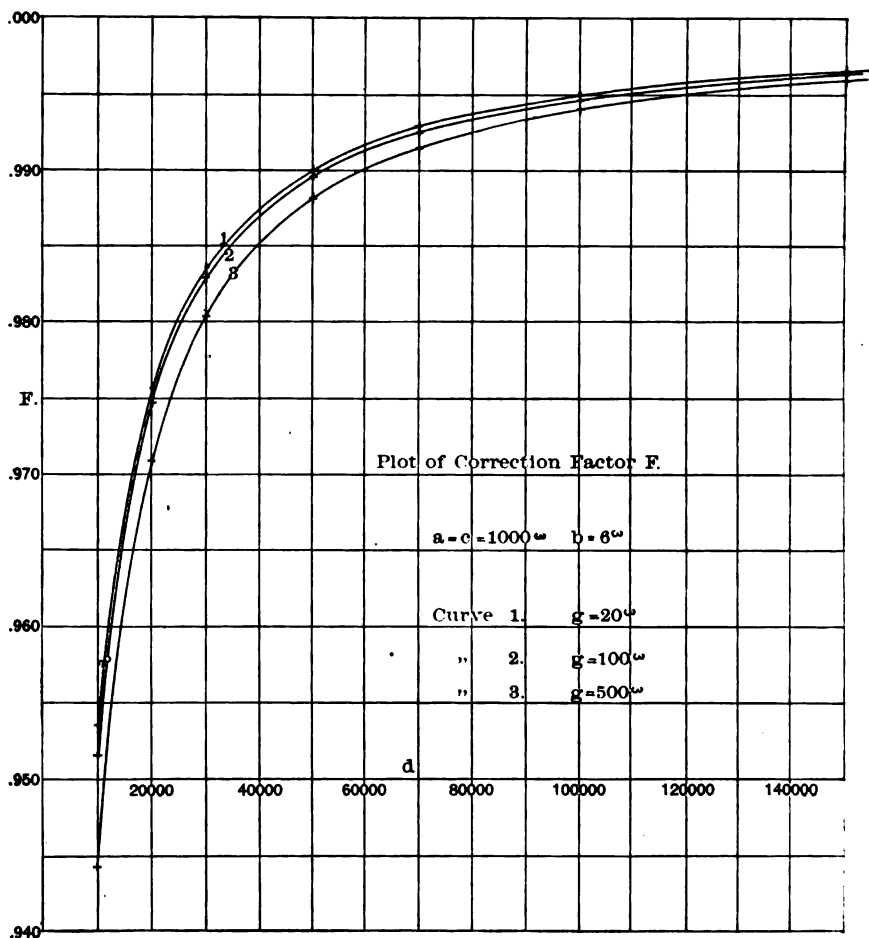
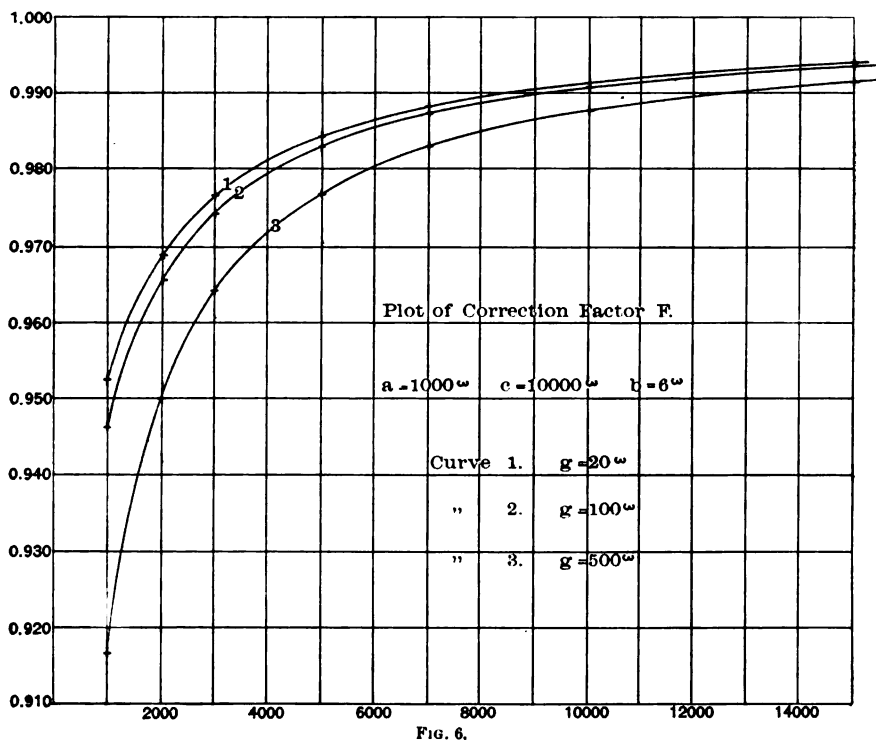


FIG. 5.

To show how much the factor  $F$  may vary from unity, if the resistances are not properly chosen, we give in fig. 5 three curves in which  $a=c=1,000$ ,  $b=6$  ohms, and  $g$  is 20, 100, and 500, respectively, in the three curves, corresponding to the curves of fig. 4. The only difference between this case and that of fig. 4 is that here the two equal ratio coils are 1,000 ohms each instead of 100. For 1 microfarad

(assuming  $n=100$ )  $d$  would be 10,000 and  $F$  would be 0.944, using the highest resistance galvanometer and 0.953 using the lowest resistance galvanometer. For 0.1 microfarad  $d=100,000$  ohms and  $F=0.995$ , approximately. In fig. 6 the ratio coils  $a$  and  $c$  have a ratio of 1 to 10,  $a$  being 1,000 as before, but  $c$  is increased to 10,000, and  $d$  therefore reduced to one-tenth of its former value, being 1,000 for 1 mf and 10,000 for 0.1 mf. The factor  $F$  is still smaller, varying from 0.916 for 1 mf ( $d=1,000$  ohms) to 0.988 for 0.1 mf ( $d=10,000$ ). It would probably be impossible to balance the bridge with these values of

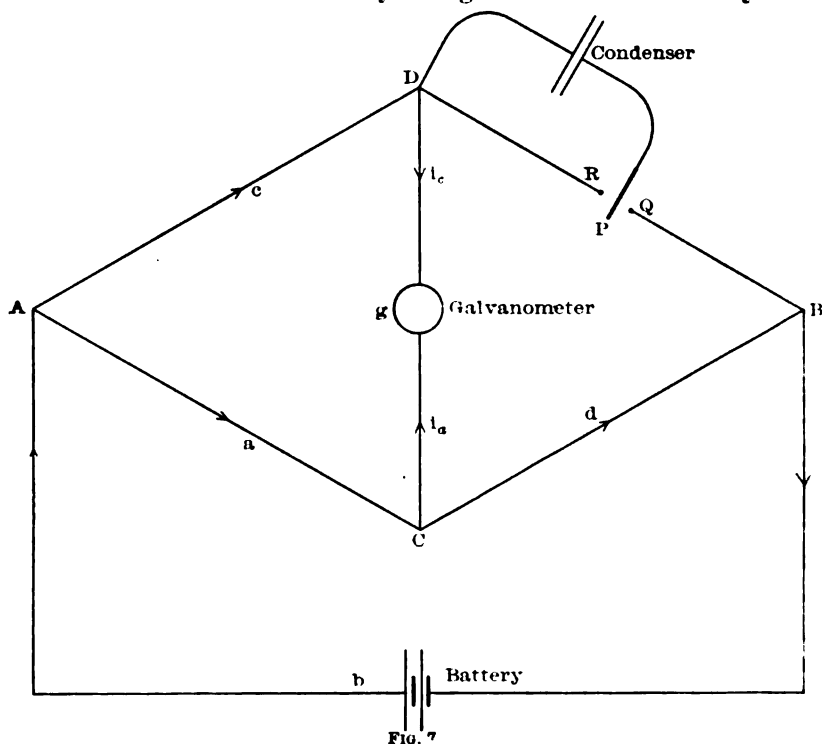


the resistances, as we shall show later, but so large a correction would be very undesirable if it were not impossible. By choosing the resistances with reference to the expression for  $F$ , however, there is no trouble in making the correction comparatively small in every case.

### 3. THE CHARGING CYCLE, AND CIRCUMSTANCES AFFECTING THE COMPLETENESS OF THE CHARGE.

When the tuning fork or commutator is running and the bridge is balanced, the condenser is alternately charged and discharged as the tongue  $P$  touches  $Q$  and  $R$ , respectively. When it touches  $R$  the

condenser is short-circuited through a negligible resistance and the discharge of the condenser is almost instantaneous, in all ordinary cases being complete. But the charge which passes into the condenser when  $P$  touches  $Q$  flows from the battery through the two parallel paths  $A D$  and  $A C D$ , and if their resistances are considerable, especially if the frequency of the fork or commutator is relatively high, it may happen that the condenser is not completely charged. Evidently this introduces an error, as the capacity is computed on the assumption that the condenser is fully charged before  $P$  leaves  $Q$ . It is



therefore necessary to determine the conditions for complete charge of the condenser, for different capacities, frequencies, and resistances.

Consider first what is occurring in the various arms of the bridge during the charging cycle.

Let  $T = \frac{1}{n}$ , the time between successive charges.

$t_1$  = duration of the charge, or time that  $P$  is in contact with  $Q$ .

$E$  = electromotive force of the battery.

$E_1$  = electromotive force on  $BD$  at the end of the charge, or final potential charging the condenser.

$i_c$ =Steady current flowing through the galvanometer from  $D$  to  $C$  when  $P$  is not touching  $Q$ .

$i_a$ =variable current flowing through the galvanometer from  $C$  to  $D$  while  $P$  touches  $Q$ .

When  $P$  is not touching  $Q$  a small steady current  $i_c$  flows through the galvanometer from  $D$  to  $C$ , joining the current through  $A C$  and thence flowing through  $C B$ , as in an ordinary Wheatstone bridge with the resistance of the fourth arm infinite. When  $P$  touches  $Q$  a rush of current into the condenser occurs, through the parallel circuits  $A D$  and  $A C D$ . The portion  $i_a$  of this passing through the galvanometer is shown in fig. 8, and when the galvanometer is balanced,

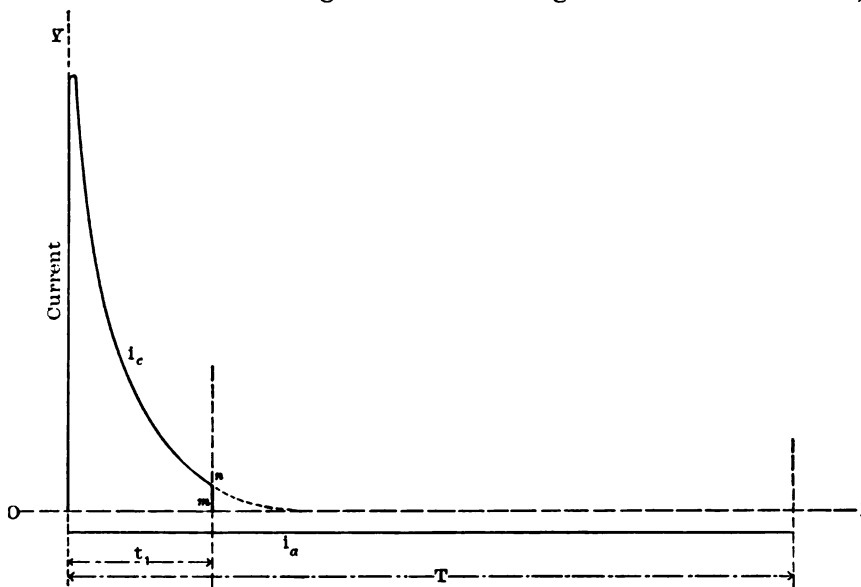


FIG. 8.

the total quantity flowing from  $C$  to  $D$  during the charge, or  $\int_0^{t_1} i_a dt$ ,

is equal to the quantity flowing from  $D$  to  $C$  or  $i_c T$ . At the moment of breaking contact between  $P$  and  $Q$  the charging current has a certain value  $m n$  which should be negligibly small, but which, as stated above, is sometimes appreciable. In the latter case the galvanometer

reading is likely to be unsteady, due to variations in  $\int_0^{t_1} i_a dt$ , since the

duration of  $t_1$  will not be always exactly the same and a wrong value of  $C$  will be found. If, however, the charging is over before contact is

broken (that is, if the remainder  $\int_{t_1}^T i_a dt$  which would flow in if, contact were continued through the remainder of a period, is negligible), then the exact duration of  $t_1$  is of no consequence, the galvanometer reading will not vary even though  $t_1$  varies, and a correct value of  $C$  will result.

Evidently the completeness of the charging of the condenser will depend on (1) the quantity  $CE_1$  which is necessary for complete charge, (2) the duration of the charge, (3) the resistance through which the condenser is charged, and (4) the electromotive force effective in producing the charging current. Of these factors the first will depend upon the capacity of the condenser and the voltage  $E_1$  to which it is charged. The latter depends not only on  $E$  the voltage of the battery, but also on the values of the resistances of the bridge. If  $d$  is very large compared with  $a$  and  $c$  the potential  $E_1$  on the condenser at the end of the charge will be nearly as great as  $E$ , but if  $d$  is small and  $c$  large this potential  $E_1$  is smaller. Hence, although the time required for complete charge of the condenser does not change if  $E$  is changed, the resistances remaining the same, it does vary if  $E_1$  is varied by altering the resistances. The duration of the charge  $t_1$  depends of course on the construction and speed of the commutator.

If the condenser is not fully charged, the quantity  $\frac{F}{nR}$  (equation 3) will not give the true capacity  $C$  but an apparent capacity  $C_o$  and we may put

$$Q_o = C_o E_1$$

where  $Q_o$  is the actual charge the condenser receives and  $E_1$  the potential to which it is charged. The expression for the quantity of electricity which flows into a condenser of capacity  $C$  in the time  $t$  is

$$Q_o = Q(1 - e^{-\frac{t}{CR}}) = C E_1 (1 - e^{-\frac{t}{CR}}) \quad (5)$$

In the present case  $t = t_1$  and  $R$ , the resistance through which the charging takes place has a value  $R_1$  yet to be determined. Hence

$$\begin{aligned} Q_o &= C_o E_1 = C E_1 (1 - e^{-\frac{t_1}{CR_1}}) \\ \therefore C_o &= C (1 - e^{-\frac{t_1}{CR_1}}) \\ &= C(1 - A) \end{aligned}$$

where  $C_o$  is the apparent capacity  $\frac{F}{nR}$  and  $A$  is the correction term depending on the completeness of the charge. Thus the true capacity is

$$C = \frac{F}{nR} \frac{1}{1-A} \quad (6)$$



and  $A$  is to be made as small as possible. Thus we may correct the value of the capacity  $C_0$  derived from the expression  $C_0 = \frac{F}{nR}$  if we know the values of the quantities  $t_1$ ,  $R_2$ , and the approximate value of  $C$ . Usually  $C$  and  $C_0$  are so nearly equal that  $C_0$  may be used for  $C$  in the exponential expression for  $A$ . We can easily determine  $t_1$  as a fraction of  $T$  by putting an ammeter in place of the condenser and observing the angle of the circumference through which the circuit remains closed in the charging position when the commutator is rotated through an angle corresponding to one cycle. We shall now determine  $R_2$  and then show not simply how to calculate the correction

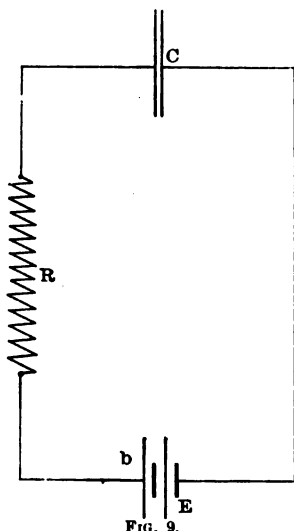


FIG. 9.

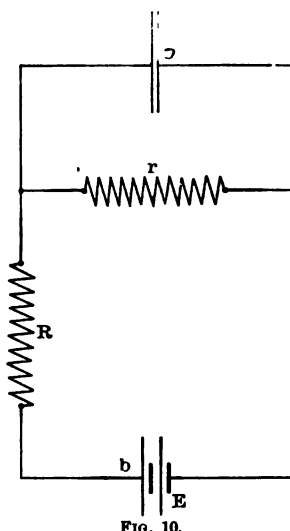


FIG. 10.

term  $A$ , but also how to make its value negligibly small in every case so that no correction need be applied.

#### 4. THE CALCULATION OF $R_2$ AND THE CORRECTION TERM $A$ .

Suppose we have a circuit including a condenser of capacity  $C$ , a resistance  $R$ , and a battery of resistance  $b$  and e. m. f.  $E$ . Then the quantity entering the condenser in time  $t$  is

$$q = CE \left( 1 - e^{-\frac{t}{C(R+b)}} \right) \quad (7)$$

The resistance through which the condenser is charged is of course  $R+b$ , the whole resistance external to the condenser.

If a resistance  $r$  short-circuits the condenser, a steady current  $i_1$  will

flow through it when the condenser is fully charged, and the potential to which the condenser is charged is therefore

$$E_1 = i_1 r$$

To find the expression for  $q$ , the quantity of electricity in the condenser at any instant, we suppose the circuit is suddenly closed at the battery, and as the condenser receives its charge the difference of potential  $e$  at the terminals of  $r$  rises gradually, being always equal to the difference of potential of the terminals of the condenser which are joined to the circuit by wires of negligible resistance. Then

$$i_1 = \frac{e}{r} \quad \text{= the current at any instant through } r.$$

$$i_2 = C \frac{de}{dt} \quad \text{= the current at any instant into the condenser.}$$

$$i = \frac{E-e}{R+b} \quad \text{= the current at any instant through the battery.}$$

Therefore, since  $i = i_1 + i_2$ ,

$$\frac{E-e}{R+b} = \frac{e}{r} + C \frac{de}{dt}$$

$$\text{or, } \frac{Er - (R+b+r)e}{(R+b)r} = C \frac{de}{dt}$$

$$\frac{de}{Er - (R+b+r)e} = \frac{dt}{C(R+b)r}$$

$$\int_0^t \frac{-(R+b+r)de}{Er - (R+b+r)e} = - \int_0^t \frac{(R+b+r)dt}{C(R+b)r} + K$$

$$\log \left( \frac{Er - (R+b+r)e}{Er} \right) = - \frac{R+b+r}{C(R+b)r} \cdot t + K$$

To find the constant of integration  $K$  we notice that when  $t=0$ ,  $e=0$ ; therefore

$$\log (Er) = K$$

Hence,

$$\log \left( \frac{Er - (R+b+r)e}{Er} \right) = - \frac{(R+b+r)}{C(R+b)r} \cdot t$$

$$1 - \frac{(R+b+r)e}{Er} = e^{-\frac{t}{R_2 C}}, \text{ putting } R_2 = \frac{(R+b)r}{R+b+r}$$

$$\text{or, } e = \frac{r}{(R+b+r)} \cdot E(1 - e^{-\frac{t}{R_2 C}}) \quad (8)$$

When the condenser is fully charged and therefore  $i=i_1$ , the value of  $e$  will be given by the expression

$$\frac{e}{E} = \frac{r}{R+b+r}$$

Calling this final value of the potential at the terminals of the condenser  $E_1$ , we have

$$e = E_1 \left(1 - e^{-\frac{t}{R_2 C}}\right) \quad (9)$$

$$\text{or } q = CE_1 \left(1 - e^{-\frac{t}{R_2 C}}\right) \quad (10)$$

This expression is of the same form as (7), and we notice that the value of  $R_2$  in this case,  $\frac{(R+b)r}{R+b+r}$ , is the joint resistance from  $B$  to  $D$  through the two paths  $r$  and  $R+b$ ; that is, the resistance external to the condenser, as in the simpler case fig. 9.

We have assumed in this demonstration that the circuit is made and broken at  $E$ , fig. 10. The result would, however, be the same if it were made and broken at  $C$ . In the latter case the current  $i_1$  would be flowing through  $r$  when contact is made at  $C$ , but instantly this current would cease and the potential of the terminals of  $r$  would become equal, since the two plates of the condenser have equal potentials when contact is made.

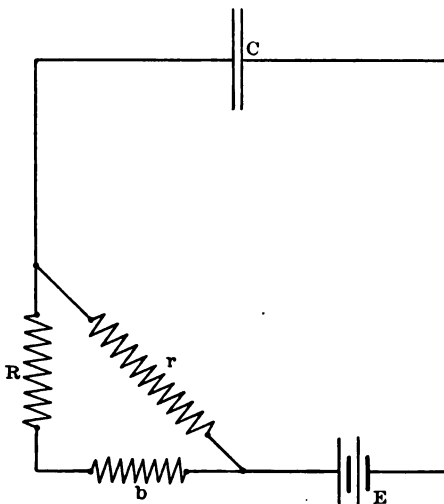


FIG. 11.

For, the inductance and electrostatic capacity of the resistance  $r$  are assumed zero, and hence joining its terminals to the plates of a condenser of finite capacity would cause it to assume instantly the potential of the condenser. The current  $i_1$  and difference of potential  $e$  would then rise logarithmically in exactly the same manner as if the circuit had been made at  $E$ .

We see from the foregoing that the charge  $q$  of the condenser rises to its final value logarithmically in exactly the same time that it would if it flowed through the parallel paths  $R+b$  and  $r$ , as shown in fig. 11, the battery resistance being assumed zero in the latter case. The cases are not alike, however, as in the latter the whole current flows into the condenser and the presence of  $r$  accelerates the charge, while

in the case of fig. 10 the resistance  $r$  shunts off a portion of the current away from the condenser. But in this case the total charge is less than in the case of fig. 11, being  $CE_1$  in one case and  $CE$  in the other,  $E_1$  being  $\frac{r}{R+b+r} E$ . Since we have proved that the time of reaching full charge is exactly the same in both cases, it is evident that these two opposing influences exactly counterbalance one another; that is, that  $r$  tends to reduce the time of charge of the condenser in

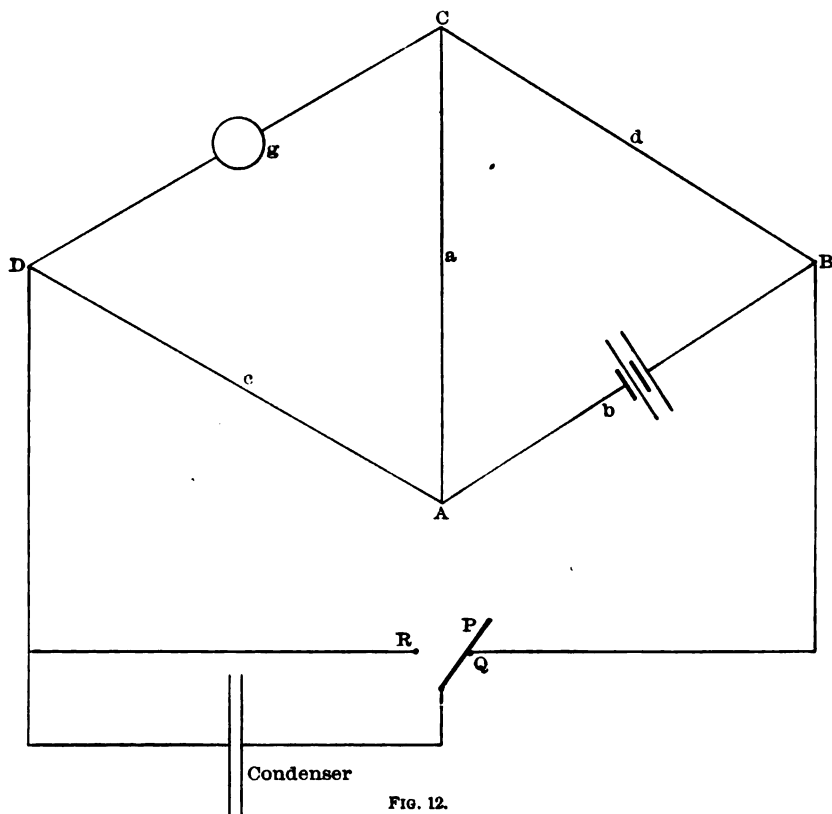


FIG. 12.

the first case (fig. 10) by reducing the total charge exactly as much as it tends to increase it by shunting away some of the battery current  $i$ . Referring, therefore, to equation (10) we see that  $R_2$  of the exponent, which is the joint resistance external to the condenser, is not strictly the resistance through which the condenser is charged, but that the time of charge of the condenser is the same as though it were if the condenser were charged to the full voltage of the battery. In the latter case the charging current is larger than in the former, but the time

required to reach any given percentage of the final charge is the same in both.

To get  $R_s$  in the case of the Wheatstone bridge, fig. 7, we must find similarly the total resistance external to the condenser—that is, from  $B$  to  $D$ .

Here again  $R_s$  will not be the resistance through which the condenser is charged, but it will be an equivalent resistance if the condenser were charged to the full voltage of the battery. Evidently the

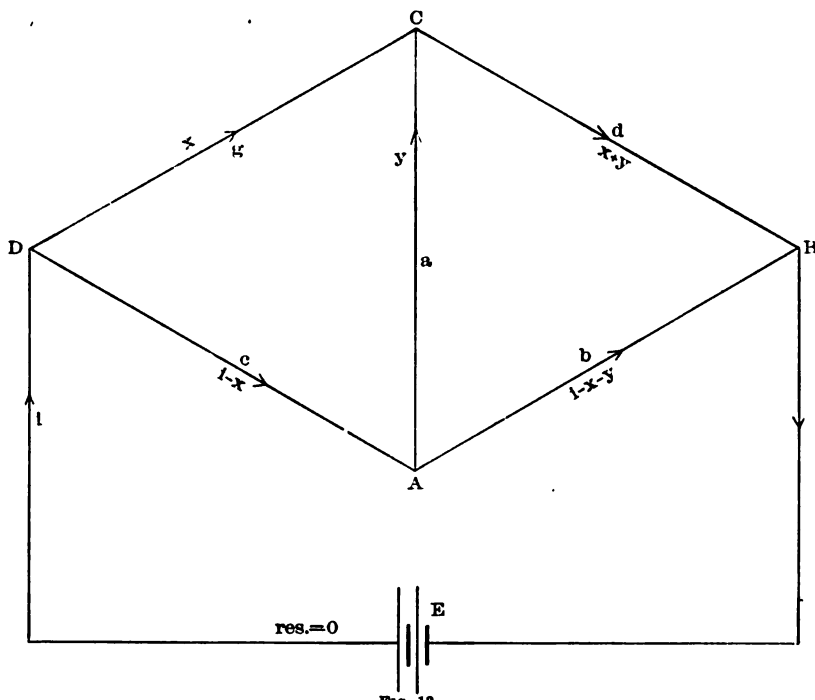


FIG. 13.

resistance  $d$  shunts away some of the battery current that would otherwise go into the condenser, but at the same time it reduces the final voltage to which the condenser is charged, and so reduces the quantity  $q$  and thus the charging current necessary. We require the network resistance  $R_s$  from  $B$  to  $D$ , and then the exponential expression (10) will give us the time required for the condenser to acquire any given fractional part of its final charge. The bridge may be rearranged as in fig. 12, and since we wish to determine the resistance of the network from  $D$  to  $B$ , outside the condenser, we may take the case

shown in fig. 13 where the condenser is omitted and the battery transferred from  $AB$  to the outside branch  $BD$ , the resistances of the five branches of the network remaining unaltered.

The distribution of currents according to Kirchhoff's first law is given in fig. 13, and by the second law

$$\left. \begin{aligned} (i-x)c + (i-x-y)b + 0 &= E \\ xg - ya - (i-x)c &= 0 \\ ya + (x+y)d - (i-x-y)b &= 0 \end{aligned} \right\} \quad (8)$$

Equations (8) rearranged give

$$\left. \begin{aligned} i(b+c) - x(b+c) - yb &= E \\ -ic + x(c+g) - ya &= 0 \\ -ib + x(b+d) + y(a+b+d) &= 0 \end{aligned} \right\} \quad (9)$$

Solving these equations for  $i$  we have

$$i = \frac{\begin{vmatrix} E & -(b+c) & -b \\ 0 & +(c+g) & -a \\ 0 & +(b+d), & (a+b+d) \end{vmatrix}}{\begin{vmatrix} (b+c), & -(b+c) & -b \\ -c & +(c+g) & -a \\ -b & +(b+d), & (a+b+d) \end{vmatrix}} = \frac{E \begin{vmatrix} (c+g), & -a \\ (b+d), & (a+b+d) \end{vmatrix}}{\Delta} \quad (10)$$

Hence, since  $R_s = \frac{E}{i}$

$$R_s = \frac{\Delta}{\begin{vmatrix} (c+g), & -a \\ (b+d), & (a+b+d) \end{vmatrix}} \quad (11)$$

$$= \frac{a(b+c)(d+g) + bc(d+g) + dg(b+c)}{a(b+c+d+g) + (b+d)(c+g)} \quad (12)$$

This is the resistance of the network from  $B$  to  $D$  of fig. 13 and hence also of fig. 7.

If  $d$  is infinite (that is, if the arm  $CB$  is removed), the value of  $R_2$  becomes, dividing numerator and denominator of (12) by  $d=\infty$ ,

$$R'_2 = \frac{bc+gc+bg+ac+ab}{a+c+g} = b + \frac{c(a+g)}{c+a+g} \quad (13)$$

This is obviously the resistance from  $B$  to  $D$  through the battery of resistance  $b$  and the two parallel circuits  $AD$  and  $ACD$  of resistances  $c$  and  $a+g$ , respectively. This is a much simpler expression than (12) and yet when  $d$  is large the value of  $R_2$ , calculated from the assumption of  $d=\infty$ , nearly agrees with the true value, and so may be used in determining whether the condenser in any given case is fully charged. The following numerical examples illustrate the calculation of  $R_2$  and of the completeness of the charge of the condenser in several special cases.

In each case  $g=20$  ohms.

$b=20$  ohms.

$T=.01$  sec.

$t_1=.002$  sec.

TABLE I.—Values of correction term  $A$ .

[Capacity=1 microfarad.]

|                                                                                                                   | By formula 12. |                     |                                  | By approximate formula 13. |                      |                                    |
|-------------------------------------------------------------------------------------------------------------------|----------------|---------------------|----------------------------------|----------------------------|----------------------|------------------------------------|
|                                                                                                                   | $R_2$          | $\frac{t_1}{R_2 C}$ | $A=\epsilon - \frac{t_1}{R_2 C}$ | $R'_2$                     | $\frac{t_1}{R'_2 C}$ | $A'=\epsilon - \frac{t_1}{R'_2 C}$ |
| Case I ... $\left\{ \begin{array}{l} a=c=1,000 \dots\dots\dots \\ d=10,000 \dots\dots\dots \end{array} \right\}$  | 499.74         | 4.02                | 0.0179                           | 524.9                      | 3.82                 | 0.022                              |
| Case II ... $\left\{ \begin{array}{l} a=d=1,000 \dots\dots\dots \\ c=10,000 \dots\dots\dots \end{array} \right\}$ | 499.74         | 4.02                | 0.0179                           | 945.6                      | 2.12                 | 0.121                              |
| Case III... $\left\{ \begin{array}{l} a=c=100 \dots\dots\dots \\ d=10,000 \dots\dots\dots \end{array} \right\}$   | 74.12          | 26.98               | $1.9 \times 10^{-12}$            | 74.54                      | 26.88                | $2.2 \times 10^{-12}$              |
| Case IV... $\left\{ \begin{array}{l} a=100 \dots\dots\dots \\ c=d=1,000 \dots\dots\dots \end{array} \right\}$     | 116.10         | 17.24               | $3.3 \times 10^{-8}$             | 127.14                     | 15.73                | $14.7 \times 10^{-8}$              |

## 6. DESCRIPTION OF THE COMMUTATOR.

The design of the commutator is shown in figs. 14, 15, and 16. Fig. 14 gives a plan,<sup>a</sup> and fig. 15 a front elevation, of the commutator, and fig. 16 an end elevation of two of the disks. The commutator consists of two separate and distinct instruments combined into one, only one being used ordinarily at once. The first part consists of the disk *Z* and the two sets of contact pieces and brushes belonging to it. This constitutes the commutator used in the bridge method of measuring capacity, as described in this paper. The second part consists of the two disks *X* and *Y* and their contact pieces and brushes, and this constitutes a secohmmeter or double commutator for use in comparing capacities or inductances with one another, or in comparing a capacity with an inductance.

These disks are mounted on a steel shaft as shown, the latter being directly coupled to a driving motor capable of maintaining a very constant speed. At the opposite end of the shaft from the coupling is a metal disk *P*, behind which is a small electric lamp *N*, fig. 14. There are four sets of fine holes in the disk each in a separate circle near the circumference of the disk, the holes of each set being equally spaced and capable of being viewed through a microscope mounted in front. The objective of this microscope is carried on one prong of an electrically driven tuning fork, and the electric lamp is so placed as to illuminate the holes of any one of the four sets as the disk revolves. There are 4, 6, 8, and 10 holes, respectively, in each of the four series, and using any one of these we have a stroboscopic method of comparing the speed of the disk (that is, of the commutator) with the rate of the fork. For example, if the fork vibrates 100 times per second and the microscope is set to observe the holes of the first series, then if the commutator is running at a speed of exactly 25 revolutions per second the vertical band of light seen in the microscope will appear stationary. Any variation in speed will be indicated by the shifting position of this band. If the lamp and microscope be shifted laterally about 7 mm the fourth series of holes will be seen and a speed of 10 revolutions per second will give the stationary band as seen before. Between these extreme positions lie the other two series of holes which may be used with intermediate speeds.

<sup>a</sup> *Plan of commutator*—*X*, *Y*, *Z*, disks of hard rubber; *h*, *k*, brass rings; *a*, *b*, *c*, *d*, *e*, projecting segments; *C*, *G*, brushes touching *a*, *b*; *D*, *H*, brushes touching *c*, *d*, *e*; *K*, brush touching ring *h*; *f*, *g*, *f'*, *g'*, insulated brass segments; *A*, *E*, *B*, *F*, rubber posts carrying brushes, latter touching segments. *T*, *U*, *T'*, *U'*, brass rings, on which brushes, *I*, *L*, *J*, *M*, bear; *P*, stroboscopic disk; *N*, lamp; *R*, *S*, bearings of shaft; *Q*, *Q'*, oil cups. (See also fig. 15.)



This method of measuring the speed of the commutator in terms of the frequency of the fork is very convenient and sensitive. We found it practicable, by recording the number of revolutions on a chronograph together with the second beats of a chronometer to redetermine the frequency of the fork as often as desired. In the course of the work, however, we found another method of controlling the speed, described below, which proved even more sensitive and more accurate, and we therefore abandoned the stroboscopic device in the later work.

The end of the shaft next this stroboscopic disk carries a worm which meshes into a gear of 50 teeth. The latter carries a stud which touches a platinum point at every revolution of the gear, corresponding to 50 revolutions of the commutator. This makes a very short electric contact which gives a sharp record on the chronograph sheet on which the second beats of the chronometer are also recorded. Reading off these contacts with a diagonal scale a very exact value of the frequency can be obtained, supposing it to be maintained constant for a few minutes at a time. Examples of these records and of the calculated frequency will be given later.

Different views of the disk *Z* appear in figs. 14, 15, and 16. This disk is of hard rubber and is 2.5 cm thick. On its perimeter two rings of brass are mounted, one carrying four equidistant projecting segments of brass and the other eight narrower segments, each about half the angular breadth of the first. These two rings are metallically connected and a brush *K* (fig. 14) bears on the edge of one ring. Two sets of wire-gauze brushes, *C*, *G*, and *D*, *H*, are supported on hard rubber posts *W* and *W'*. Only one pair of brushes is used at a time, *C* and *G* or *D* and *H*. In the first case contact will be made on the four-segment ring and in the second case on the eight-segment ring. The brushes are set at such angular positions that only one touches a segment at once, the other being between two segments during contact of the first. Referring to fig. 7, the brush *K* corresponds to the tongue *P*, and the brush *C* or *D* to the point *R*, while *G* or *H* represents *Q*. Thus, as the commutator revolves and the brush *K* is alternately joined to *C* and *G*, the condenser is discharged and charged four times per revolution. The angular breadth of segments *a*, *b* is such that contact between *K* and *Q* continues for about one-fifth of a period, and if the commutator is running at 25 revolutions per second the frequency of charge and discharge is 100 per second ( $T = .01$ ) and the duration of charges is 0.002 sec. (See p. 169.) Using the brushes *D* and *H* the frequency would be doubled at the same speed of the commutator.

The object of this particular construction was to secure the highest possible insulation between  $Q$  and  $R$ ; that is, between the brushes  $C$  and  $G$ . When brushes bear continuously on a cylindrical surface of any insulating medium in which metal segments are inserted, moisture, dirt, and metallic particles are liable to gather on the insulating segments and reduce the insulation resistance. This is the more noticeable when measuring small capacities, as those of air condensers, and under these circumstances precise measurements are difficult. With the brushes touching only the metallic segments and one brush entirely off the disk while the other makes contact, we have the insulation afforded by the hard-rubber columns  $W$ ,  $W'$  to separate  $R$  and  $Q$ . These columns are undercut (like so-called petticoat insulators), and the length of the leakage path over the surface is thereby considerably increased. The connecting wires are clamped by the screws at the tops of the hard-rubber columns, and hence the circuits are thoroughly insulated from the base of the instrument. The brush  $K$  is of phosphor bronze and laminated. The brushes  $C$ ,  $D$ ,  $G$ ,  $H$ , are of woven copper-wire gauze beveled at the end, so that the tips are quite flexible. Each brush is supported by a phosphor-bronze strip at the back, which is adjusted by means of the screw  $u$ . The brush can thus be so adjusted as to make good contact on the projecting segments throughout their entire breadth without bearing so hard against them as to wear out the brush too rapidly. The commutator runs quietly and is, on the whole, very satisfactory.

Each of the disks  $X$  and  $Y$  carries four insulated brass segments, as shown in fig. 16. The two opposite brushes of each disk ( $A$ ,  $E$ , and  $B$ ,  $F$ ) make contact simultaneously on adjacent segments. Two of these segments are joined to the ring  $T$  and the other two to  $U$ , the opposite ones being joined to the same ring. Thus, as in a secohmmeter, the two brushes  $I$ ,  $L$  are joined successively to  $A$ ,  $E$  and  $E$ ,  $A$  as the disks revolve, so reversing any electric current through them. In comparing capacities or inductances by a bridge method one disk will of course be in the battery circuit and the other in the galvanometer circuit.

The brushes  $I$ ,  $L$ ,  $J$ ,  $M$  are of phosphor bronze and laminated; the brushes  $A$ ,  $B$ ,  $E$ ,  $F$  are of woven-wire gauze like the brushes  $C$ ,  $D$ ,  $G$ ,  $H$ , and are mounted similarly. The insulation is made thorough by using hard-rubber columns, as before.

The angular position of one disk with respect to the other may be altered so as to give any desired phase relation between the commutation of the two circuits.

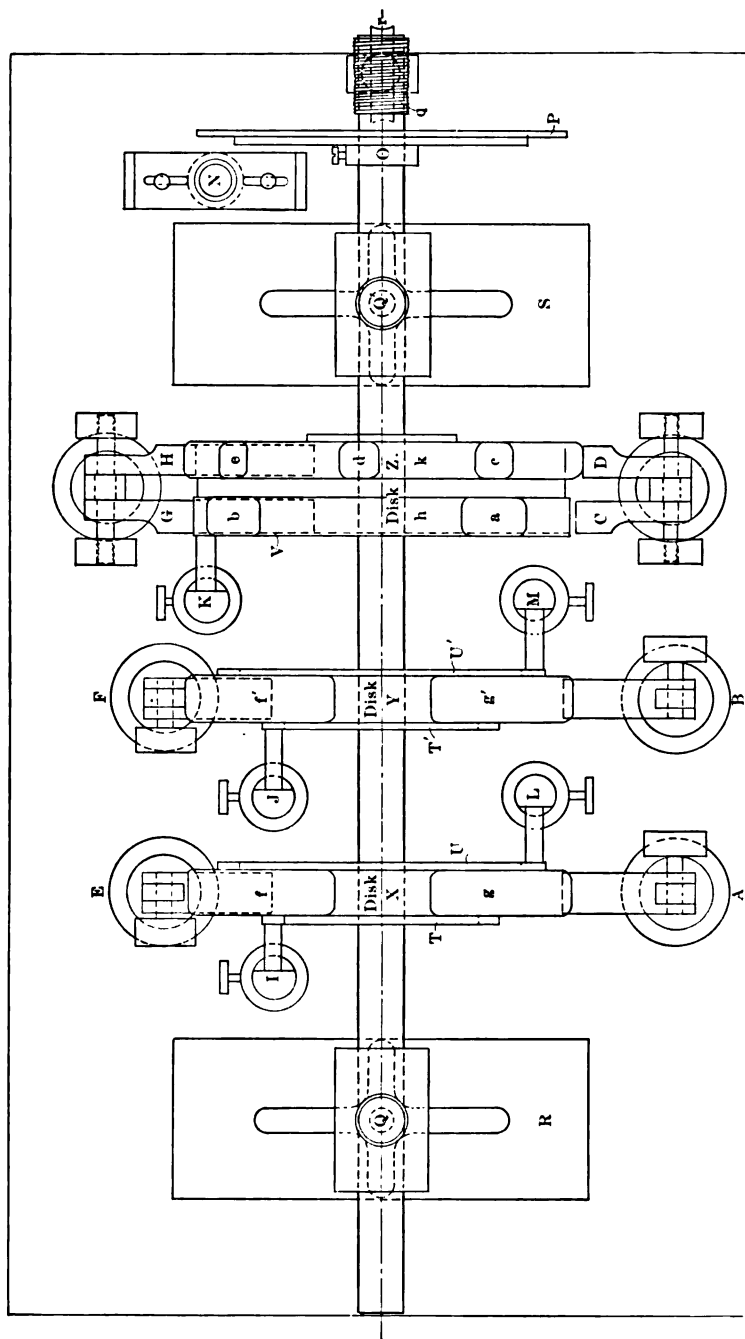
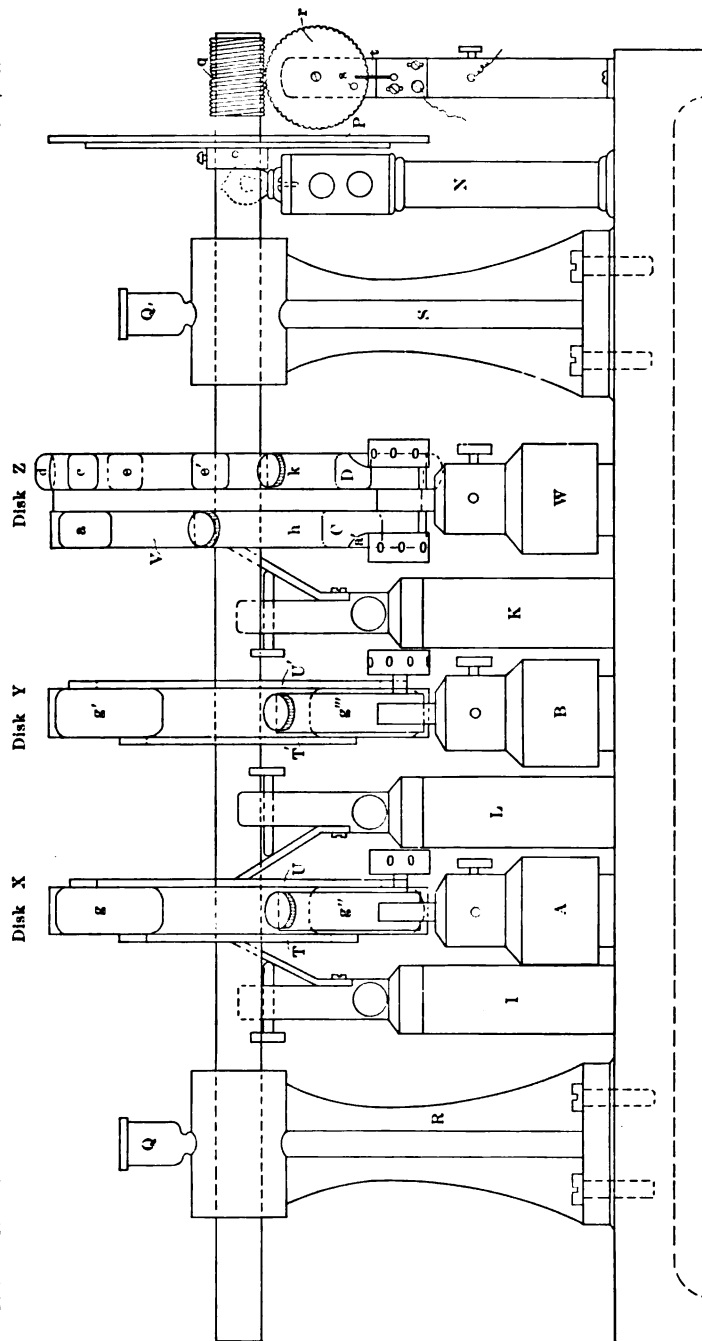


FIG. 14. ROTATING COMMUTATOR. PLAN.







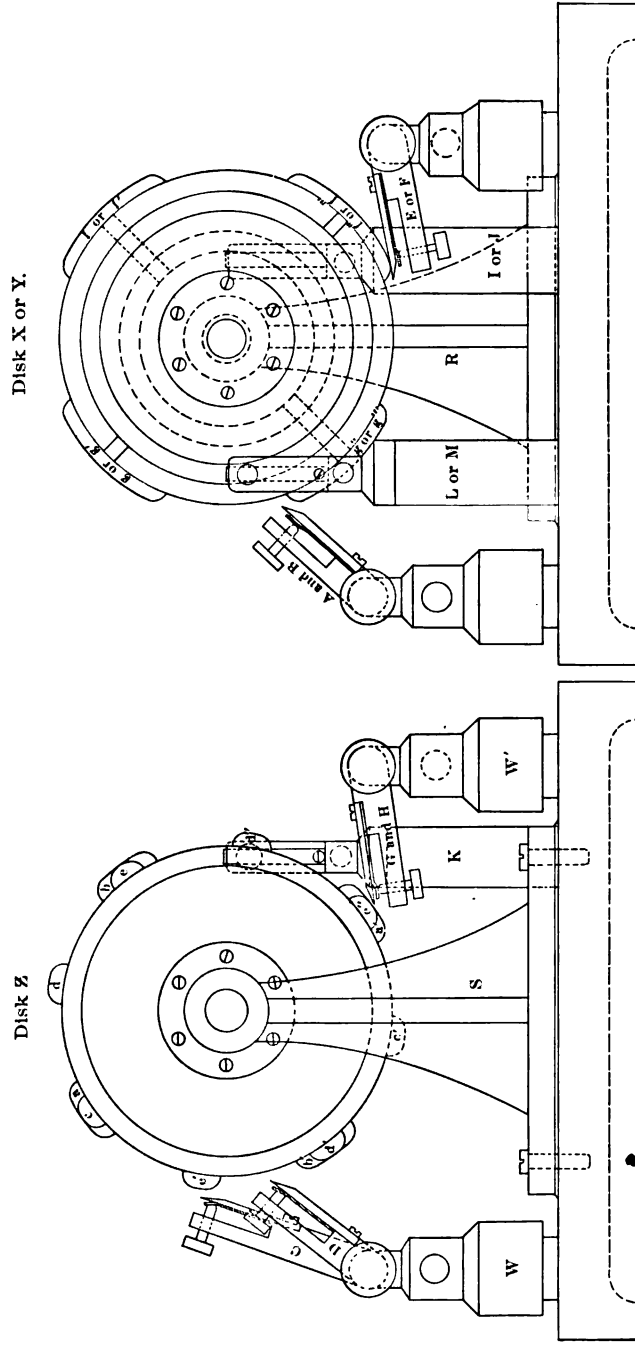


FIG. 16. ROTATING COMMUTATOR. END ELEVATION.





## 7. MEASUREMENT OF THE FREQUENCY.

As already stated, the original intention was to determine the frequency of the charge and discharge of the condenser by means of an electrically driven tuning fork, the rate of the fork being redetermined from time to time. The stroboscopic attachment to the commutator indicated when the speed of the commutator and of the fork were alike. It was, however, found in practice that the speed could be regulated by observing the deflections of the galvanometer much more readily than by watching the movements of the band of light in the microscope of the stroboscopic device. In the latter case the band of light would often be at an extreme position to the right or left where the sensibility is a minimum and insufficient; a second difficulty was that when the speed varied considerably it was not always easy to decide whether it was too fast or too slow, while a third objection is that it requires two observers constantly—one at the galvanometer telescope, the other at the microscope.

On the other hand, abandoning the stroboscopic device altogether the observer at the galvanometer telescope could, by controlling the speed of the driving motor, keep the galvanometer deflection continuously at or near zero, and the number of revolutions of the commutator and the beats of a chronometer being simultaneously recorded on the chronograph, the mean frequency for the period of the run could be afterwards calculated with great precision. The only disadvantage of this method is the labor of taking off the record from the chronograph sheet; but, on account of changes in the frequency of the fork, it is necessary to do this frequently even if the stroboscopic device is employed. We therefore abandoned the use of the tuning fork entirely and determined the speed of the commutator for every separate measurement by a sufficiently long record on the chronograph to give the desired accuracy.

In the earlier part of our work we used a small driving motor and found difficulty in securing sufficient constancy of speed to keep the galvanometer balanced. Small variations in the friction in bearings or brushes of commutator or motor varied the load and so the speed. We have since used two different driving arrangements which have been satisfactory. One was a testing set consisting of a 10-horsepower motor and two direct-connected generators, the commutator being directly coupled to the shaft at one end of the set. With this arrangement there was large momentum and very uniform friction, and the speed was controlled by a carbon rheostat in the armature circuit of

the driving motor. The brushes and bearings being kept in good condition, the speed could be maintained remarkably uniform for any desired period. The most serious objection to this arrangement was that the machines were too heavy to take into the laboratory, and the work was done in the dynamo room. The driving machine employed at present is a small motor-generator set (0.5 kw. motor and a 0.3 kw. alternator), which are mounted conveniently on a wooden base which also accommodates the commutator. A comparatively heavy fly wheel is used to steady the speed, and a light load is usually added by lighting up two electric lamps from the generator. The galvanometer is so sensitive that the slightest fluctuations in speed are noticeable, and hence bearings and brushes must always be kept in good condition and end thrust of the shaft avoided.

It is of course not possible to hold the speed exactly constant, nor the galvanometer exactly on zero. The observer keeps his hand constantly on the wheel of the carbon rheostat and quickly checks any tendency toward increase or decrease of speed as indicated by the galvanometer deflection going to the right or left. Such small deflections to the right and left are balanced against each other during the run so that the average deflection is as nearly zero as possible, and this can be done so well that there is no advantage in actually recording these deflections and then computing a correction from the average deflection, as the correction is insignificant. By this method of keeping the resistances constant and holding the deflection as nearly zero as possible for three or four minutes while the number of revolutions is being recorded on the chronograph we get nearly the same result as though the speed had been kept strictly constant and the mean of a number of settings of the resistance were taken.

We give below two tables showing the determination of frequency in a number of cases for each of the driving systems employed, that of March 30 using the heavy machines, when measuring the capacities of air condensers, and that of July 27 using the lighter machines, when measuring the capacities of some mica condensers.

TABLE III.—*Examples of the determination of the frequency in six runs of March 30.*

| Run 1.                                  |                  |                 | Run 3.                                  |                  |                 |
|-----------------------------------------|------------------|-----------------|-----------------------------------------|------------------|-----------------|
| Chronograph record.                     |                  | Interval.       | Chronograph record.                     |                  | Interval.       |
| Beginning.                              | End.             |                 | Beginning.                              | End.             |                 |
| <i>Min. sec.</i>                        | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                        | <i>Min. sec.</i> | <i>Seconds.</i> |
| 0 20.44                                 | 3 58.38          | 217.94          | 0 12.63                                 | 3 58.58          | 225.95          |
| 22.43                                   | 4 0.38           | .95             | 14.63                                   | 4 0.58           | .95             |
| 24.44                                   | 2.39             | .95             | 16.63                                   | 2.57             | .94             |
| 26.44                                   | 4.39             | .95             | 18.62                                   | 4.57             | .95             |
| 28.44                                   | 6.39             | .95             | 20.62                                   | 6.57             | .95             |
| 30.44                                   | 8.39             | .95             | 22.62                                   | 8.57             | .95             |
| 32.44                                   | 10.39            | .95             | 24.62                                   | 10.57            | .95             |
| 34.44                                   | 12.39            | .95             | 26.61                                   | 12.57            | .96             |
| 36.44                                   | 14.38            | .94             | 28.61                                   | 14.56            | .95             |
| 38.44                                   | 16.38            | .94             | 30.61                                   | 16.56            | .95             |
| 21,800 charges= 217.947 seconds.        |                  |                 | 22,600 charges= 225.950 seconds.        |                  |                 |
| $n = \frac{21,800}{217.947} = 100.024.$ |                  |                 | $n = \frac{22,600}{225.950} = 100.022.$ |                  |                 |
| Run 2.                                  |                  |                 | Run 4.                                  |                  |                 |
| <i>Min. sec.</i>                        | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                        | <i>Min. sec.</i> | <i>Seconds.</i> |
| 5 18.35                                 | 9 14.68          | 236.33          | 4 50.57                                 | 8 46.75          | 236.18          |
| 20.35                                   | 16.68            | .33             | 52.57                                   | 48.75            | .18             |
| 22.36                                   | 18.68            | .32             | 54.58                                   | 50.76            | .18             |
| 24.37                                   | 20.69            | .32             | 56.58                                   | 52.76            | .18             |
| 26.37                                   | 22.69            | .32             | 58.58                                   | 54.76            | .18             |
| 28.38                                   | 24.69            | .31             | 5 0.58                                  | 56.76            | .18             |
| 30.38                                   | 26.69            | .31             | 2.59                                    | 58.76            | .17             |
| 32.38                                   | 28.69            | .31             | 4.59                                    | 9 0.76           | .17             |
| 34.39                                   | 30.70            | .31             | 6.59                                    | 2.77             | .18             |
| 36.39                                   | 32.70            | .31             | 8.59                                    | 4.77             | .18             |
| 23,600 charges= 236.317 seconds.        |                  |                 | 23,600 charges= 236.178 seconds.        |                  |                 |
| $n = \frac{23,600}{236.317} = 99.868.$  |                  |                 | $n = \frac{23,600}{236.178} = 99.925.$  |                  |                 |

TABLE III.—*Examples of the determination of frequency—Continued.*

| Run 5.                                  |                  |                 | Run 6.                                 |                  |                 |
|-----------------------------------------|------------------|-----------------|----------------------------------------|------------------|-----------------|
| Chronograph record.                     |                  | Interval.       | Chronograph record.                    |                  | Interval.       |
| Beginning.                              | End.             |                 | Beginning.                             | End.             |                 |
| <i>Min. sec.</i>                        | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                       | <i>Min. sec.</i> | <i>Seconds.</i> |
| 1 2. 27                                 | 4 56. 19         | 233. 92         | 5 46. 19                               | 9 56. 33         | 250. 14         |
| 4. 27                                   | 58. 20           | . 93            | 48. 19                                 | 58. 34           | . 15            |
| 6. 28                                   | 5 0. 19          | . 91            | 50. 19                                 | 10 0. 34         | . 15            |
| 8. 27                                   | 2. 19            | . 92            | 52. 19                                 | 2. 34            | . 15            |
| 10. 28                                  | 4. 19            | . 91            | 54. 20                                 | 4. 34            | . 14            |
| 12. 27                                  | 6. 19            | . 92            | 56. 20                                 | 6. 34            | . 14            |
| 14. 28                                  | 8. 19            | . 91            | 58. 20                                 | 8. 34            | . 14            |
| 16. 28                                  | 10. 19           | . 91            | 6 0. 20                                | 10. 34           | . 14            |
| 18. 27                                  | 12. 19           | . 92            | 2. 20                                  | 12. 35           | . 15            |
| 20. 27                                  | 14. 19           | . 92            | 4. 21                                  | 14. 35           | . 14            |
| 23,400 charges = 233.917 seconds.       |                  |                 | 25,000 charges = 250.144 seconds.      |                  |                 |
| $n = \frac{23,400}{233.917} = 100.035.$ |                  |                 | $n = \frac{25,000}{250.144} = 99.943.$ |                  |                 |

TABLE IV.—*Examples of determination of the frequency from runs made July 27.*

| Run 1.                                                                      |                  |                 | Run 3.                                                                      |                  |                 |
|-----------------------------------------------------------------------------|------------------|-----------------|-----------------------------------------------------------------------------|------------------|-----------------|
| Chronograph record.                                                         |                  | Interval.       | Chronograph record.                                                         |                  | Interval.       |
| Beginning.                                                                  | End.             |                 | Beginning.                                                                  | End.             |                 |
| <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> |
| 2 38.71                                                                     | 6 5.37           | 206.66          | 15 46.51                                                                    | 19 17.23         | 210.72          |
| 40.50                                                                       | 7.18             | .68             | 48.26                                                                       | 18.98            | .72             |
| 42.29                                                                       | 8.98             | .69             | 50.03                                                                       | 20.73            | .70             |
| 44.10                                                                       | 10.78            | .68             | 51.73                                                                       | 22.47            | .74             |
| 45.90                                                                       | 12.58            | .68             | 53.48                                                                       | 24.20            | .72             |
| 47.69                                                                       | 14.38            | .69             | 55.22                                                                       | 25.95            | .73             |
| 49.49                                                                       | 16.17            | .68             | 56.96                                                                       | 27.69            | .73             |
| 51.28                                                                       | 17.97            | .69             | 58.71                                                                       | 29.43            | .72             |
| 53.08                                                                       | 19.76            | .68             | 16 0.45                                                                     | 31.17            | .72             |
| 54.89                                                                       | 21.56            | .67             | 2.18                                                                        | 32.90            | .72             |
| 23,000 charges= 206.680 seconds.<br>$n = \frac{23,000}{206.680} = 111.283.$ |                  |                 | 24,200 charges= 210.722 seconds.<br>$n = \frac{24,200}{210.722} = 114.844.$ |                  |                 |
| Run 2.                                                                      |                  |                 | Run 4.                                                                      |                  |                 |
| <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> |
| 8 50.27                                                                     | 12 16.09         | 205.82          | 21 25.77                                                                    | 24 55.48         | 209.71          |
| 52.07                                                                       | 17.84            | .77             | 27.49                                                                       | 57.23            | .74             |
| 53.80                                                                       | 19.60            | .80             | 29.24                                                                       | 58.96            | .72             |
| 55.56                                                                       | 21.35            | .79             | 30.97                                                                       | 25 0.69          | .72             |
| 57.34                                                                       | 23.12            | .78             | 32.70                                                                       | 2.42             | .72             |
| 59.08                                                                       | 24.87            | .79             | 34.45                                                                       | 4.16             | .71             |
| 9 0.84                                                                      | 26.63            | .79             | 36.17                                                                       | 5.90             | .73             |
| 2.60                                                                        | 28.38            | .78             | 37.90                                                                       | 7.62             | .72             |
| 4.37                                                                        | 30.16            | .79             | 39.63                                                                       | 9.35             | .72             |
| 6.12                                                                        | 31.90            | .78             | 41.37                                                                       | 11.08            | .71             |
| 23,400 charges= 205.789 seconds.<br>$n = \frac{23,400}{205.789} = 113.708.$ |                  |                 | 24,200 charges= 209.720 seconds.<br>$n = \frac{24,200}{209.720} = 115.392.$ |                  |                 |

TABLE IV.—*Examples of determination of the frequency from runs made July 27—Cont'd.*

| Run 5.                                                                      |                  |                 | Run 6.                                                                      |                  |                 |
|-----------------------------------------------------------------------------|------------------|-----------------|-----------------------------------------------------------------------------|------------------|-----------------|
| Chronograph record.                                                         |                  | Interval.       | Chronograph record.                                                         |                  | Interval.       |
| Beginning.                                                                  | End.             |                 | Beginning.                                                                  | End.             |                 |
| <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> | <i>Min. sec.</i>                                                            | <i>Min. sec.</i> | <i>Seconds.</i> |
| 27 44.45                                                                    | 31 12.18         | 207.73          | 32 39.65                                                                    | 38 10.34         | 210.69          |
| 46.16                                                                       | 13.90            | .74             | 41.38                                                                       | 12.07            | .69             |
| 47.86                                                                       | 15.60            | .74             | 43.10                                                                       | 13.76            | .66             |
| 49.59                                                                       | 17.32            | .73             | 44.79                                                                       | 15.48            | .69             |
| 51.31                                                                       | 19.04            | .73             | 46.52                                                                       | 17.19            | .67             |
| 53.05                                                                       | 20.76            | .71             | 48.24                                                                       | 18.90            | .66             |
| 54.75                                                                       | 22.48            | .73             | 49.94                                                                       | 20.62            | .68             |
| 56.47                                                                       | 24.19            | .72             | 51.65                                                                       | 22.34            | .69             |
| 58.19                                                                       | 25.91            | .72             | 53.37                                                                       | 24.07            | .70             |
| 59.91                                                                       | 27.64            | .73             | 55.08                                                                       | 25.75            | .67             |
| 24,200 charges= 207.728 seconds.<br>$n = \frac{24,200}{207.728} = 116.498.$ |                  |                 | 24,600 charges= 210.680 seconds.<br>$n = \frac{24,600}{210.680} = 116.764.$ |                  |                 |

Ten readings from the chronograph sheet at the beginning of the run are given in the first column and ten at the end of the run in column two; the differences in column three give the interval of time corresponding (in run 1, Table III) to 5,450 revolutions of the commutator or 21,800 charges of the condenser, since the condenser was charged and discharged four times for every revolution of the commutator. This gives a frequency of 100.024. The resistance of the third arm of the bridge was then increased by 500 ohms (from 342,460 to 342,960) and the speed decreased until the bridge was balanced, when run 2 was made, and 23,600 charges and discharges were effected in 236.317 seconds, or a frequency of 99.866. Condenser No. 2 was then put in place of 1, and the speed of the motor having been restored to its initial value, run 3 was made, giving a frequency of 100.022. The resistance was then increased by 350 (from 337,960 to 338,310), the speed reduced until the bridge was again balanced, and run 4 was made, showing a frequency of 99.925. Runs 5 and 6 were similarly made with the two condensers in parallel. The further data of these runs are given in Table VII.

Table IV gives the data for determining the frequency for six runs of July 27, using the smaller driving motor while measuring the capacity of some mica condensers of 0.01 mf each. The first five runs were made on five different condensers, and no attempt was made to have the speed the same for the different runs. The data given show that the mean speed of the motor, and thus the mean frequency of the charge and discharge of the condenser, can be determined with very high accuracy, the variations in the separate determinations of the interval being scarcely more than the error of reading the chronograph record. Of course there are slight variations of speed during the interval, but they are exceedingly small, and the galvanometer deflections show their magnitude. The actual deflection of course depends upon the sensibility of the galvanometer and the magnitude of the capacity being measured. Taking a special case as an example of running under favorable conditions, a change in the speed of one part in ten thousand produced a deflection of 1.0 mm; the maximum deflection during the run was not more than 1.0 mm, and hence the maximum deviation of the speed from uniformity was not more than one part in ten thousand. The average variation was probably not more than one part in twenty-five thousand. The error in the result due to variations in speed would be even less than this, since the positive and negative deflections are balanced against each other, and therefore the average speed will correspond to the setting of the bridge, probably to within one part in fifty thousand. Other sources of error are more troublesome than the uncertainty in the frequency of charge of the condenser.

It occurred to us that such a combination of a Wheatstone bridge and condenser could be used as an auxiliary apparatus for holding the speed of a motor steady for other purposes, and we have used it in that way for some recent absolute measurements of inductance.<sup>a</sup> There are other important physical quantities the determination of which involve the accurate measurement of angular velocity, as the velocity of light, and Lorenz's method of determining resistance in absolute measure, where this method of controlling the speed might be used advantageously.

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<sup>a</sup>The Absolute Measurement of Inductance, read at the International Electrical Congress, St. Louis, Mo. (See this Bulletin, p. 125.)

## 8. THE MEASUREMENT OF CAPACITY OF MICA CONDENSERS.

The Bureau of Standards possesses a mica condenser made by Siemens & Halske, of Berlin, which has been frequently measured during the past six months. It consists of four parts, of 0.5, 0.2, 0.2, and 0.1 mf, respectively. We give in Table V the detailed data of one determination of each section, and in Table VI the results of all the measurements which have been made on the four sections of the condenser. The temperature coefficient of a mica condenser is considerable, and it is difficult to ascertain what its temperature is unless it has been in a region of constant temperature for some hours. Some of the measurements given were made when there was an uncertainty of a degree or more in the temperature of the condensers, and hence the agreement of the results is not as good as the capacity determinations would demand. Nevertheless, the results show a very satisfactory concordance, and indicate that the capacity of the condensers (except possibly the smallest section) has remained very constant.

TABLE V.—*Determination of capacity of mica condensers.*[ $a=c=100$  ohms;  $b=3$ ,  $g=20$ .]

| Date.         | No. | Con-<br>denser. | $R$   | $F$     | $n$     | $C_1 = \frac{F}{nR}$ | $c$      | $C = C_1 - c$ |
|---------------|-----|-----------------|-------|---------|---------|----------------------|----------|---------------|
| 1904.         |     | mf              |       |         |         |                      |          |               |
| April 2 ..... | 1   | 0.2             | 49714 | 0.99884 | 99.980  | 0.200958             | 0.000087 | 0.200921      |
| Do .....      | 2   | 0.2             | 49914 | 0.99684 | 99.943  | 0.200226             | 0.000087 | 0.200189      |
| Do .....      | 3   | 0.1             | 99407 | 0.99942 | 100.073 | 0.100465             | 0.000037 | 0.100428      |
| Do .....      | 4   | 0.5             | 19793 | 0.99711 | 100.035 | 0.503593             | 0.000037 | 0.503556      |



TABLE VI.—*Summary of determinations of capacity of mica condensers.*

[One microfarad in four sections—0.5, 0.2, 0.2, and 0.1.]

| Condenser and temperature coefficient. | No. | Date.   | Temperature. | Measured values of $C$ . | Correction to $20^\circ$ . | $C_{20}^\circ$ . | Average deviation from mean.       |
|----------------------------------------|-----|---------|--------------|--------------------------|----------------------------|------------------|------------------------------------|
| 0.5 mf<br>-0.00020                     |     | 1904.   | °            | mf                       | mf                         | mf               | 0.000028 mf<br>=5.6 in<br>100,000  |
|                                        | 1   | Jan. 5  | 22.5         | 0.50330                  | +0.00025                   | 0.50355          |                                    |
|                                        | 2   | Jan. 27 | 22.8         | 0.50335                  | +0.00028                   | 0.50363          |                                    |
|                                        | 3   | Jan. 29 | 23.0         | 0.50328                  | +0.00030                   | 0.50358          |                                    |
|                                        | 4   | Apr. 2  | 20.0         | 0.50356                  | 0                          | 0.50356          |                                    |
|                                        | 5   | Apr. 12 | 15.5         | 0.50401                  | -0.00045                   | 0.50356          |                                    |
|                                        | 6   | May 20  | 24.0         | 0.50824                  | +0.00040                   | 0.50364          |                                    |
|                                        |     |         |              | Mean .....               |                            | 0.50359          |                                    |
| 0.2 mf<br>-0.000165                    | 7   | Jan. 5  | 22.5         | 0.200100                 | +0.000082                  | 0.200182         | 0.000027 mf<br>=13.5 in<br>100,000 |
|                                        | 8   | Feb. 3  | 22.5         | 0.200070                 | +0.000082                  | 0.200152         |                                    |
|                                        | 9   | Mar. 26 | 22.5         | 0.200050                 | +0.000082                  | 0.200132         |                                    |
|                                        | 10  | Mar. 29 | 23.0         | 0.200133                 | +0.000099                  | 0.200232         |                                    |
|                                        | 11  | Mar. 30 | 23.5         | 0.200011                 | +0.000115                  | 0.200126         |                                    |
|                                        | 12  | Apr. 2  | 20.0         | 0.200201                 | 0                          | 0.200201         |                                    |
|                                        | 13  | Apr. 12 | 15.5         | 0.200315                 | -0.000148                  | 0.200167         |                                    |
|                                        | 14  | May 20  | 24.0         | 0.200025                 | +0.000132                  | 0.200157         |                                    |
|                                        |     |         |              | Mean .....               |                            | 0.200169         |                                    |
| 0.2 mf<br>-0.00015                     | 15  | Jan. 5  | 22.5         | 0.200780                 | +0.000075                  | 0.200855         | 0.000027 mf<br>=13.5 in<br>100,000 |
|                                        | 16  | Feb. 3  | 22.5         | 0.200810                 | +0.000075                  | 0.200885         |                                    |
|                                        | 17  | Mar. 26 | 22.5         | 0.200854                 | +0.000075                  | 0.200929         |                                    |
|                                        | 18  | Mar. 29 | 23.0         | 0.200870                 | +0.000090                  | 0.200960         |                                    |
|                                        | 19  | Mar. 30 | 23.5         | 0.200770                 | +0.000105                  | 0.200875         |                                    |
|                                        | 20  | Apr. 2  | 20.0         | 0.200933                 | 0                          | 0.200933         |                                    |
|                                        | 21  | Apr. 5  | 21.5         | 0.200863                 | +0.000045                  | 0.200908         |                                    |
|                                        | 22  | Apr. 9  | 22.5         | 0.200840                 | +0.000075                  | 0.200915         |                                    |
|                                        | 23  | Apr. 11 | 14.0         | 0.201020                 | -0.000180                  | 0.200840         |                                    |
|                                        | 24  | Apr. 12 | 15.5         | 0.201038                 | -0.000135                  | 0.200903         |                                    |
|                                        | 25  | May 20  | 24.0         | 0.200758                 | +0.000120                  | 0.200878         |                                    |
|                                        |     |         |              | Mean .....               |                            | 0.200898         |                                    |
| 0.1 mf<br>-0.000175                    | 26  | Feb. 3  | 22.5         | 0.100410                 | +0.000043                  | 0.100453         | 0.000033 mf<br>=33 in<br>100,000   |
|                                        | 27  | Mar. 29 | 23.0         | 0.100379                 | +0.000052                  | 0.100431         |                                    |
|                                        | 28  | Apr. 2  | 20.0         | 0.100431                 | 0                          | 0.100431         |                                    |
|                                        | 29  | Apr. 11 | 14.0         | 0.100447                 | -0.000105                  | 0.100342         |                                    |
|                                        | 30  | Apr. 12 | 15.5         | 0.100470                 | -0.000078                  | 0.100392         |                                    |
|                                        | 31  | May 20  | 24.0         | 0.100310                 | +0.000070                  | 0.100380         |                                    |
|                                        |     |         |              | Mean .....               |                            | 0.100405         |                                    |

Table V is self-explanatory. The correction factor  $A$  was of course negligible. The quantity  $c$  is the capacity of the commutator, lead wire joining  $P$  to the condenser, and the terminal on the condenser to which the wire is joined. As this varies with the length and position of the wire joining the condenser to  $P$ , the correction capacity  $c$  is determined for every set of observations. In making this measure-

ment the plugs are removed, both wires remaining connected to the terminals of the condenser.

If  $l_1$  is disconnected from  $B$  (fig. 17) the capacity falls off from 0.0000366 to 0.0000256, due to the large reduction of the capacity from the terminal  $A$ , including the bar  $A'$  to the blocks  $B, C, D, E, F$ . If  $l_1$  is detached from  $A$  (leaving it as nearly as possible in its former position), the capacity  $c$  falls to 0.0000196, a further reduction due to subtracting the capacity of  $A A'$ . Adding 15 cm of wire, the capacity

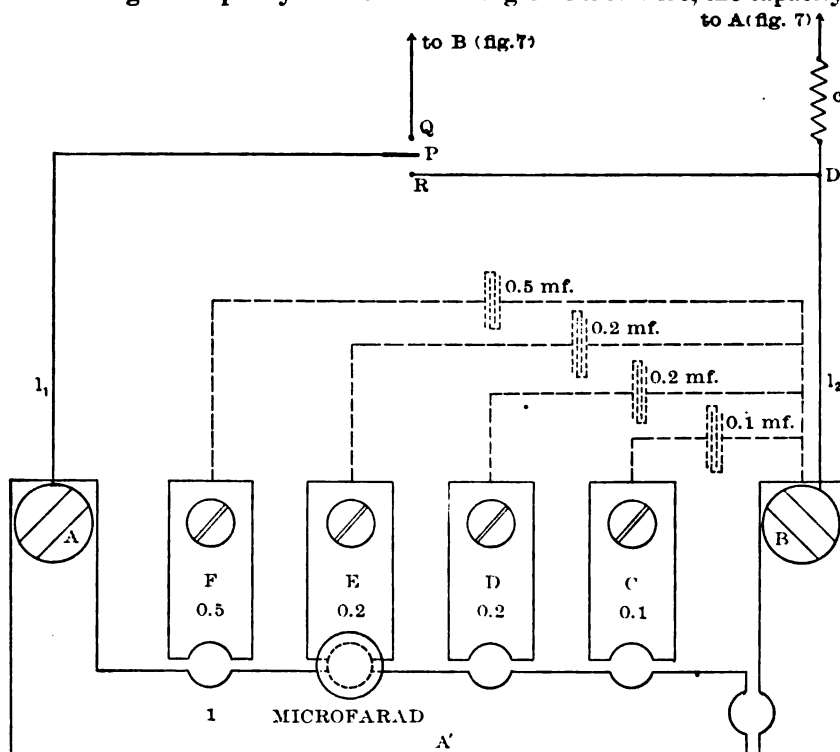


FIG. 17.—Connections for mica condenser.

$c$  increased to 0.0000215. These figures illustrate the importance of taking the capacity  $c$  always in the same way.

The chief cause of variation in the capacities of the mica condensers given in Table VI is the uncertainty in their temperature, as already stated. The measurements were made in the temporary laboratory of the Bureau of Standards where the diurnal change of temperature was excessive. The condenser was usually kept in a thermostat for some hours before measuring, but it was necessary to remove it from the thermostat and carry it to the dynamo room in the basement for

measurement, and an uncertain change in temperature resulted, in some cases probably amounting to a degree or more. Moreover, some of these measurements were made in testing the commutator and perfecting the method and were considered preliminary; more care would have been taken in ascertaining the temperature of the condenser if it had been supposed the results were to be of permanent value. We mean to make a new series of measurements under more favorable temperature conditions in the near future.

Plotting out the results for the four sections with temperatures and values of  $C$  as abscissas and ordinates, respectively, we find a slightly different temperature coefficient for each section, ranging from 15 to 20 parts in 100,000 per degree. Reducing the values of all to  $20^{\circ}\text{C}$ . by means of these temperature coefficients, we have the values given in column 7 of the table. The average deviation of a single determination from the mean is 5.6 parts in 100,000 for the 0.5 mf section, 13.5 for each of the two 0.2 mf sections, and 33 for the 0.1 mf section. The last is excessive and may possibly be due to an actual change in the value, the last three determinations differing considerably from the first three. With the exception of the smallest section these mean deviations amount to less than the variation in the capacity for  $0^{\circ}.7\text{C}$ . It is thus evident that if a series of determinations are to agree closely among themselves the temperature must be carefully determined. It is also evident that if the temperature is constant a good mica condenser is an instrument of precision and can be trusted to hold its value.

#### 9. THE MEASUREMENT OF CAPACITY OF AIR CONDENSERS.

The first measurements made were on a so-called air leyden, purchased by the Bureau from Kelvin & Jas. White. The results are shown in the first part of Table VII. The correction capacity  $c$  was very large in this case, because the commutator was in an adjoining room from the bridge and condenser, and the connecting wires were several meters in length. In this determination three values of the resistance  $d$  were used (388,100; 386,000; 384,000 ohms) and the speed varied to balance the bridge in each case.

The two larger air condensers, No. 1 and No. 2, were designed by us and built in the instrument shop of the Bureau. We desired a sufficiently large air capacity to use as a standard in obtaining the constant of a ballistic galvanometer, which latter was to be employed in determining the capacity of mica condensers with relatively long charging periods. Each condenser has 200 nickel-aluminum plates, about 20 cm

TABLE VII.—Summary of determinations of capacities of air condensers.

| Date.                                      | No. | Con-<br>denser.       | R       | F       | n       | $G_1 = \frac{F}{nR}$ | c          | C          | Mean of C           | C' (calc.)          | (obs.—calc.)        |
|--------------------------------------------|-----|-----------------------|---------|---------|---------|----------------------|------------|------------|---------------------|---------------------|---------------------|
| Jan. 27, 1904<br>a=100, c=1,000, b=6, g=20 | 1   | {<br>Air leyden.<br>} | 3831000 | 0.99996 | 100.045 | 0.00257582           | 0.00010603 | 0.00246979 | {<br>0.0024698<br>} | {<br>0.0024698<br>} | {<br>0.0024698<br>} |
|                                            | 2   |                       | 3860000 | 0.99996 | 100.590 | 0.00257580           | 0.00010603 | 0.00246977 |                     |                     |                     |
|                                            | 3   |                       | 3840000 | 0.99996 | 101.112 | 0.00257585           | 0.00010603 | 0.00246982 |                     |                     |                     |
| Apr. 12<br>a=c=100, b=6, g=20              | 1   | No. 1.....            | 340610  | 0.99982 | 100.144 | 0.0298116            | 0.0000203  | 0.0292913  | {<br>0.0292913<br>} | {<br>0.0292913<br>} | {<br>0.0292913<br>} |
|                                            | 2   | No. 1.....            | 341610  | 0.99982 | 99.851  | 0.0298116            | 0.0000203  | 0.0292913  |                     |                     |                     |
|                                            | 3   | No. 2.....            | 338711  | 0.99982 | 100.060 | 0.0295006            | 0.0000201  | 0.0294905  | {<br>0.0294905<br>} | {<br>0.0294905<br>} | {<br>0.0294905<br>} |
|                                            | 4   | No. 2.....            | 337711  | 0.99982 | 100.359 | 0.0295000            | 0.0000201  | 0.0294799  |                     |                     |                     |
|                                            | 5   | {Two in par-<br>}     | 169919  | 0.99964 | 100.070 | 0.0587983            | 0.0000238  | 0.0587655  | {<br>0.0587655<br>} | {<br>0.0587655<br>} | {<br>0.0587655<br>} |
|                                            | 6   | {allel.....<br>}      | 170418  | 0.99964 | 99.770  | 0.0587984            | 0.0000238  | 0.0587696  |                     |                     |                     |
|                                            | 7   | {Two in ser-<br>}     | 678098  | 0.99991 | 100.095 | 0.0147188            | 0.0000201  | 0.0146987  | {<br>0.0146990<br>} | {<br>0.0146990<br>} | {<br>0.0146990<br>} |
|                                            | 8   | {ies.....<br>}        | 679700  | 0.99991 | 99.944  | 0.0147194            | 0.0000201  | 0.0146993  |                     |                     |                     |
| May 21<br>a=c=100, b=6, g=20               | 1   | No. 1.....            | 287218  | 0.99979 | 116.500 | 0.0298796            | 0.0000212  | 0.0298583  | {<br>0.0298571<br>} | {<br>0.0298571<br>} | {<br>0.0298571<br>} |
|                                            | 2   | No. 1.....            | 286218  | 0.99979 | 116.915 | 0.0298771            | 0.0000212  | 0.0298559  |                     |                     |                     |
|                                            | 3   | No. 2.....            | 284217  | 0.99979 | 117.623 | 0.0299064            | 0.0000205  | 0.0298859  | {<br>0.0298876<br>} | {<br>0.0298876<br>} | {<br>0.0298876<br>} |
|                                            | 4   | No. 2.....            | 284217  | 0.99979 | 117.611 | 0.0299097            | 0.0000205  | 0.0298892  |                     |                     |                     |
|                                            | 5   | {Two in par-<br>}     | 140563  | 0.99957 | 118.947 | 0.0597704            | 0.0000270  | 0.0597434  | {<br>0.0597447<br>} | {<br>0.0597447<br>} | {<br>0.0597447<br>} |
|                                            | 6   | {allel.....<br>}      | 140093  | 0.99957 | 119.379 | 0.0597680            | 0.0000270  | 0.0597410  |                     |                     |                     |
|                                            | 7   | {Two in ser-<br>}     | 570518  | 0.99989 | 117.144 | 0.0149611            | 0.0000218  | 0.0149383  | {<br>0.0149386<br>} | {<br>0.0149386<br>} | {<br>0.0149386<br>} |
|                                            | 8   | {ies.....<br>}        | 568518  | 0.99989 | 117.547 | 0.0149622            | 0.0000218  | 0.0149404  |                     |                     |                     |

square, 0.064 cm thick, and 0.218 cm apart. They are mounted in a similar manner to the Kelvin "air leyden," being thoroughly insulated on hard-rubber columns and inclosed in a cylindrical case.

In the measurements the terminal which is joined to the case of the condenser is always connected to the point  $D$ , the other terminal not connected to any exposed surface being joined to  $P$ . If these connections are reversed the measured capacity depends upon the surroundings of the condenser, increasing if a person approaches the condenser. The correction capacity  $c$  must be made with even greater care than for mica condensers, since it is relatively larger. In joining the condensers in parallel and in series the cases of the condensers are joined together, in the former case the two interior terminals being connected to  $P$ .

The capacity in series is one-fourth the capacity in parallel, since the two are nearly equal. The object of measuring them in parallel and in series was to check the determination with the values calculated from the separate determinations. The agreement is very satisfactory.

The increase in the capacity of these condensers between April 12 and May 21 was due to a change in their adjustment due to moving. They were not designed to be portable.

We are indebted to Dr. N. E. Dorsey, assistant physicist of the Bureau, for valuable assistance.



## OPTICAL PYROMETRY.

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By C. W. WAIDNER and G. K. BURGESS.

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### INTRODUCTION.

It is becoming generally recognized by engineers and technical men in charge of industrial processes carried out at high temperatures that it is usually necessary to measure and control the temperatures of these processes, and many instances might be cited where a change of less than  $20^{\circ}$  C. in the heat treatment radically alters the resulting product, and often such a small temperature change occurring unnoticed necessitates later the rejection of the completed product.

For a long time the problem of estimating high temperatures was dependent on the trained eye of the workman, but with the high degree of accuracy with which temperatures must be controlled to-day in many specialized lines of work, the requirements are such as can be fulfilled only by the use of a sensitive pyrometer. The two great advantages resulting from the use of the pyrometer, which are at once evident, are:

(1) Once the proper method of working a particular product has been found, this operation can be indefinitely repeated, thus rendering possible the exact duplication of products.

(2) The reproduction of any particular product is no longer locked up in the experience of a few workers, but becomes a matter of permanent record, which may be consulted at any time.

In this connection should be emphasized the advantage arising from the use of the same standard scale of temperature whatever type of pyrometer is employed, for this alone renders possible that important factor in the advance of scientific and technical knowledge—the interchange of experience among men.

There are many instances in practice where it is impossible to make use of any form of pyrometer which must be brought into contact with the substance operated upon, whether it be from the inaccessibility of

the object, its being in motion, or because the contact may be detrimental to the object or pyrometer. For all these cases and also where a rapid examination for the uniformity of temperature over a considerable area is required, some form of pyrometer entirely separated from the substance or furnace and thus acting at a distance is required, that is an optical or radiation pyrometer; and again, for the estimation of very high temperatures, such a pyrometer is the only form of instrument available.

There is an impression current that an optical pyrometer is of necessity a very delicate, mysterious piece of laboratory apparatus, not fitted for shop practice, and not to be trusted except in the hands of an expert, and even then giving results of uncertain reliability; but one of the primary objects of this paper is to show that there are several trustworthy optical pyrometers available, simple in operation, and suited to the most varied and exacting requirements of scientific laboratories and technical works.

In response to numerous inquiries which have been addressed to the Bureau of Standards on the availability and choice of pyrometric methods for particular problems, an experimental investigation of all the leading types of optical pyrometers obtainable has been carried out. This investigation has also been stimulated by the great advances that have been made recently in the development of optical pyrometry, advances resulting in the production of several simple and trustworthy instruments, called into existence on the one hand by the pressing industrial need of them, and rendered possible on the other hand largely by the great progress made during the past ten years in our knowledge of the laws of radiation from incandescent bodies.

These questions will be treated under the following headings:

- (1) General discussion of optical pyrometry;
- (2) Laws of radiation;
- (3) Methods of optical pyrometry;
- (4) Description of instruments, including their calibration, range, sources of error, and precision;
- (5) Comparison of various types of optical pyrometers;
- (6) Special problems in optical pyrometry.

It may be well to state at this point by way of explanation of the method of treatment adopted in this paper, that it has been the aim of the authors to discuss the subject from the point of view of its application primarily to industrial processes, and to answer those questions, that their observations in the shop and consultations with the experts in charge of these processes have shown nearly always arise when the applications of optical pyrometers are considered. Their experience



with these instruments has also strongly impressed them with the wide field of usefulness of these pyrometers in scientific laboratories for many lines of research.

A résumé of the most important work done in recent years on the laws of radiation has been added for the two-fold reason that it is the basis of the entire subject of radiation pyrometry and that this work has not hitherto been available to English readers.<sup>a</sup>

### 1. GENERAL DISCUSSION OF OPTICAL PYROMETRY.

The temperature of bodies may be estimated from the radiant energy emitted, either in the form of visible light radiation or of the longer infra red waves that are studied by their thermal effects. For the estimation of temperature in this way use is made of the so-called laws of radiation. It would be beyond the scope of this paper to more than briefly outline the researches that have been made in recent years bearing on the laws of radiation. All that will be attempted here will be a statement of these laws, with a brief outline of the experimental evidence on which they are based, and the way in which they have been applied to give an idea of temperatures beyond the range of all ordinary pyrometers that have to be exposed to the temperatures to be measured, e. g., the temperatures of the filament of an incandescent lamp, the electric arc, the electric furnace, and the boiling points of metals.

A number of excellent pyrometers have been introduced into practice that are based on the photometric measurement of the intensity of the light emitted by incandescent bodies.

Most of these pyrometers measure photometrically the intensity of the red radiation. This is done for two reasons, first, because the color of the light from the incandescent source will undergo wide variations as the temperature changes and it will thus be difficult to compare it with the light from some standard source, so that by passing the radiation from both sources through a red glass (or prism) the photometry is reduced to the comparison of two lights of the same color; and secondly, the use of the red radiation enables the measurements to be carried down to lower temperatures, as red light is the first to become visible.

When we consider the enormous increase in the intensity of the light with rise in temperature, this method appears especially well adapted to the measurement of high temperatures. Thus, for example, if the intensity of the red light  $\lambda = 0.656\mu$ , emitted by a body at

<sup>a</sup>Since the beginning of this work an excellent discussion of the laws of radiation by A. L. Day and C. E. Van Orstrand has appeared in the *Astrophys. J.*, 19, p. 1; 1904.

1000° C. is called 1, at 1500° C. the intensity will be over 130, and at 2000° C. over 2100 times as great. The rapid increase of the photometric intensity of the light in comparison with that of the temperature is also shown by the following table from a paper by Lummer and Kurlbaum,<sup>a</sup> for light emitted by incandescent platinum. The strip of platinum, made in the form of a hollow cylinder, was heated electrically and the intensity of the light emitted from its surface was measured photometrically. The corresponding temperatures were measured by a thermocouple placed inside the platinum cylinder. If  $I_1$  and  $I_2$  are the intensities of the light emitted at the absolute temperatures  $T_1$  and  $T_2$  (not differing many degrees from one another), then if we write

$$\frac{I_1}{I_2} = \left( \frac{T_1}{T_2} \right)^x$$

the values of  $x$  at various absolute temperatures ( $t^\circ\text{C} + 273^\circ$ ) are as follows:

| $T^\circ \text{ abs.}$ | $x$ |
|------------------------|-----|
| 900°                   | 30  |
| 1000°                  | 25  |
| 1100°                  | 21  |
| 1200°                  | 19  |
| 1400°                  | 18  |
| 1600°                  | 15  |
| 1900°                  | 14  |

From this table<sup>b</sup> it will be seen that at 1000° absolute (727° C.) the intensity of the light increases twenty-five times as rapidly as the temperature; at 1900° absolute (1627° C.) fourteen times as rapidly.

It is this rapid change in intensity of light with change in temperature that makes it possible for the trained eye of the workman to estimate the approximate temperature in the many industrial operations dependent on temperature control.

It would therefore appear that a system of pyrometry based on the intensity of the light emitted by incandescent bodies would be an ideal one, inasmuch as a comparatively rough measurement of the photometric intensity would measure the temperature quite accurately. This, however, is only partly true; it is limited somewhat by the fact that different bodies, although at the same temperature, emit very different amounts of light. Thus the intensity of the radiation from

<sup>a</sup> Lummer and Kurlbaum: *Verh. d. Deutsch. Phys. Ges.*, 11, p. 89; 1900.

<sup>b</sup> Also these data give approximately  $Tx=25,000$ . See E. Rasch. *Ann. d. Phys.*, 14, p. 198; 1904.

incandescent iron or carbon at  $1000^{\circ}\text{C.}$ , for example, is many times greater than that emitted by magnesia or polished platinum at the same temperature. In other words, the intensity of the light emitted is not a function of the temperature alone, but is dependent on other properties of the body, such as the condition of its surface and its composition. Consequently, if any conclusions were drawn as to the temperatures of these bodies from the light that they emit, it might lead to large errors.

The possibility of erroneous conclusions that might be drawn from such a system of optical pyrometry is still further emphasized in considering the result to which it would lead if applied to the measurement of the temperatures of flames. Here the intensity of the light would vary with the thickness of the flame in the line of sight. If applied to the measurement of the temperature of a colorless Bunsen flame the method would entirely fail.

At first sight the difficulties here enumerated might seem insurmountable, and sufficient to condemn the use of optical pyrometry. Such, however, is fortunately not the case. It therefore becomes necessary to examine more in detail the principles on which optical pyrometry is based, its limitations, advantages, and the necessary precautions that must be observed in overcoming the difficulties enumerated above. For this purpose it will be necessary to briefly review the recent progress that has been made in the study of the radiation laws giving a connection between temperature and intensity of radiation.

Kirchoff<sup>a</sup> in one of the most important contributions to the theory of radiation was led to the important conception of what he termed a "black body," which he defined as one which would absorb all radiations falling on it, and would neither reflect nor transmit any. He further pointed out clearly the important fact that the radiation from such a body is a function of the temperature alone, and is identical with the radiation inside an inclosure all parts of which have the same temperature. The first experimental realization of a black body as a practical laboratory apparatus, was made by Wien and Lummer<sup>b</sup> by heating the walls of a hollow opaque inclosure as uniformly as possible and observing the radiation coming from the inside through a very small opening in the walls of the inclosure.

It is evident at once that such a body will absorb practically all the radiation incident through the small opening, and this whatever the material of the walls, for, unless the walls are totally reflecting, all

<sup>a</sup> Kirchoff: *Pog. Ann.*, 109, p. 275; 1860. The important contributions of Balfour Stewart in this field must also be considered.

<sup>b</sup> Wien and Lummer: *Wied. Ann.*, 56, p. 451; 1895.

the radiation will be absorbed, except that portion which might again escape through the opening; the presence of the opening therefore causes a slight departure from ideal black body conditions.

The radiation from a black body being a function of the temperature only, makes it of great importance in the experimental study of the laws of radiation, where a relation is sought between the temperature of the body and the energy radiated, whether the total energy or the energy associated with some particular wave length.

No body is known whose surface radiation is exactly that of a black body. The radiation from carbon and iron approximates fairly near to black body radiation, while the radiation from polished platinum and the white oxides departs very far from it. Black body radiation corresponding to temperatures from that of liquid air or lower up to  $1600^{\circ}\text{C}$ . (or much higher if suitable materials are chosen)<sup>a</sup> are now available in the laboratory. For temperatures up to about  $600^{\circ}\text{C}$ ., this is realized by immersing a metallic or other vessel in a constant temperature bath (liquefied gas, vapor, or fused salt) and observing the radiation from the interior through a small opening in the walls. At higher temperatures it is very difficult to heat the walls of the inclosure uniformly, especially with gas flames. Lummer and Kurlbaum<sup>b</sup> have very satisfactorily overcome this difficulty in their electrically heated black body, which is shown in section in fig. 1.

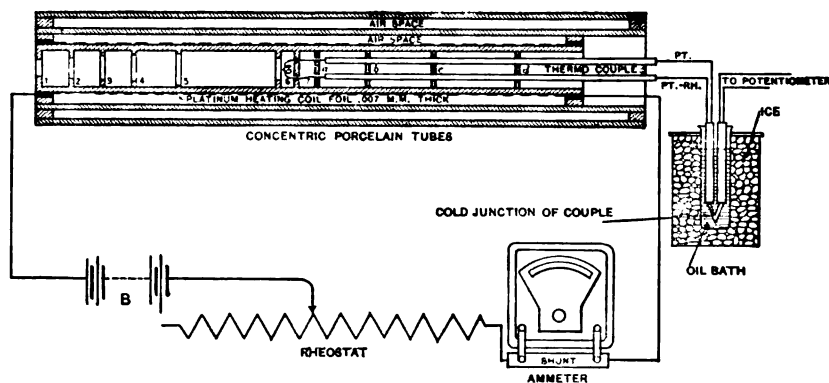


FIG. 1.—Electrically heated black body.

The central porcelain tube is wound over with thin platinum foil through which passes an electric current, which can be adjusted to

<sup>a</sup> Iridium, carbon, or refractory tubes that become conducting at high temperatures.

<sup>b</sup> Lummer and Kurlbaum: Verh. d. Deutsch. Phys. Ges., 17, p. 106-111; 1898. Ann. d. Phys. 117 (4), 5, p. 829-836; 1901.

maintain any desired temperature up to  $1600^{\circ}\text{C}$ . This tube is provided with a number of diaphragms to minimize the disturbing effects of air currents. To protect this inner tube from external influences and to diminish unnecessary heat losses, it is surrounded by several porcelain tubes and air spaces, as shown in the figure. The radiation from the uniformly heated region near the center and which passes out through the end of the tube, is a very close approximation to the ideal black body radiation of Kirchoff. The temperature of this central region is measured by means of a carefully calibrated thermocouple.

As has already been stated, if magnesia, porcelain, platinum, iron, etc., are heated to the same temperature they will emit very different amounts of light. If, however, these bodies<sup>a</sup> are heated inside a black body, they will all emit equal radiation, and on looking into the small opening all details of their contour will be lost, the whole region being of uniform brightness.

An electrically heated black body is used in the calibration of the various radiation pyrometers which will be described later. The question which naturally arises at this point is, what is the use of calibrating an optical pyrometer to measure the temperatures of a black body when in one case the instrument is to be used to measure the temperature of a piece of steel, and in another case the temperature of a piece of porcelain? If the porcelain were actually at a temperature of  $1200^{\circ}\text{C}$ ., say, such a calibrated optical pyrometer would give something like  $1100^{\circ}\text{C}$ ., because a black body at  $1100^{\circ}\text{C}$ . radiates as much as the porcelain does at  $1200^{\circ}\text{C}$ . On the other hand, for iron at  $1200^{\circ}\text{C}$ . the same optical pyrometer would give about  $1140^{\circ}\text{C}$ ., because iron approximates more nearly to a black body. Again, suppose one experimenter has found that in hardening a particular grade of steel a temperature of  $700^{\circ}\text{C}$ . ( $1290^{\circ}\text{F}$ .), as measured by a thermocouple, gives the desired product. Then if another experimenter used an optical pyrometer (calibrated against a black body) to measure the temperature of the same grade of steel just before hardening and found it  $700^{\circ}\text{C}$ ., ( $1290^{\circ}\text{F}$ .) he might get a product very different from that obtained by the first process, and for the reason that the steel would in reality be at a temperature of about  $720^{\circ}\text{C}$ . (at which temperature its radiation is equal to that of a black body at  $700^{\circ}\text{C}$ .).

It is clear that the answer to these questions, or at least a proper understanding of them, is of considerable importance. The most evi-

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<sup>a</sup>It is here assumed that the radiation is purely thermal and that no part is due to luminescence, as the laws of radiation are only directly applicable where such is the case.

dent solution is to make the body whose temperature is to be measured as nearly a black body as possible. Then the optical pyrometer will read true temperatures—i. e., the same as a calibrated thermocouple (gas thermometer scale).

For example, if a porcelain tube with closed bottom be immersed into the lead-hardening bath, the radiation coming from the bottom of this tube will be a close approximation to that of a black body, especially if the inside of the tube is blackened with iron or nickel oxide, for example. Again, such a tube, or a metal one, projecting into the gas heating or annealing furnace would solve the problem very satisfactorily. Moreover, in many of the practical cases that arise the conditions fortunately approximate those of a black body. Thus, the temperature of a piece of porcelain is not measured in the open air, but generally inside a furnace whose walls are very often at a temperature not very different from that of the porcelain. The same approximation to black body radiation arises in many cases, e. g., a pot of glass in the furnace, interior of kilns, annealing furnaces, etc. In many other cases, however, the radiation is very different from that of a black body, as for the white-hot gases from the Bessemer converter for which an optical pyrometer would give temperatures several hundred degrees too low. Nevertheless, its indications would always be consistent and would serve as a valuable check on the chemical transformations going on down in the heart of the furnace.

To summarize, the difficulties of and the objections to the use of optical pyrometers, which have been referred to, are then answered in the way already indicated, that in many of the technical operations the radiation is an approximation to black body radiation; in many others it can easily be made so, in which cases the various radiation pyrometers will give true temperatures, and when such ideal conditions can not be approximated, these pyrometers at least give consistent results and will likely be just as useful in most cases as if they recorded true temperatures. Thus in the problem already referred to when the temperature of a hardening bath was measured in one case with a thermocouple and in the other with an optical pyrometer the observer using the latter would soon learn the proper temperature at which to set his optical pyrometer and would then be able to repeat his conditions after that with an accuracy satisfying every requirement.

The term "*black body temperature*" has come into quite extensive use and is of great convenience in the discussion of pyrometric problems. The temperatures indicated by a radiation pyrometer that has been calibrated against a black body are known as black body tempera-

tures. Thus, in the example already given, where a piece of iron and a piece of porcelain were both at  $1200^{\circ}\text{C.}$ , the optical pyrometer, which used the red light emitted, gave as the temperature of these bodies  $1140^{\circ}$  and  $1100^{\circ}\text{C.}$ , respectively. This means that iron and porcelain at  $1200^{\circ}\text{C.}$  emit red light of the same intensity as is emitted by a black body at  $1140^{\circ}$  and  $1100^{\circ}\text{C.}$ , respectively. The "black body temperature" of these materials for green light would differ quite appreciably from that for red light. It is evident that if the "black body temperature" of different bodies, e. g., carbon and platinum, are equal, their actual temperatures may differ considerably ( $80^{\circ}\text{C.}$  or so, at  $1500^{\circ}\text{C.}$ ). This violates the ordinary conception of equal temperatures, which is based on thermal equilibrium between the bodies if brought into contact.

The results of our experiments on radiation from iron and copper oxides, porcelain, fire clay, and platinum are discussed on pages 243–250, under the head of "Departure from black body radiation."

The temperature of any body, therefore, as measured by an optical pyrometer, will in general (luminescence effects excluded) be lower than its true temperature by an amount depending on the departure of its radiation from that of a black body. There is another source of error, however, that may act in the direction of making the pyrometer read too high, namely, due to light reflected by the body whose temperature is being measured coming from surrounding flames and hotter objects. Experiments giving the magnitude of the errors that may arise from this cause and methods of eliminating them are discussed on page 253, under the head of "Diffuse and reflected light."

## 2. LAWS OF RADIATION.

### STEFAN-BOLTZMANN'S LAW.

Naturally the first relation sought between intensity of radiation and temperature was one for the total radiant energy sent out by a body. Numerous attempts to express such a relation were made by Newton, Dulong and Petit, Rosetti, and others. These attempts, however, merely resulted in empirical expressions that held only through narrow ranges of temperature. The first important step was made by Stefan,<sup>a</sup> who, in examining some of the experimental data of Tyndall on the radiation from incandescent platinum wire in the interval  $525^{\circ}\text{C.}$  to  $1200^{\circ}\text{C.}$ , was led to the conclusion that the energy

<sup>a</sup> Stefan: Ber. d. K. Akad. d. Wiss., Wien, 79 B, 2 Abth., p. 391–428; 1879.

radiated was proportional to the fourth power of the absolute temperatures. This relation seemed to be further supported by the best experimental data of other observers, at least to within the limit of accuracy of their observations. Stefan was under the impression that this was a perfectly general relation that held for the radiation of any solid body. Subsequent work has shown that this is not the case, and that it is only true for the energy of total radiation from a black body. This relation received independent confirmation from Boltzmann,<sup>a</sup> who deduced it from thermodynamic reasoning, assuming the existence of the light pressure in the direction of propagation of the wave, as was predicted by Maxwell and by Bartoli, making one further assumption as to the spectral composition of radiation undergoing change of density. The existence of the light pressure has since been determined experimentally by Lebedew<sup>b</sup> and by Nichols and Hull.<sup>c</sup> The remaining assumption made by Boltzmann concerning the spectral composition of the radiation was subsequently proved by Wien. The conditions imposed by Boltzmann, in his discussion on the nature of the radiation, were such as are fulfilled by the radiation from a black body. This relation, which has now come to be generally known as the *Stefan-Boltzmann radiation law*, may then be stated as follows:

*The energy radiated by a black body is proportional to the fourth power of the absolute temperature, or*

$$E = K(T^4 - T_0^4)$$

where  $E$  is the total energy radiated by the body at absolute temperature  $T$  to the body at absolute temperature  $T_0$ , and  $K$  is a constant.

This law has received abundant experimental support throughout the widest range within which temperature measurements can be made from the researches of Lummer, Kurlbaum, Pringsheim, Paschen, and others. Physicists have therefore come to regard this as something more than a purely empirical expression.

An illustration of the experimental evidence in support of this law is given in the following table taken from the experiments of Lummer and Kurlbaum.<sup>d</sup>

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<sup>a</sup> Boltzmann: Wied. Ann., **22**, p. 291; 1884; p. 31; 1884.

<sup>b</sup> Lebedew: Ann. d. Phys., **6**, p. 433; 1901.

<sup>c</sup> E. F. Nichols and G. F. Hull: Phys. Rev., **13**, p. 307; 1901: **17**, pp. 26, 91; 1903.

<sup>d</sup> Lummer and Kurlbaum: Verh. d. Deutsch. Phys. Ges., p. 110; 1898.



| Absolute temperature. |       | $\frac{E}{T^4 - T_0^4}$ |                    |             |
|-----------------------|-------|-------------------------|--------------------|-------------|
| $T$                   | $T_0$ | Black body.             | Polished platinum. | Iron oxide. |
| 372.8                 | 290.5 | 108.9                   |                    |             |
| 492                   | 290   | 109.0                   | 4.28               | 33.1        |
| 654                   | 290   | 108.4                   | 6.56               | 33.1        |
| 795                   | 290   | 109.9                   | 8.14               | 36.6        |
| 1108                  | 290   | 109.0                   | 12.18              | 46.9        |
| 1481                  | 290   | 110.7                   | 16.69              | 65.3        |
| 1761                  | 290   |                         | 19.64              |             |

It will also be seen from this table that while the intensity of the total radiation of iron oxide is four or five times that of polished platinum, it is still considerably less than that emitted by a black body. The total radiation from other than black bodies increases more rapidly than the fourth power of the absolute temperature, so that as the temperature is raised the radiation of all bodies approaches that of the black body.

#### LAWS OF ENERGY DISTRIBUTION.

Among the first facts to be noticed about the nature of the radiations sent out by bodies were that at low temperatures these radiations consisted of ether waves too long to affect the human eye. As the temperature was raised, shorter and shorter waves were added which could finally be detected by the eye, the first of the visible radiations producing the sensation termed red, then orange, etc., until the violet waves were reached, which were the shortest waves that the eye could detect.

It has long been known that if light from an incandescent source is passed through a prism the waves are spread out into a spectrum with the longer red waves at one end and the shorter violet waves at the other end. When instruments like the thermopile, bolometer, etc., for detecting minute quantities of radiant energy were exposed in the dark region of the spectrum beyond the red, it was found that a very considerable part, indeed the largest part, of the energy of the radiation was present in this region which is termed the *infra-red* part of the spectrum. Similarly by exposing fluorescent substances or photographic plates in the region of the spectrum beyond the violet, it was

found that radiations were also present in the *ultra-violet* part of the spectrum. Müller was among the first to map out the energy corresponding to different wave lengths in the infra-red region of the solar spectrum by exposing a thermopile in different parts of the spectrum. Tyndall, Desains, Curie, Lamansky, and others examined in a similar manner the spectrum of the sun, the electric arc, the lime light, the incandescent platinum lamp, etc. Soon after Langley brought out the bolometer, which was so admirably adapted to the measurement of minute energy of radiations, a great mass of valuable experimental data was obtained bearing on the spectral distribution of the energy of the radiation emitted by various bodies. Among the most important of these contributions must be mentioned the painstaking series of researches of Paschen, who examined the distribution of energy in the emission and absorption spectra of various substances.

Among the experimental facts established by these researches were, that by far the largest portion of the energy in the spectrum was found in the infra-red region, that the position of the wave length having the maximum energy depended on the temperature of the body, and that as the temperature was raised the energy of all the waves emitted increased, but the shorter waves more rapidly than the longer, so that the position (wave length) of maximum energy in the spectrum shifted toward shorter wave lengths. These facts are well illustrated by the curves shown in fig. 2 taken from a paper by Lummer and Pringsheim, in which the ordinates are proportional to the intensity of radiation emitted by a black body, and the abscissæ are wave lengths (in thousandths of a millimeter). Each curve was obtained by keeping the temperature of the radiating source constant, and passing the radiation through a fluorite prism, which has no appreciable selective absorption in the region of the spectrum here studied. The narrow platinum strip of a linear bolometer was then exposed in different parts of the spectrum and the intensity of the radiation, corresponding to different wave lengths, measured by the observed deflections of the galvanometer. These curves show that at temperature  $723^{\circ}$  abs., the wave length having the maximum energy is  $4.08\mu$ , and as the temperature of the radiating source is raised the position of the maximum is steadily shifted toward shorter wave lengths until at  $1646^{\circ}$  abs. it is at wave length  $1.78\mu$ . Such curves as are here shown, where the temperature is constant and the energy is measured corresponding to radiations of different wave lengths emitted by a body, are called *energy curves*, i. e., the relation determined is  $J=f(\lambda)$  for  $T=\text{constant}$ , where  $J$ =energy corresponding to wave length  $\lambda$  (strictly

the energy comprised in the region of the spectrum between  $\lambda$  and  $\lambda+d\lambda$  and  $T$  is the absolute temperature of the radiating source.

It is also interesting to study the change in the intensity of some particular wave length as the temperature of the radiating source is

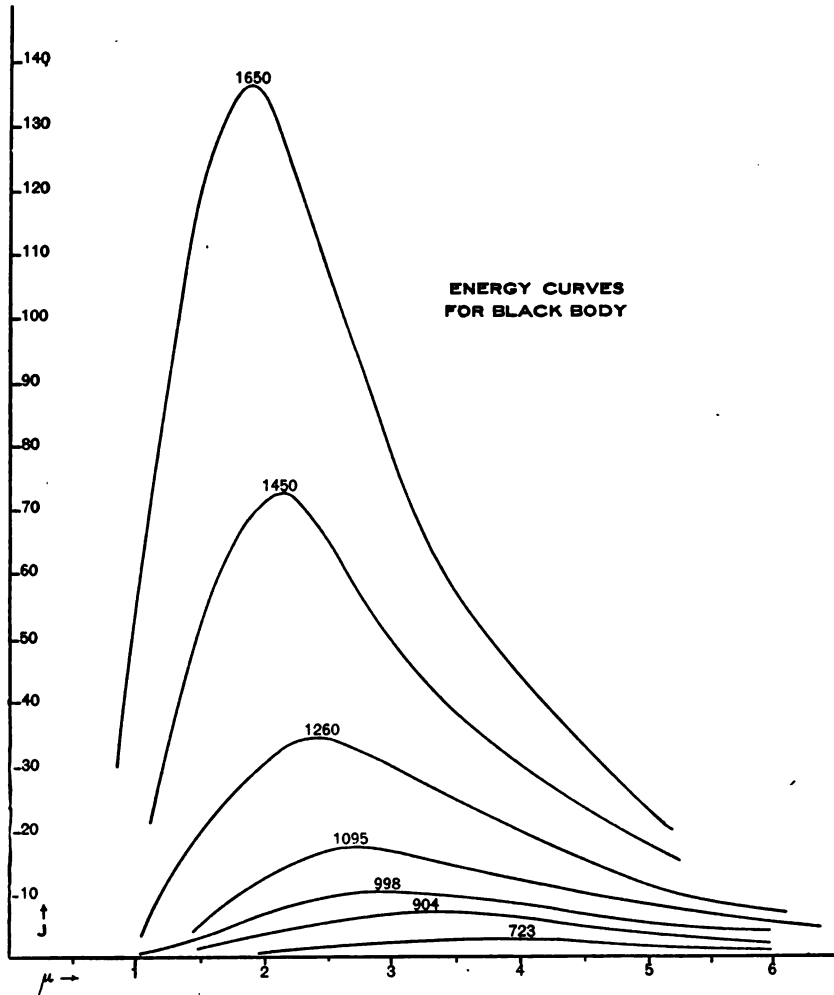


FIG. 2.—Energy distribution in black body radiation.

changed, i. e., to find  $J=F'(T)$  for  $\lambda=\text{constant}$ . This can be done by exposing the bolometer strip in a fixed part of the spectrum and observing the galvanometer deflections as the temperature is changed. The curves obtained in this way for  $J=F(T)$  are called *isochromatic*

*curves.* Both of these types of spectral radiation curves were first studied in a systematic and thorough manner for a large number of bodies by Paschen, who found that the general form of the radiation curves was the same for all bodies examined by him, i. e., the mathematical relation or form of function giving  $J = F(\lambda, T)$  that expressed his experimental results was of the same general type, although the constants in the equations were different for every different body. Under these circumstances it was evidently difficult, if not impossible, to arrive at any general law that would give a relation between  $J$ ,  $\lambda$ , and  $T$ .

#### WIEN'S LAWS.

This was in brief the experimental state of our knowledge concerning the spectral radiation of bodies, when Wien was led from theoretical considerations to most important generalizations. Starting with the well-grounded assumptions that (1) according to the electromagnetic theory the pressure of the radiation was equal to the energy in unit volume, (2) that the second law of thermodynamics, and (3) that Doppler's principle were applicable, and by postulating the existence of walls (inclosing radiant energy) that were completely black and others that were completely reflecting and diffusing, Wien<sup>a</sup> was led to the conclusion that "when the temperature increases, the wave length of every monochromatic radiation diminishes in such a way that the product of the temperature and the wave length is a constant," i. e.,

$$\lambda T = \lambda_0 T_0$$

Hence for the wave length of the maximum energy,  $\lambda_m$ ,

$$(I) \quad \lambda_m T = \text{const} = A$$

This is known as the "*displacement law*" and is simply a mathematical statement of the fact that as the temperature of the radiating source is changed the wave length having maximum energy in the spectrum will be changed in such a way that the product of this wave length  $\lambda_m$  and the corresponding absolute temperature of the source  $T$  is equal to a constant, i. e., if  $T$  increases  $\lambda_m$  shifts toward shorter wave lengths, and if  $T$  diminishes the shift is toward longer wave lengths. Wien then combined the above relation with the Stefan-Boltzmann law and was led to the relation that

$$(II) \quad J_{\max} T^{-5} = \text{constant} = B$$

<sup>a</sup> Wien: Ber. d. K. Akad. d. Wiss., Berlin, p. 55; 1893. Wied. Ann., 46, p. 633; 1893; 52, p. 132, 1894.

in which  $J_{max}$  indicates the energy corresponding to the wave length  $\lambda_m$  having maximum energy and  $T$  is the absolute temperature of the radiating source (black body).

W. Michelson<sup>a</sup> has criticized the application of the Stefan-Boltzmann law in the way it has been applied by Wien to monochromatic radiation, and takes exception to the use of the term temperature as applied to such radiation existing in the free ether. The above criticisms of Michelson, it seems, take away much of the theoretical support for this law (II). Notwithstanding this, however, both of these generalizations of Wien for the radiation emitted by a black body have received the most convincing experimental verification throughout the widest ranges of measurable temperatures that are at present available to the experimentalist.

For the radiation from other solids that have been examined experimentally it has been found that the "displacement law"

$$(Ia) \quad \lambda_m T = \text{const} = A_1$$

still holds true, although the radiation may depart far from that for a black body. In this case, however, the value of the constant is different from that for a black body. Thus for polished platinum Lummer and Pringsheim found  $A_1 = 2626$ .

For the radiation from other than a black body the law of maximum energy applies only in the modified form

$$J_{max} T^{-\alpha} = \text{const.}$$

Thus for polished platinum Paschen has found  $\alpha = 6.42$ , while Lummer and Pringsheim find  $\alpha = 6.00$ .

As an illustration of the experimental evidence in support of these two laws of radiation, the following table has been added, taken from a paper by Lummer and Pringsheim<sup>b</sup> on the radiation from a black body.

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<sup>a</sup> W. Michelson: J. Rus. Phys. Chem. Soc., 34 (5), p. 155; 1902. See Day and Van Orstrand: Astrophys. J., 19, p. 1; 1904.

<sup>b</sup> Lummer and Pringsheim: Verh. d. Deutsch. Phys. Ges., I, p. 218; 1899.

| Absolute temperature. | $\lambda_m$ | $E_m$ | $A = \lambda_m T$ | $B = E_m T^{-5}$      | $T = \sqrt[5]{E_m/B_{mean}}$ | Diff. |
|-----------------------|-------------|-------|-------------------|-----------------------|------------------------------|-------|
| °                     |             |       |                   |                       | °                            | °     |
| 1646                  | 1.78        | 270.6 | 2928              | 2246.10 <sup>17</sup> | 1653.5                       | +7.5  |
| 1460.4                | 2.04        | 145.0 | 2979              | 2184                  | 1460.0                       | -0.4  |
| 1259.0                | 2.35        | 68.8  | 2959              | 2176                  | 1257.5                       | -1.5  |
| 1094.5                | 2.71        | 34.0  | 2966              | 2164                  | 1092.3                       | -2.2  |
| 998.5                 | 2.96        | 21.50 | 2956              | 2166                  | 996.5                        | -2.0  |
| 908.5                 | 3.28        | 13.66 | 2980              | 2208                  | 910.1                        | +1.6  |
| 723.0                 | 4.08        | 4.28  | 2950              | 2166                  | 721.5                        | -1.5  |
| 621.2                 | 4.53        | 2.026 | 2814              | 2190                  | 621.3                        | +0.1  |
| Means                 |             |       | 2940              | 2188.10 <sup>17</sup> |                              |       |

As will be seen the results of experiment are in most satisfactory agreement with these laws, when one considers the experimental difficulties that are involved in the measurements. In the value for  $B$  the temperature enters to the fifth power, so that a small error in the temperature produces a very marked effect on the value of  $B$ .

The results of the above table are also shown by the curves in fig. 2, which show very clearly the shift of the maximum toward the visible end of the spectrum with rise in temperature.

Neither the displacement law,  $\lambda_m T = \text{const.}$ , nor the maximum energy law,  $J_m T^{-5} = \text{const.}$ , of Wien gives us any information as to the distribution of the energy in the spectrum of a black body at a given temperature. As has already been stated the problem was attacked experimentally by Paschen, who, inasmuch as the black body had not yet been experimentally realized, hoped to arrive at a generalization by measuring the distribution of the energy in the spectra of a number of bodies. As he stated it, "Gesetzmässigkeiten aufzufinden sucht, welche entweder allen Körpern gemeinsamen sind oder umso mehr zu Tage traten, je näher der Körper dem absolut schwarzen kommt."

About this time Wien<sup>a</sup> published the result of a further theoretical investigation on the spectral distribution of energy in the radiation of a black body, in which he was led to the conclusion that the energy corresponding to any wave length was represented by

$$(III) \quad J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

<sup>a</sup> Wien: Wied. Ann., 58, p. 662; 1896.

where  $J$  is the energy corresponding to wave length  $\lambda$  (region of spectrum  $\lambda$  to  $\lambda+d\lambda$ ),  $T$  is the absolute temperature of the radiating black body,  $e$  is the base of the natural system of logarithms, and  $c_1$  and  $c_2$  are constants.

In general the proof given by Wien is based on thermodynamic reasoning, but it involves some purely arbitrary assumptions as to the radiations sent out by vibrating gaseous molecules, so that, invaluable as has been the result to which this reasoning has led, it can not be claimed that this spectral distribution law of Wien has any strong theoretical basis.

This formula seemed to satisfy very well the experimental data of Paschen<sup>a</sup> and received independent confirmation by Planck,<sup>b</sup> who deduced Wien's formula in an entirely different way by theoretical reasoning based on the electromagnetic theory of light. Planck, however, in his proof was also compelled to make somewhat arbitrary assumptions (in his definition of the entropy of his electric resonators) so that his conclusions are open to the same criticism as those of Wien.

The further experiments of Paschen,<sup>c</sup> carried out with black bodies of different forms, seemed to be in most satisfactory agreement with this law. Paschen in summing up the results of these investigations says:

Wien's law may accordingly be regarded as demonstrated as well as the difficulties of the experiments admit, within the range of wave lengths from  $9.2\mu$  to  $0.7\mu$  (and in the research carried on in conjunction with Wanner<sup>d</sup> to  $0.5\mu$ ) and within the range of temperature from  $1300^\circ$  C. to  $100^\circ$  C.

With the theoretical confirmation of this law derived from such entirely different points of view, by Wien and by Planck, and with the strong experimental support afforded by the investigations of Paschen, it seemed at the time that the question of distribution of energy in the spectrum of a black body (the so-called Kirchhoff function) was fairly solved.

Subsequent experimental work, however, has shown that Wien's

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<sup>a</sup> Paschen: Wied. Ann., 58, p. 455; 1896.

<sup>b</sup> Planck: Ber. d. Akad. d. Wiss., Berlin, I, p. 440; 1899; pp. 57, 715, 1122; 1897; p. 449; 1898. Ann. d. Phys., I, pp. 69, 719; 1900.

<sup>c</sup> Paschen: Ber. d. K. Akad. d. Wiss., Berlin, pp. 405, 959; 1899. Astrophys. J., 10, p. 40; 1899; 11, p. 288; 1900.

<sup>d</sup> Paschen u. Wanner: Ber. d. K. Akad. d. Wiss., Berlin, 108, p. 5, 1899. Wanner: Ann. d. Phys., 2, p. 141; 1900.

distribution law does not hold for long-wave lengths. Thus, Lummer and Pringsheim<sup>a</sup> found for the so-called constant  $c_2$  the following:

| $\lambda$ | 1.21 $\mu$ | 4.56 $\mu$ | 8.3 $\mu$ | 12.3 $\mu$ | 17.9 $\mu$ |
|-----------|------------|------------|-----------|------------|------------|
| $c_2$     | 13510      | 16510      | 18500     | 24800      | 31700      |

Beckmann,<sup>b</sup> using the long-wave lengths, 24 $\mu$ , obtained by multiple reflections from fluorite, after the experiments of Rubens and Nichols,<sup>c</sup> found for  $c_2$  24250 (or 26000, as corrected by Rubens<sup>d</sup>). The experiments of Rubens and Kurlbaum<sup>e</sup> and of Paschen<sup>f</sup> confirm these results.

Rayleigh<sup>g</sup> has also criticised the Wien formula on the ground that the function has a maximum, which leads to the improbable result that the energy corresponding to a definite wave length would not go on increasing with rise in temperature beyond a certain point.

A number of other formulæ were proposed which represented empirically the results of experiments for certain regions of the spectrum and for certain temperature ranges somewhat better than the Wien formula. Among these formulæ may be mentioned those of Thiesen,<sup>h</sup> Lummer and Jahnke,<sup>i</sup> and others.

#### PLANCK'S LAW.

Planck<sup>j</sup> in a further theoretical consideration of the distribution of energy in the radiation of a black body has modified his assumptions and has been led to the following relation:

$$J = C_1 \lambda^{-5} \frac{1}{e^{\frac{c_2}{\lambda T}} - 1}$$

<sup>a</sup>Lummer: Rap. d. Cong. Int., Paris, 2; 1900. Lummer and Pringsheim: Verh. d. Deutsch. Phys. Ges., 2, p. 163; 1900. ZS. f. Instrk., 20, p. 149; 1900.

<sup>b</sup>Beckmann: Inaug. Dissert., Tübingen, 1898.

<sup>c</sup>Rubens and Nichols: Wied. Ann., 60, p. 418; 1897; Phys. Rev., 4, p. 314; 5, pp. 98, 152; 1897.

<sup>d</sup>Rubens: Wied. Ann., 69, p. 576; 1899.

<sup>e</sup>Rubens and Kurlbaum: Ber. d. K. Akad. d. Wiss., Berlin, p. 929; 1900. Ann. d. Phys., 4, p. 649, 1901.

<sup>f</sup>Paschen: Verh. d. Deutsch. Phys. Ges., 2, p. 202; 1900. Ann. d. Phys. 4, p. 277; 1901.

<sup>g</sup>Rayleigh: Phil. Mag., 49, p. 539; 1900.

<sup>h</sup>Thiesen: Verh. d. Deutsch. Phys. Ges., 2, p. 65; 1900.

<sup>i</sup>Lummer and Jahnke: Ann. d. Phys., 3, p. 283; 1900.

<sup>j</sup>Planck: Verh. d. Deutsch. Phys. Ges., 2, p. 202; 1900; Ber. d. k. Akad. d. Wiss., Berlin, p. 544; 1901.



An illustration of the remarkable manner in which this equation satisfies the results of experiments throughout the largest range of measurable temperatures and wave lengths is given by the following table taken from the experiments of Rubens and Kurlbaum.<sup>a</sup> This table shows the results of the measurements of the energy of radiation of wave length  $51.2\ \mu$ , obtained by multiple reflection (Reststrahlen) from rock salt surfaces, and the comparison of the several radiation formulæ with these measurements.

*Reststrahlen from rock salt,  $\lambda=51.2\mu$ .*

| Temper-<br>ature in<br>°C.<br><i>t</i> | Absol-<br>utetem-<br>pera-<br>ture.<br><i>T</i> | <i>E</i><br>obs. | <i>E</i><br>Wien. | <i>E</i><br>Thiesen. | <i>E</i><br>Rayleigh. | <i>E</i><br>Lumner<br>and<br>Jahnke. | <i>E</i><br>Planck. |
|----------------------------------------|-------------------------------------------------|------------------|-------------------|----------------------|-----------------------|--------------------------------------|---------------------|
| - 273                                  | 0                                               | —                | -121.5            | - 44                 | - 20                  | - 27                                 | - 23.8              |
| - 188                                  | 85                                              | - 20.6           | -107.5            | - 40                 | - 19                  | - 24.5                               | - 21.9              |
| - 80                                   | 193                                             | - 11.8           | - 48.0            | - 21.5               | - 11.5                | - 13.5                               | - 12.0              |
| + 20                                   | 293                                             | 0                | 0                 | 0                    | 0                     | 0                                    | 0                   |
| + 250                                  | 523                                             | + 31.0           | + 63.5            | + 40.5               | + 28.5                | + 31                                 | + 30.4              |
| + 500                                  | 773                                             | + 64.5           | + 96              | + 77                 | + 62.5                | + 65.5                               | + 63.8              |
| + 750                                  | 1023                                            | + 98.1           | +118              | +106                 | + 97                  | + 99                                 | + 97.2              |
| +1000                                  | 1273                                            | +132.0           | +132              | +132                 | +132                  | +132                                 | +132                |
| +1250                                  | 1523                                            | +164.5           | +141              | +154                 | +167                  | +165.5                               | +166                |
| +1500                                  | 1773                                            | +196.8           | +147.5            | +175                 | +202                  | +198                                 | +200                |
| + $\infty$                             | $\infty$                                        | —                | +194              | + $\infty$           | + $\infty$            | + $\infty$                           | + $\infty$          |

The experiments with the "Reststrahlen" from fluorite ( $\lambda=24.0\mu$  and  $31.6\mu$ ) are in equally good agreement with the Planck equation. It is interesting to note that for short wave lengths Planck's equation practically reduces to Wien's, and for long wave lengths to Rayleigh's equation. Wien's equation gives  $c_2=5\lambda_m T$  and Planck's  $c_2=4.965\lambda_m T$ . The difference between Wien's and Planck's equations, for such temperatures and wave lengths whose product  $\lambda T$  does not exceed 3000, is within 1 per cent. So that if we make use of a wave length  $\lambda=0.65\mu$  in an optical pyrometer, the results would be expressed by Wien's equation with every required degree of accuracy for the highest attainable temperatures ( $4000^\circ\text{C}$ . or more), and even for the longer waves in the infra red, including by far the largest

<sup>a</sup>Rubens and Kurlbaum: Ber. d. k. Akad. d. Wiss., Berlin, p. 929; 1900; Ann. d. Phys., 4, p. 649; 1901.

part of the energy of total radiation, this equation (Wien's) represents the results of experiment with a very fair degree of approximation.

The remarkable agreement of the Planck equation with the results of experiment throughout the extreme ranges in which it has been tested at least justifies the hope that it is something more than a purely empirical relation, which may some day earn the title of a "law" of nature.

### III. METHODS OF OPTICAL PYROMETRY.

The subject of Optical Pyrometry might perhaps be more generally classed under the heads of *pyrometry at a distance*, or *radiation pyrometry*, so as to include all methods in which the pyrometer is separated from the body whose temperature is to be measured; but we shall in general use the single expression, Optical Pyrometry, in what follows.

There are numerous methods which suggest themselves as forming a feasible basis for a system of optical pyrometry, and the fact that certain only of the many possible ways of measuring temperature from a distance have been chosen results naturally from a process of elimination either before or after their attempted application; and again, the fact that there is more than one type of optical pyrometer in general use is evidence of the adaptability of special forms to particular needs.

#### I. COLOR ESTIMATION.

The most widely used method for the estimation of high temperatures is still, unfortunately, by means of the unaided eye, which was naturally the first one to be employed.

The earliest attempt at gauging temperatures of incandescent bodies was made by Pouillet in 1836,<sup>a</sup> who suggested his color scale, giving a name to represent each hundred-degree interval from dull red, 600°-700° C., to dazzling white, 1600°-1700° C., as estimated by the unaided eye. This scale is still quoted as authoritative in text-books and tables of constants to-day, which is a deplorable fact, as the estimation of color change with temperature is largely a personal matter, and, more than that, varies for the same person with the exterior lighting; and although Pouillet's scale was undoubtedly the best to be had in 1836 and for many years after, yet it is at least 100° C. out over almost its whole range, as interpreted by H. M. Howe and others, as is shown by the following table<sup>b</sup> in which the values of Howe and of White and Taylor were obtained quite independently. The column  $\Delta$ , gives

<sup>a</sup>Pouillet: C. R., 3, p. 784; 1836.

<sup>b</sup>Howe: Eng. & Min. J., 69, p. 75; 1900. White and Taylor: *ibid.*, Dec. 23; 1899.

Pouillet's values less White and Taylor's in degrees Fahrenheit, and column  $\Delta$ , the difference between Howe's and White and Taylor's estimations.

| White and Taylor.             | C.   | F.   | Pouillet.          | C.    | F.    | $\Delta_1$ | Howe.                           | C.    | F.    | $\Delta_2$ |
|-------------------------------|------|------|--------------------|-------|-------|------------|---------------------------------|-------|-------|------------|
|                               |      |      |                    |       |       |            | Lowest red visible in dark .    | 470   | 878   | { -90      |
|                               |      |      | Lowest red....     | 525   | 977   | .....      | Lowest red visible in daylight. | 475   | 887   | <P. ....   |
| Dark red, blood red, low red. | 566  | 1050 | Dark red.....      | 700   | 1292  | +242       | } Dull red.....                 | 550   | 1022  | -28        |
| Dark cherry red.              | 635  | 1175 | Lowest cherry.     | 800   | 1472  | +297       |                                 | 625   | 1157  | -18        |
| Cherry, full red.             | 746  | 1375 | Cherry .....       | 900   | 1652  | +277       | Full cherry....                 | 700   | 1292  | -83        |
| Light red, light cherry.      | 843  | 1550 | Bright cherry.     | 1000  | 1832  | +282       | Light red .....                 | 850   | 1562  | +12        |
| Orange .....                  | 899  | 1650 | Dark orange ..     | 1100  | 2012  | +362       | .....                           | ..... | ..... | .....      |
| Light orange...               | 941  | 1725 | Bright orange.     | 1200  | 2192  | +467       | .....                           | ..... | ..... | .....      |
| Yellow .....                  | 996  | 1825 | .....              | ..... | ..... | .....      | Full yellow....                 | 950   | 1742  | -83        |
| Light yellow...               | 1079 | 1975 | .....              | ..... | ..... | .....      | to to                           | 1000  | 1832  | + 7        |
| White .....                   | 1205 | 2200 | White .....        | 1300  | 2372  | +172       | Light yellow...                 | 1050  | 1922  | -53        |
|                               |      |      | Very bright white. | 1400  | 2552  | .....      | White .....                     | 1150  | 2108  | -98        |
|                               |      |      | Dazzling white     | 1500  | 2732  | } to to    | .....                           | ..... | ..... | .....      |
|                               |      |      |                    | 1600  | 2912  |            | .....                           | ..... | ..... | .....      |

It is evident that skilled observers may disagree by  $50^\circ$  C. ( $90^\circ$  F.) or more in the estimation of relatively low temperatures by color, and that beyond  $1200^\circ$  C. ( $2200^\circ$  F.) it is practically impossible to make eye estimations with any certainty whatever.

Cobalt glasses have sometimes been used to aid the eye to exaggerate the changes in hue but this method is not reliable and varies with the observer and glass used. Similarly, matching the hues against pigments or colored yarns, as has been tried, gives hardly better results. A polarizing apparatus utilizing approximately a single hue was devised by Mesuré and Nouel, which will be described later.

## II. PHOTOMETRIC METHODS.

Becquerel,<sup>a</sup> in 1862, was probably the first to attempt the measurement of high temperatures by optical means. Studying the irradiation of incandescent bodies, he suggested the possibility of estimating

<sup>a</sup> Becquerel: C. R., 55, p. 821-829; 1862. Ann. de Chem. et de Phys., (3) 68, p. 49-143; 1863.

their temperatures by means of photometric measurements of the intensity of the red radiation, although he did not develop an instrument suitable for that purpose and many of his observations on the brightness of bodies were incorrect and some of his conclusions in contradiction with Kirchoff's law.

The principle suggested by Becquerel gave rise to the first practicable optical pyrometer, and to-day in the various modified forms still furnishes our most trustworthy and convenient method.

In practice a standard of intensity is chosen, and the brightness of the photometric field illuminated by the red light emitted by the body under observation is varied until it is equal in intensity to that of the standard either by use of an iris diaphragm (Le Chatelier), an absorbing wedge (Féry), or by a polarizing device (Wanner), or the standard light may itself be varied according to some known or empirical relation (Holborn-Kurlbaum, Morse).

For monochromatic black-body radiation, and for the wave lengths of the visible spectrum, Wien's law

$$J = C_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

is directly applicable to this type of pyrometer.

The above formula may be written

$$\log_{10} J = K_1 + K_2 \frac{1}{T},$$

$$\left\{ \begin{array}{l} \text{where } K_1 = \log c_1 - 5 \log \lambda \text{ and} \\ K_2 = c_2 \frac{\log e}{\lambda} \end{array} \right.$$

which is a linear relation between  $\log J$  and  $\frac{1}{T}$ , so that it requires measurements of the intensity at two known temperatures only to calibrate an instrument.

If the radiation is not black-body radiation, the same formula holds for calibration and use, but with different values of the constants  $K_1$  and  $K_2$ .

Assuming Wien's law to hold, instruments calibrated in terms of it may be used to estimate temperatures indefinitely high.

The pyrometers of Holborn-Kurlbaum and of Morse are based upon an empirical relation between the brightness of a filament and the electric current traversing it when the filament is of the same brightness as the incandescent body under observation. Their calibration, therefore, is not so simple as in the preceding case, as none of the radiation laws apply.

## III. RATIO OF INTENSITIES OF TWO WAVE LENGTHS.

Crova, in 1878,<sup>a</sup> devised a method based on the relative variation in intensity of two different monochromatic radiations, red and green; but as Violle has shown, the ratio of intensities of  $\lambda = .656 \mu$  (red) and  $\lambda = .482 \mu$  (blue) varies only in the ratio 1 to 4.5 over a temperature interval of  $700^{\circ} \text{C.}$ , and again the lower limit of temperature measurement is given by the green (about  $1200^{\circ} \text{C.}$ ) rather than by the red, so that Crova can not be said to have made use of a practicable principle, although he actually used his spectrophotometer in industrial works.

## IV. UPPER LIMIT OF THE SPECTRUM.

Another method of hardly greater precision than the last but susceptible of being used at much lower temperatures was also suggested by Crova (loc. cit.); namely, using the upper limit of the spectrum of an incandescent body as a measure of its temperature. Hempel<sup>b</sup> has recently tried this method with a special form of spectroscope and using a luminescent screen for observing when the upper spectrum limit is beyond the visible radiations, but compared with other photometric and radiation pyrometers, only crude results can be obtained.

## V. MAXIMUM ENERGY IN SPECTRUM.

(a) *Displacement of  $\lambda_m$ , Wien's Law I.*—Use might also be made of the displacement law,

$$\lambda_m T = \text{const.}$$

for the determination of temperatures, the constant varying from 2930 for a black body to 2620 for polished platinum; some form of bolometer or thermopile could be used in connection with a dispersion apparatus to locate the position of  $\lambda_m$  in the spectrum.

For substances whose radiation is intermediate between that of a black body and polished platinum the temperature determined by this relation will be between the values

$$\frac{2930}{\lambda_m} \quad \text{and} \quad \frac{2620}{\lambda_m}$$

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<sup>a</sup> Crova: *Journal de Phys.*, 7, p. 357; 1878; 9, p. 196; 1879. *Ann. de Chem et de Phys.*, (5) 19, p. 472; 1880.

<sup>b</sup> Hempel: *Angewandte Chem.*, 14, p. 237; 1901.

As an illustration of this range of limiting temperatures, the results of measurements due to Lummer and Pringsheim<sup>a</sup> may be cited:

|                         | $\lambda_m$ | $T_{max}$  | $T_{min}$  |
|-------------------------|-------------|------------|------------|
| Arc light .....         | 0.7 $\mu$   | 4200° abs. | 3750° abs. |
| Nernst lamp .....       | 1.2         | 2450       | 2200       |
| Welsbach mantle.....    | 1.2         | 2450       | 2200       |
| Incandescent lamp ..... | 1.4         | 2100       | 1875       |
| Candle .....            | 1.5         | 1960       | 1750       |
| Argand burner.....      | 1.55        | 1900       | 1700       |

The displacement law is among the best established of the radiation laws, both from the theoretical and experimental side, so that this method is of considerable theoretical interest, yet it is not capable of a high degree of precision on account of the difficulty of locating  $\lambda_m$ , due to the flattened form of the energy curve and the effect of atmospheric absorption.

(b) *Magnitude of maximum energy, Wien's Law II.*—The application of the maximum energy law

$$T = C\sqrt{J_{max}}$$

requires the same elaborate installation as the preceding method, and no pyrometer has been constructed utilizing either of these two laws.

## VI. TOTAL RADIATION.

Another method available for the determination of temperatures is the measurement of the energy of total radiation, visible and invisible, emitted by a body. It is evident in this case that, with a suitable recording device, the temperature of bodies could be determined even before they glow, and indeed theoretically the method might be applied throughout the whole temperature range, in which the sensibility would be least at temperatures near that of the instrument.

Many instruments have been constructed for the measurement of the energy of total radiation, such as the bolometer of Langley, Crooke's radiometer as modified by Rubens and Nichols, the actinometers of Pouillet and Violle, the thermopile, and the radiomicrometer as developed by Boys.

The use of these instruments in general involves elaborate and special installations, and they have found only limited application in

<sup>a</sup> Lummer and Pringsheim: Verh. d. Deutsch. Phys. Ges., 4, p. 36; 1901.

the laboratory for the measurement of temperature. Féry<sup>a</sup> has recently brought out a convenient instrument called a thermoelectric telescope, based on the energy of total radiation, which is measured by the rise in temperature produced at one junction of a very sensitive thermocouple joined to a direct-reading galvanometer.

If the radiation is that emitted by a black body, the calibration of these instruments may be based on the Stefan-Boltzmann law

$$E = K(T^4 - T_o^4)$$

and they may then be used to measure the temperature of black bodies, and by extrapolation temperatures may be estimated far beyond the upper limit attainable with thermocouples. For bodies departing from the black-body conditions the measurements may be expressed either in black-body temperatures or the calibration must be carried out empirically for every different radiating body observed.

#### VII. INTERFERENCE METHOD.

D. Berthelot,<sup>b</sup> making use of the law of variation of index of refraction of gases with density on the one hand and with temperature on the other, has devised an interferential gas thermometer at constant pressure which is independent of the containing envelope,<sup>c</sup> and although he has obtained results in admirable agreement with the ordinary gas scale his method of measurement requires a very elaborate installation whose description can only be most briefly given here.

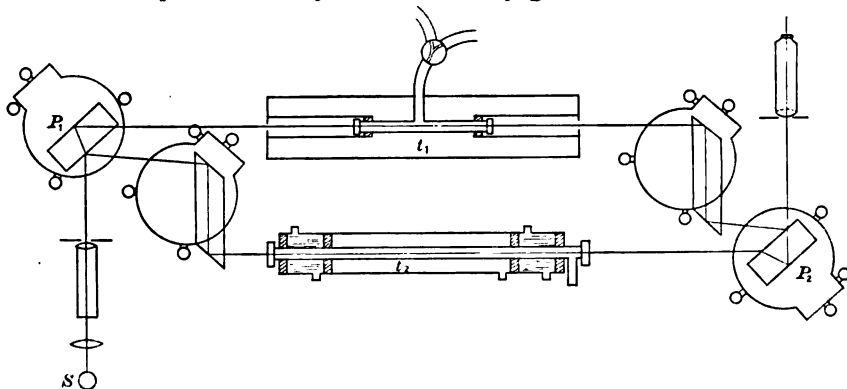


FIG. 3.—Berthelot interference method.

<sup>a</sup> Féry: C. R., **134**, p. 977; 1902.

<sup>b</sup> D. Berthelot: C. R., **120**, p. 831; 1895. Ann. de Chem. et de Phys., (7) **26**, p. 58; 1902.

<sup>c</sup> Provided the envelope does not give off gases or vapors that may influence the results.

Light from a source  $S$  is divided into two beams by a thick plate  $P_1$ . One beam is passed through a tube  $t_1$  at constant temperature, in which the pressure may be varied; the other passes through the tube  $t_2$ , which is heated electrically. The two beams are reunited at the plate  $P_2$  and give a series of interference bands in the eye-piece. The manipulation consists in diminishing the pressure in  $t_1$  as the temperature rises in  $t_2$ , so as to always keep the fringes stationary. Then from the relation connecting the temperature of a gas with its pressure, density, and index of refraction, proved to hold throughout a considerable range and assumed to hold for higher temperatures, it can be shown that the temperature of the gas in the hot tube is defined in terms of the *equivalent* lengths of the two tubes and the temperature and pressure of the gas in the cold tube.

This method is evidently an independent system of gas thermometry, and while not in the strict sense of the word a method of optical pyrometry, it may eventually be of service in the extension of the gas scale to higher temperatures than can be obtained by the ordinary forms of gas thermometer, and so serve as an independent check on the radiation laws at these high temperatures.

#### 4. DESCRIPTION OF INSTRUMENTS AND INVESTIGATION OF THEIR CALIBRATION, RANGE, SOURCES OF ERROR, AND PRECISION.

##### LE CHATELIER OPTICAL PYROMETER.<sup>a</sup>

The introduction of a serviceable and simple optical pyrometer was reserved to Le Chatelier in 1892, after he had perfected the thermoelectric pyrometer. Realizing the limitations of the latter for very high temperatures and for inaccessible bodies, he devised his optical pyrometer, which is a modification of Cornu's microphotometer, an instrument for the determination of the brightness of flames. Le Chatelier recognized that he could adjust to equality the two photometric fields from the standard lamp and the incandescent body respectively, either by cutting down the objective aperture, or interposing absorbing screens, or by using a polarizing device. He discarded the last on account of the resulting loss in light and the consequent inability to read relatively low temperatures, and also because the light from incandescent bodies being partially polarized would introduce complications, and he therefore adopted a cat's-eye diaphragm before the objective.

<sup>a</sup>Le Chatelier: C. R., 114, pp. 214, 470; 1892. J. de Phys., (3) 1, p. 185; 1892. High Temperature Measurements (Wiley & Sons). This instrument is made by M. Pellin, 21, Rue de l'Odéon, Paris.



Light from the central portion of the flame of a small gasoline lamp  $L$ , after passing through the lens  $L_1$ , is reflected from the  $45^\circ$  mirror  $M$  through a small stop and is brought to a focus in the focal plane of the eyepiece, where it is observed through a red glass. This gives a red comparison field of constant intensity. The lamp  $L$  is mounted eccentrically and may be turned so that the image of the flame is exactly bisected by the edge of the mirror  $M$ . Light from the incandescent source under observation is focused by the objective and, passing by the edge of the  $45^\circ$  mirror, forms a red field immediately beside and touching the first.

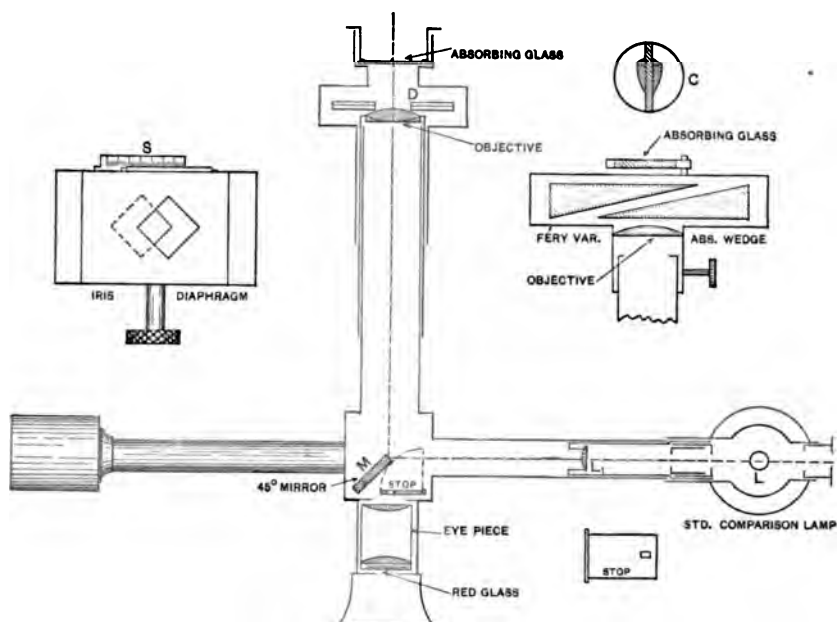


FIG. 4.—Le Chatelier optical pyrometer—section.

A measurement of temperature is made photometrically by bringing the two red fields to the same brightness. This is effected by opening or closing the iris diaphragm  $D$  before the objective, thus letting in more or less light from the body whose temperature is sought. For very high temperatures one or more absorbing glasses whose coefficient of absorption is known are placed before the diaphragm opening, and for relatively low temperatures before the comparison lamp. The opening of the iris diaphragm is read on an attached scale, the square of whose readings is a measure of the intensity of the light from the incandescent body.

If  $K$  is the coefficient of absorption of the glass before the objective,  $n'$  the scale reading of the iris diaphragm when the pyrometer is sighted upon a primary standard of light, as the amyl-acetate lamp—a measurement which may be made once for all—and  $f'$  the distance from objective to image, read off the graduated draw tube, and if  $n$  and  $f$  are the corresponding quantities when viewing the body whose temperature is sought, then the intensity of light from this body in terms of the amyl-acetate flame, taken as unity, is given by the expression

$$J = \left(\frac{n'}{n}\right)^2 \cdot \left(\frac{f}{f'}\right)^2 \cdot \left(\frac{1}{K}\right)^p$$

where  $p$  absorbing glasses are used.<sup>a</sup>

When absorbing glasses are placed before the gasoline lamp this formula becomes

$$J = \left(\frac{n'}{n}\right)^2 \cdot \left(\frac{f}{f'}\right)^2 \cdot K^p$$

which applies to temperatures below about 1000° C.

When the pyrometer is always used with the same arrangement of glasses, and at the same distance from the source observed, the expression for  $J$  reduces to

$$J = \text{const.} \cdot \left(\frac{1}{n}\right)^2$$

*Calibration.*—It is now necessary to translate the intensity  $J$  into temperature. In this operation consists the calibration of the pyrometer. At the time Le Chatelier brought out his instrument in 1892, the laws of monochromatic radiation were not known, so he expressed this calibration by an empirical relation of the form

$$J = 10^a T^{-\frac{b}{T}}$$

where  $a$  and  $b$  are constants, whose values depend upon the unit of intensity chosen, and  $T$  is the absolute temperature.

---

<sup>a</sup> The absorption factor  $\left(\frac{1}{K}\right)^p$  is somewhat too large, but owing to the great variation of light in the temperature, this expression is sufficiently accurate for all ordinary ranges of temperature.

As shown on page 210, the calibration of such a pyrometer may be made in terms of Wien's equation (III),

$$\log J = K_1 + K_2 \frac{1}{T}$$

so that if the temperature is known for only two intensities the instrument is completely calibrated for all temperatures, since the relation

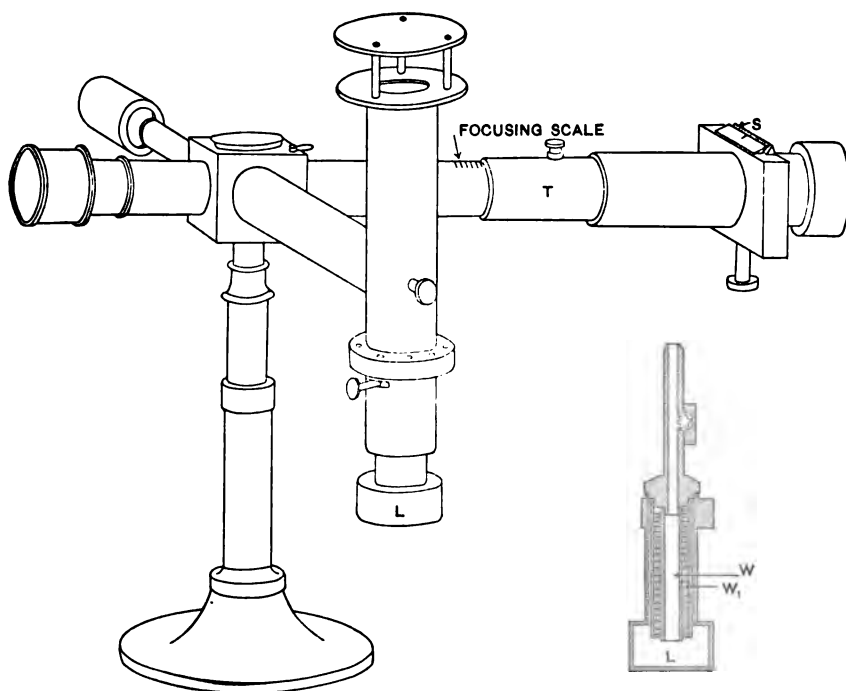


FIG. 5.—Le Chatelier optical pyrometer.

between  $\log I$  and  $\frac{1}{T}$  is linear, which may be plotted graphically and then a table may readily be constructed giving  $T$  in terms of  $J$  or scale readings. Such a table will serve indefinitely for the use of the instrument. Calibration by using Le Chatelier's empirical formula is more tedious and complicated.

The accompanying table shows the results of our calibration of a pyrometer by both methods:

| Observed temperature (absolute). | Temperature calculated by Le Chatelier's equation. | (Le Chatelier)<br>$T_{obs} - T_{calc}$ | Temperature calculated by Wien's equation. | (Wien)<br>$T_{obs} - T_{calc}$ |
|----------------------------------|----------------------------------------------------|----------------------------------------|--------------------------------------------|--------------------------------|
| 975°                             | 975°                                               | 0°                                     | 982°                                       | - 7°                           |
| 995                              | 994                                                | + 1                                    | 1001                                       | - 6                            |
| 1147                             | 1147                                               | 0                                      | 1150                                       | - 3                            |
| 1154                             | 1148                                               | + 6                                    | 1150                                       | + 4                            |
| 1191                             | 1190                                               | + 1                                    | 1189                                       | + 2                            |
| 1211                             | 1214                                               | - 3                                    | 1213                                       | - 2                            |
| 1278                             | 1276                                               | + 2                                    | 1274                                       | + 4                            |
| 1296                             | 1290                                               | + 6                                    | 1287                                       | + 9                            |
| 1398                             | 1400                                               | - 2                                    | 1396                                       | + 2                            |
| 1459                             | 1464                                               | - 5                                    | 1460                                       | - 1                            |
| 1587                             | 1602                                               | -15                                    | 1597                                       | -10                            |

The central portion of the flame of a Hefner standard was used as the unit of intensity. The data satisfy the following equations:<sup>a</sup>

$$\log J = 6.697 - 9983 \frac{1}{T} \quad (\text{Wien's Eq.})$$

$$J = 10^{7.77} T^{-\frac{3660}{T}} \quad (\text{Le Chatelier's Eq.})$$

The calibration was carried out by sighting the pyrometer upon the the bottom of a small fire-clay crucible placed at the center of a nickel wound electric furnace 60 cm long and 2 cm opening. Temperatures were measured by two thermocouples with junctions in contact with the crucible.

The effective wave length of the red light transmitted by the glass used before the eye piece was determined as  $0.64\mu$ . As shown on page 210, the value of  $c_2$  in the Wien equation is given by

$$K_2 = \frac{c_2 \log_{10} e}{\lambda} = 9983$$

$$\therefore c_2 = 14700.$$

This would indicate that the radiation from the interior of the electric furnace used in this work is a close approximation to black body radiation.

<sup>a</sup>In the computation of the constants of the equations the observations found in the table have not all equal weights.

tion, the constant  $c_2$  for which is about 14500. Covering with iron oxide the radiating fire-clay surface found at the center of the furnace produced no very appreciable increase in the "blackness" of the radiation (not greater than  $5^\circ$  or  $10^\circ$ ).

*Sources of error.*—The sources of error of this instrument may be those due to the Hefner amyl-acetate standard, the oil comparison lamp, the focusing system, the red glass before the eye piece, and the absorption glasses used. The first of these affects only comparative results with different instruments, while the others, if they exist, may be of considerable importance in work with a single instrument. We shall consider them in the order named.

As only the central portion of the amyl-acetate flame is used, variations in height and fluctuations in total intensity, due to various causes, such as moisture and carbonic acid in the atmosphere and changes due to differing samples of acetate, become almost, if not quite, insignificant, so that when using only a small central area of the amyl-acetate flame, it is a very reproducible standard under the most varying conditions of burning. Again the effects of any slight fluctuations in light intensity are further greatly reduced when transformed into temperature changes, as has been shown, page 192. Thus, a variation of one mm in the height of the Hefner flame, which produces a variation of 10 per cent in the total intensity, causes a change of less than 1 per cent in the intensity of light from the central area, which is equivalent to less than  $1^\circ$  C. change in temperature.

When used at intervals, the Hefner serves well enough as an ultimate standard by means of which the indications of all photometer-pyrometers may be reduced to a common basis, yet it is not suited for continuous use as a working standard, for various reasons, an important one being that with continued burning there is considerable charring and an accumulation of a troublesome deposit about the top of the burner.

The comparison lamp used in the pyrometer is a small round-wick lamp (fig. 5) of special construction, burning gasoline preferably, and having an auxiliary feed-wick, which maintains the constancy of burning. This lamp is cleaner than the Hefner, burning for three hours without charring or appreciable accumulation about the burner top. It also burns more steadily, has a more uniform brightness over the flame surface, and remains of constant brightness for considerable variations in the nature of the oil burned.

The other factors which may change the brightness of this flame which we have studied are the effects of time of burning, height of

flame, resetting flame by different observers, kind of oil and depth of oil in lamp.

In order to obtain a perfectly constant source of light with which to compare the oil flame, a 32 c. p. incandescent electric lamp was placed in a fixed position before the objective of the pyrometer and a glass diffusing screen inserted before the objective. The voltage across the lamp terminals was kept rigorously constant, thus giving an arbitrary but invariable standard of illumination.

To control accurately the flame height in the gasoline lamp, a sight was inserted consisting of a horizontal scratch 2 millimeters above the window before the flame and a very fine platinum wire in the same horizontal plane but in a collar behind the flame. With this improvement an observer can set and control the flame height to 0.2 mm. The effect of varying the flame height is shown in the following table:

| Flame height.   | Cats-eye reading. | $\Delta$ | $\Delta$ in $J$ in per cent. | $\Delta$ in $T$ |
|-----------------|-------------------|----------|------------------------------|-----------------|
| 1 mm high ..... | 27.2              | 0.6      | 4.6                          | 2.5° C.         |
| Normal.....     | 26.6              |          |                              |                 |
| 1 mm low .....  | 25.9              | 0.7      | 5.2                          | 2.9° C.         |

This shows that for most purposes changes of over 2 mm may occur in the flame height with unimportant changes resulting in the temperature estimation.

The concordance of results obtained by different observers setting the flame and observing is shown below:

| Observer.              | No. 1. | No. 2. | No. 3. |
|------------------------|--------|--------|--------|
| Cat's-eye reading..... | 26.9   | 26.6   | 26.5   |
|                        | 26.5   | 26.3   |        |

Here the greatest variation corresponds to less than one degree in temperature at 1000° C.

Considering the time effect of burning upon the flame height and intensity due to local heating and change of depth of oil, we find the flame ceases creeping up after ten minutes and will then remain at

constant height to within 0.5 mm until the oil is used up (or in three hours).

The constancy of brightness with time is illustrated by the following set of observations, where the flame height was kept constant:

|                                          | Cat's-eye<br>reading. | $\Delta J$ in<br>percentage. | $\Delta$ in $^{\circ}\text{C}$ |
|------------------------------------------|-----------------------|------------------------------|--------------------------------|
| After 25 minutes' burning .....          | 26.03                 | 1.7                          | $0^{\circ}.9$                  |
| After 45 minutes' burning .....          | 26.26                 | 7.5                          | $4^{\circ}.2$                  |
| After 2 hours' burning.....              | 27.03                 | 5.0                          | $2^{\circ}.75$                 |
| After $2\frac{1}{2}$ hours' burning..... | 26.68                 |                              |                                |

Thus there was never a variation corresponding to much greater than  $4^{\circ}\text{C}$ . arising from change of intensity with time, showing that the lamp is fed at nearly a constant rate for all stages of burning.

It might be expected that oils of different grades would give widely differing results, yet such did not prove to be the case.

Different samples of gasoline were used and gasoline mixed with a heavy, pure kerosene having a flash point of  $135^{\circ}\text{C}$ ., when the following results were obtained, the numbers being the readings of the cat's-eye scale, readings being taken over a considerable period of time.

| Gasoline. |           |           | Gasoline + x per cent kerosene. |             |              |              |
|-----------|-----------|-----------|---------------------------------|-------------|--------------|--------------|
| Sample 1. | Sample 2. | Sample 3. | 2 per cent.                     | 5 per cent. | 10 per cent. | 25 per cent. |
| 26.3      | 27.3      | 26.4      | 26.1                            | 27.8        | 26.4         | 26.7         |
| 26.4      | 27.9      | 26.1      | 26.3                            | 28.3        |              |              |
| 27.1      |           |           |                                 | 27.3        |              |              |
| 26.1      |           |           |                                 | 26.7        |              |              |

It appears, then, that with a base of any ordinary gasoline there may be considerable variations in the composition of the oil burned with a practicably negligible change resulting in the intensity of the light emitted by the flame. This is of importance in the practical use of the instrument, as it shows that the scale of the instrument is by no means locked up in a single can of gasoline.

From the above it is clear that variations in brightness of the comparison flame due to all possible causes need not produce errors in temperature measurement of over  $5^{\circ}$  C., that is, as will be shown, within the limits of making the photometric settings.

Considering now the sources of error due to focusing upon the object whose temperature is sought, it is first to be noticed that there is a minimum distance from the object at which the pyrometer can be focused, this distance being somewhat over a meter, depending, of course, upon the focal length of the objective and length of draw tube. There is also a minimum area which can be sighted upon and give an image of sufficient size to completely cover the desired photometric field. This minimum size of object is about 6 mm on a side when the instrument is at its least distance; for greater distances a larger area must be viewed.

The draw tube can easily be set to 2 mm when focusing, and as the image is over 20 cm from the objective in all cases, the resulting error in intensity due to focusing is not greater than 2 per cent, since

$$I = k f^2$$

$$\text{or } \frac{\delta I}{I} = 2 \frac{\delta f}{f} = \frac{2 \times 0.2}{20} = 0.02$$

This corresponds to less than  $1^{\circ}$  C. in temperature, showing that an error of even 5 mm in focusing the draw tube will hardly produce an appreciable error in temperature estimation.

Often, in use, the distance of the instrument from the objects studied needs to be changed considerably, and in rapid work it is not always convenient to refocus. We find that a change in this distance of a fourth its value—i. e., from 120 cm to 150 cm—will produce only an apparent change in intensity of 9 per cent, or about  $5^{\circ}$  C. in temperature, at  $1000^{\circ}$  C., showing that no undue precautions in focusing are needed.

The nonmonochromatism of the red glass before the eyepiece produces no appreciable error in temperature measurement up to  $1300^{\circ}$  C., although if this glass is not very nearly monochromatic the differences in hue in the two adjacent photometric fields—from the comparison lamp and other source—are very troublesome, and the strain on the eye in matching them is considerable. The three red glasses used were of different make, the best<sup>a</sup> of which let through light between the limits  $\lambda = 0.60 \mu$  and  $\lambda = 0.69 \mu$ ,<sup>a</sup> this one proving entirely satisfactory from the photometric point of view. For very high temperatures it would not be safe to assume that Wien's law applies exactly, as there is an appreciable displacement toward the yellow, giving too high

<sup>a</sup> Jena glass No. 2745, obtained from Schott and Genossen.



results for the computed values of the intensity, and so for the temperatures.

There remains to consider the error due to uncertainty in the coefficient of absorption of the absorbing glasses. If an observation ( $N^1$ ) is taken with and then, at the same temperature, one ( $N$ ) without an absorption glass, we have

$$K = \left( \frac{N^1}{N} \right)^2$$

so that the accuracy in determining  $K$  depends directly upon the precision of setting and reading the cat's-eye opening. The following sets of observations show how closely single observations may be repeated.

| Observer .....             | Without absorption glass. |      |      |      | With absorption glass. |       |
|----------------------------|---------------------------|------|------|------|------------------------|-------|
|                            | 1                         | 2    | 3    | 4    | 1                      | 3     |
| Cat's-eye scale, readings. | 7.4                       | 7.8  | 7.6  | 7.3  | 25.7                   | 25.8  |
|                            | 7.4                       | 7.9  | 7.8  | 7.0  | 24.0                   | 24.8  |
|                            | 7.2                       | 7.7  | 7.6  | 8.0  | 23.6                   | 26.0  |
|                            | 7.8                       | 7.8  | 7.7  | 7.1  | 24.1                   | 25.8  |
|                            | 7.7                       | 7.7  | 7.8  | 8.3  | 25.4                   | 24.8  |
|                            | 7.8                       | 7.7  | 7.4  | 8.0  | 24.8                   | 24.9  |
| Mean.....                  | 7.55                      | 7.73 | 7.65 | 7.60 | 24.63                  | 25.34 |

Observers Nos. 2 and 4 had no experience in the use of the instrument. Observers 1 and 3, over a series of several hundred observations, always differ in the same way as above and by almost exactly the same amount. This constant personal difference is evidently a physiological effect excited by the slight difference in hue of the two halves of the photometric field.

The greatest difference in the means corresponds to  $2.5^\circ$  C. in temperature, and as a set of five observations may easily be taken in two minutes, it is safe to say that the precision of readings is well within  $5^\circ$  C. at  $1000^\circ$  C. The values of  $K$ , computed from the results of observers Nos. 1 and 3, differ by 3 per cent, which would introduce a like error into the value of the intensity, but its effect on temperature estimation is equivalent to a difference of less than  $2^\circ$  C. at  $1000^\circ$  C.

In general, it may be said that, with reasonable care, this pyrometer

may be depended on to an order of accuracy of about 1 per cent in the measurement of temperature.

#### MODIFICATIONS OF THE LE CHATELIER PYROMETER.

For use in technical works and other places where there are strong drafts of air, causing flickering and waving about of the flame of the oil comparison lamp, it seems that the Le Chatelier pyrometer would be improved by the substitution of an electric incandescent lamp of low voltage placed before a uniformly ground diffusing glass screen, which, illuminated by the incandescent lamp, becomes the constant comparison source. The reliability of such a method of producing comparison light of invariable intensity will be discussed when describing the Wanner instrument. We have successfully used this method with the Le Chatelier pyrometer. The electric lamp may be mounted in a vertical arm, which serves at the same time as a handle, and then the instrument becomes as portable as an opera glass.

#### FÉRY ABSORPTION PYROMETER.<sup>a</sup>

This is identical with Le Chatelier's instrument, except that a pair of absorbing glass wedges replace the iris diaphragm (see fig. 4), and the 45° mirror *M*, with parallel faces, is silvered over a narrow vertical strip giving a photometric field of form shown at *C*, when looking at a crucible. The instrument has a fixed angular aperture, so that no correction has to be made for focusing or for varying distance from furnace. The comparison light plays the same rôle as in Le Chatelier's pyrometer, and the range of the instrument may be similarly extended by the use of auxiliary absorbing glasses. Féry has in addition made his instrument movable about a horizontal axis, which is a convenience.

The calibration is equally simple. If *x* is the thickness of the wedges, read off on a scale, when the light from the comparison lamp and furnace is of the same brightness, then the relation between brightness *J* and thickness of wedge is

$$J = i e^{Kx}$$

where *K* is the coefficient of absorption of the glass of the wedges for the red light used, and *i* is a constant.

But by Wien's law, assuming it to apply here,

$$J = A e^{-\frac{B}{T}}$$

<sup>a</sup> Féry: *J. de Phys.*, (4) 3, p. 32; 1904. This instrument is made by M. Pellin, 21 Rue de l'Odeon, Paris.

or, combining these two equations,

$$\frac{A}{x} e^{-\frac{B}{T}} = e^{Kx}$$

whence

$$Kx = \frac{A}{x} - \frac{B}{T} = M - \frac{B}{T} \text{ by putting } \frac{A}{x} = M$$

Thus it follows that the thickness of the wedge is inversely proportional to the absolute temperature, so that the calibration may be effected by finding the thickness of wedge for two temperatures only, and plotting a straight line and constructing a table giving  $\frac{1}{T}$  and  $T$ , respectively, in terms of  $x$ .

It is questionable if there is any gain in substituting the wedge for the cat's-eye, in the desire to extend the range over which the instrument may be used, without employing the auxiliary absorbing glasses, for thereby the sensibility is somewhat reduced, and, more important still, the wedge instrument can not be used at such low temperatures as the original Le Chatelier form, nor is there any gain in simplicity of calibration and ease of manipulation. The shape of the photometric field, the use of an aperture of constant angle, and making the instrument movable about a horizontal axis, however, are improvements which may be applied with advantage to the Le Chatelier instrument.

#### MESURÉ AND NOUËL'S PYROMETRIC TELESCOPE.<sup>a</sup>

In this instrument an attempt is made to control temperatures by taking advantage of the rotation of the plane of polarization of light passing through a quartz plate cut perpendicular to its axis. The angle of rotation is directly proportional to the thickness of the quartz, and approximately inversely proportional to the square of the wave length.

Light from an incandescent object, passing through the slightly ground diffusing glass  $G$ , enters a polarizing nicol  $P$ , and, traversing the quartz plate  $Q$ , strikes the analyzer  $A$ , and is viewed through the eye-piece  $OL$ . In general, the field of view will be colored, and by turning the analyzer the color observed will change, because the light, all polarized in the same plane emerging from  $P$ , is polarized in various planes, depending on the wave length, by the quartz  $Q$ , so that the angle between the nicols  $A$  and  $P$  determines the color that will be seen.

<sup>a</sup> Made by M. Ducretet, 75 Rue Claude-Bernard, Paris.

There is a color to which the eye is particularly sensitive, the transition hue between the red and green, to which the analyzer is turned in taking observations. This sensitive hue is of a lemon-yellow color.

If, then, it is desired to control the constancy of temperature of a heated incandescent body, it is viewed and the instrument set to the sensitive hue, when any change in temperature will cause a change in composition of the transmitted light, and consequently a change in color as viewed through the eye-piece.

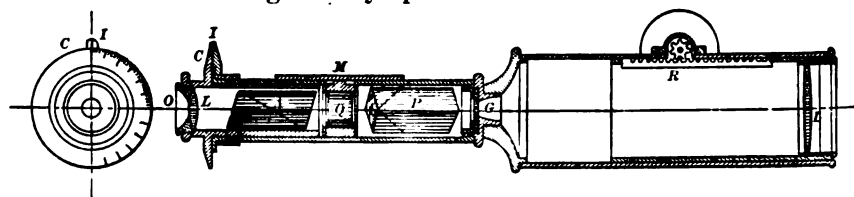


FIG. 6.—Mesuré and Nouel pyrometric telescope.

Besides being used to control heating at a constant temperature, this instrument is sometimes used for estimating the temperatures themselves. For this purpose a circle *C* graduated in degrees is attached to the rotating analyzer and the readings given by the index *I*, when the sensitive hue is reached, are empirically reduced to degrees of temperature. The adjustment of the zero of the scale may be made by removing the quartz at *M*. This pyroscope is rendered more sensitive, especially at low temperatures, by addition of the objective *L*, which is focused by the rack and pinion *R*.

This instrument gives a temperature scale varying not only with different observers but with the same observer at different times, due to the uncertainty in recognizing the sensitive hue. This hue, moreover, is not fixed in color but varies with the temperature, so that the uncertainties of observation easily amount to over  $100^{\circ}\text{C}.$ , and even after considerable practice an observer can not be sure of his readings to better than  $50^{\circ}\text{C}.$  at temperatures above  $1000^{\circ}\text{C}.$  This instrument has the advantage over all others, in that it has no auxiliary apparatus whatever, but its lack of sensibility debars it from a place among accurate measuring instruments. It replaces the unaided eye to advantage and its lower limit is about  $750^{\circ}\text{C}.$  It has found its widest use in the ceramic industries.

#### WANNER PYROMETER.<sup>a</sup>

In 1901 Wanner,<sup>b</sup> making use of the polarizing principle which had been considered but not used by Le Chatelier, brought out a

<sup>a</sup> This instrument is made by Dr. R. Hase, Hanover.

<sup>b</sup> Wanner: *Phys. Z. S.*, 3, p. 112; 1902. *Iron Age*, Feb. 18, p. 24; 1904. *Stahl und Eisen*, 22, p. 207; 1902.

photometer-pyrometer which is a modification, suited to temperature measurements, of König's spectrophotometer.<sup>a</sup>

The comparison light is a 6-volt incandescent lamp illuminating a glass mat surface. Monochromatic red light is produced by means of a direct vision spectroscopy and screen cutting out all but a narrow band in the red, and the photometric comparison is made by adjusting to equal brightness both halves of the photometric field by means of the polarizing arrangement shown in fig. 7.

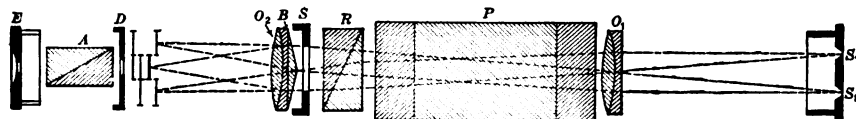


FIG. 7.—Wanner optical pyrometer.

The slit  $S_1$  is illuminated by light from the comparison source reaching  $S_1$  after diffuse reflection from a right-angled prism placed before  $S_1$ . Light from the object whose temperature is sought enters the slit  $S_2$ . The two beams are rendered parallel by the lens  $O_1$  and each dispersed into a continuous spectrum by the direct-vision prism  $P$ . Each of these beams is next separated by a Rochon prism  $R$  into two beams polarized in planes at right angles. Considering only the red light there would now be four images formed by the lens  $O_2$  and distributed about the slit  $D$ . In order to bring two red images oppositely polarized exactly before this slit, a biprism  $B$  is interposed whose angle is such as to effect this for two images only, at the same time increasing the number of images to eight. There is now in the field of view before the nicol analyzer  $A$  two contiguous red fields composed of light polarized in planes at right angles, the light of one coming from  $S_1$  and of the other from  $S_2$ . All the other images are cut off from the slit  $D$ . If the analyzer is at an angle of  $45^\circ$  with the plane of polarization of each beam, and if the illumination of  $S_1$  and  $S_2$  is of the same brightness, the eye will see a single red field of uniform brightness. If one slit receives more light than the other, one-half of the field will brighten, and the two may be brought to equality again by turning the analyzer carrying a graduated scale, which may be calibrated in terms of temperature.

If the analyzer is turned through an angle  $\phi$  to bring the two halves of the field to the same brightness, the relation between the two intensities from  $S_1$  and  $S_2$  is:

$$(a) \quad \frac{J}{J_o} = \tan^2 \phi.$$

<sup>a</sup> König: Wied. Ann., 53, p. 785; 1894.

Since monochromatic light is used, and the comparison beam and that from the object examined undergo the same optical changes, Wien's equation (III) forms the basis of the calibration.

If  $J_o$  is the intensity of the light from the standard, and  $J$  that from the object whose temperature is sought, Wien's equation gives

$$(b) \quad \log_{10} \left( \frac{J}{J_o} \right) = \frac{c_2}{\lambda} \log_{10} \varepsilon \left( \frac{1}{T_o} - \frac{1}{T} \right).$$

Since the constant  $c_2$  equals  $14500$  for a black body and  $\lambda = 0.656\mu$  as the instrument is usually constructed, a knowledge of the apparent black-body temperature of the standard source, together with the reading of the analyzer scale at the normal point when  $J = J_o$ , is all the data required for the calibration of such an instrument, as any temperature may then be calculated, by means of equations (a) and (b), in terms of the scale readings. This instrument may also, of course, be empirically calibrated against a thermocouple, using a black body to sight upon.

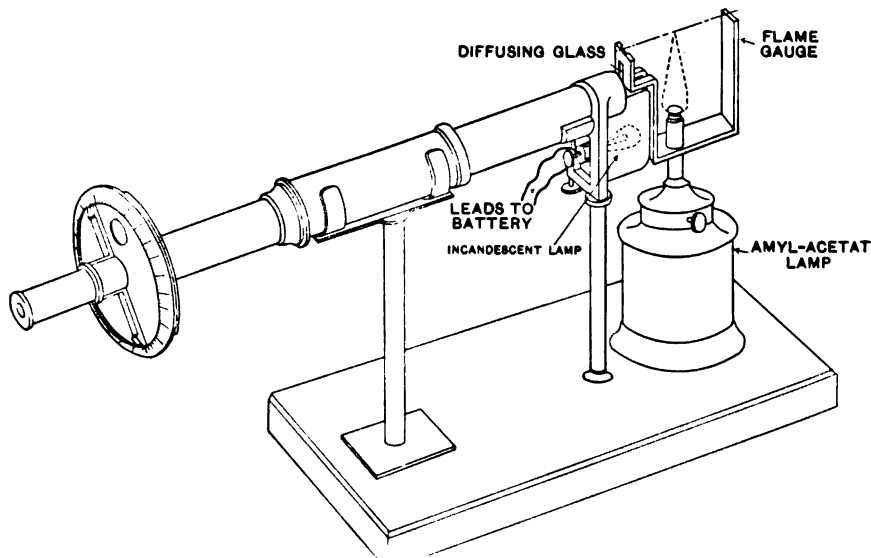


FIG. 8.—Wanner optical pyrometer in position for standardizing.

It is evidently necessary to be able to always reproduce exactly the standard intensity  $J_o$ . The brightness of an electric lamp will vary with the current through it, so it is necessary to check frequently the constancy of illumination of the slit  $S_1$  against a standard and constant source of light. An amyl-acetate lamp and a ground-glass diffusing screen can be placed before the slit  $S_1$  thus furnishing the standard

light required. The analyser is then set at the previously determined normal point and the distance of the electric lamp from  $S_1$  and the current through it adjusted until the two fields appear of the same brightness.

*Sources of error.*—The sensibility of this instrument varies with change in the angle and is so adjusted as to be the greatest between  $1000^\circ$  and  $1500^\circ$  C and is about as follows:

0.1 scale div.  $\equiv 1^\circ$  C at  $1000^\circ$  C.

0.1 scale div.  $\equiv 2^\circ$  C at  $1500^\circ$  C.

0.1 scale div.  $\equiv 7^\circ$  C at  $1800^\circ$  C.

The reproducibility of the brightness of the amyl-acetate flame as viewed through the ground-glass diffusing screen is a measure of the ability of the instrument to repeat its indications. It is very important that this diffusing screen be always placed in exactly the same position relative to the flame and slit  $S_2$ , and further, that it be free from dust and finger marks. These requirements can only be satisfactorily met by protecting this screen by a cover-glass and providing an adjustment for setting it exactly in place between the flame and slit.

The constancy of the amyl-acetate flame as used with this pyrometer under ordinary conditions of burning is illustrated by the following set of observations, taken at different times when the current through the electric comparison lamp was kept rigorously constant by means of a milliammeter and rheostat:

| Reading of instrument. <sup>a</sup> | Deviations. |
|-------------------------------------|-------------|
| 39.9                                | -0.28       |
| 39.9                                | -0.28       |
| 40.1                                | -0.48       |
| 39.9                                | -0.28       |
| 39.1                                | +0.52       |
| 39.2                                | +0.42       |
| 39.8                                | -0.18       |
| 39.0                                | +0.62       |
| Mean 39.62                          | 0.38        |

<sup>a</sup> 39.6 divs. on the graduated circle  $\equiv 1157^\circ$  C, and at this part of the scale 1 div.  $\equiv 7^\circ$  C.

This shows that the flame can be relied upon to give an intensity of illumination whose constancy expressed in terms of temperature is better than 0.5 per cent. Variations in height of the flame if they do

not exceed 3 mm, together with fluctuations in atmospheric conditions, will not produce errors in temperature estimation exceeding 1 per cent.

The uncertainty of setting the Nicol, due to lack of sensitiveness of the eye to exactly match the two halves of the photometric field, is also about 1 per cent, or somewhat better with practice.

The adjustment of the electric lamp to standard intensity at the point on the scale chosen as normal point, when proper care is taken regarding the diffusing screen, can be made to within 1 per cent expressed in temperature. This source of error does not affect relative results in any one series for one setting to the normal point.

The most serious source of error, except when special precautions are taken, is the variation in brightness of the electric comparison lamp due to variation in the current furnished by the (three cell) storage battery, the percentage change in light being of the order of six times the percentage change in current through the lamp.

The behavior of a battery of 10 ampere-hours capacity furnished with a Wanner instrument is shown by the accompanying plot in which the variations of voltage with time are shown; it will be seen that after making circuit the electromotive force drops by about 2 per cent in two minutes and then falls off slowly, but nearly recovers the original voltage after remaining on open circuit even for a very short time.

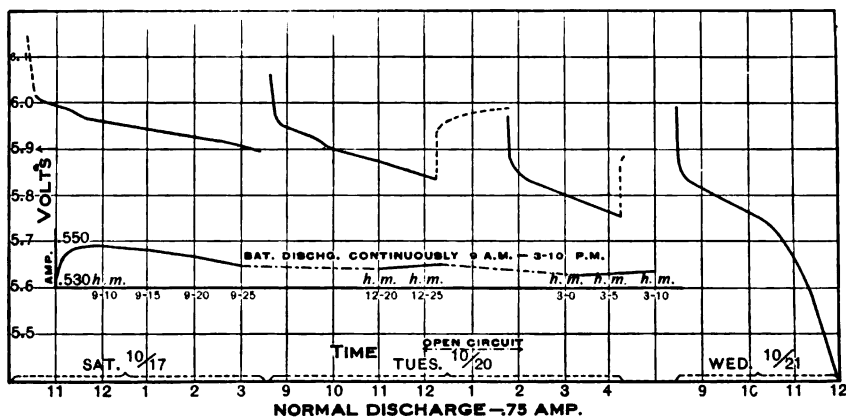


FIG. 9.—Discharge curves of storage battery.

When the battery is in good condition the variation in three hours at normal discharge (0.75 ampere) is about 0.08 volt, and somewhat less for the current (0.55 ampere) taken by the lamp; with the battery in poor condition these changes are much accentuated as shown by the plot.



The following table illustrates the effect of slight variations in current through the lamp on the apparent temperature of the amyl-acetate flame, both for the small 10-ampere-hour battery furnished with the instrument and for a battery of 75-ampere-hours capacity.

| SMALL BATTERY. |                                 |                          |                                                |                                                   |
|----------------|---------------------------------|--------------------------|------------------------------------------------|---------------------------------------------------|
| Time.          | Wanner scale<br>r'dg.           | Current<br>through lamp. | Per cent change<br>in current<br>through lamp. | Apparent<br>change in<br>temperature<br>observed. |
| 15 min.        | 31.2                            | 0.5645                   |                                                |                                                   |
| 20 "           | 31.8                            | 0.5640                   | 0.1                                            | 1° C.                                             |
| 27 "           | 32.7                            | 0.5550                   | 1.7                                            | 10                                                |
| 37 "           | 34.6                            | 0.5400                   | 4.3                                            | 25                                                |
| 38 "           | Disconnected battery 2-minutes. |                          |                                                |                                                   |
| 40 "           | 32.5                            | 0.5570                   | 1.5                                            | 7                                                 |
| 42 "           | 31.7                            | 0.5610                   | 0.6                                            | 3                                                 |
| 45 "           | 32.5                            | 0.5560                   | 1.5                                            | 15                                                |
| 47 "           | 33.1                            | 0.5405                   | 4.1                                            | 24                                                |
| LARGE BATTERY. |                                 |                          |                                                |                                                   |
| 15 min.        | 37.8                            | 0.5610                   |                                                |                                                   |
| 17 "           | 37.9                            | 0.5590                   | 0.4                                            | 2° C.                                             |
| 21 "           | 39.1                            | 0.5550                   | 1.1                                            | 7                                                 |
| 26 "           | 39.3                            | 0.5515                   | 1.7                                            | 10                                                |
| 33 "           | 40.0                            | 0.5455                   | 2.8                                            | 17                                                |
| 45 "           | 40.9                            | 0.5390                   | 3.9                                            | 23                                                |
| 52 "           | 41.1                            | 0.5385                   | 4.0                                            | 23                                                |
| 53 "           | Disconnected battery 9-minutes. |                          |                                                |                                                   |
| 63 "           | 37.2                            | 0.5610                   | 0.0                                            | 0                                                 |
| 73 "           | 39.9                            | 0.5460                   | 2.8                                            | 17                                                |

The above results give abundant evidence of the need of maintaining the current through the lamp quite constant in work of precision. A series of experiments has shown that in the range 1000°–1500° C. one division on the Wanner scale corresponds to about 0.009 amperes, or 1° C. apparent change in temperature is produced by a fluctuation of 0.0012 amperes through the lamp; hence to obtain a precision of 5° C. the current must be kept constant to 1 per cent. The above table and plot show that this is by no means effected by using the battery

without regulating the current, for even with the battery in the best condition the current increases by 2 per cent in the first eight or nine minutes of discharge and then falls off 1 per cent in the next twenty minutes, as shown by the plot. The temperature coefficient of the battery would produce only insignificant changes. The table shows further that even with a large-cell battery, breaking the circuit and then making it again may cause an apparent temperature change of over  $20^{\circ}\text{C}$ . For work of precision, therefore, it is essential to keep the current constant by means of a milliammeter and rheostat.

*Range and limitations.*—The above description of the Wanner pyrometer has shown the great loss of light due to the optical system employed. This prevents measuring temperatures below about  $900^{\circ}\text{C}$ . ( $1650^{\circ}\text{F}$ .) with this instrument. There is no method of sighting this pyrometer exactly upon the spot desired, except by trial, as no image of the object examined is formed in the eyepiece, but this inconvenience is in part compensated by not having to focus with varying distance from the object.

There is another limitation which may in certain cases become a serious source of error. Light from incandescent surfaces is in general partially polarized, and as the Wanner instrument is a polarizing pyrometer, care must be taken to eliminate this source of error when it exists. The magnitude of the error arising from this cause is entirely negligible for all practical purposes for such substances as iron, porcelain, and glass. The results of experiments on the polarization of light from incandescent surfaces are given on page 251.

A review of the sources of error and limitations of the Wanner pyrometer shows that they may exert a relatively large effect on the temperature measurements, and it was therefore thought worth while to emphasize them, but, on the other hand, they may all be kept quite small with reasonable care, and the instrument then becomes one of great precision and convenience.

#### HOLBORN-KURLBAUM<sup>a</sup> AND MORSE<sup>b</sup> PYROMETERS.

About the same time Holborn and Kurlbaum<sup>c</sup> in Germany and Morse<sup>d</sup> in this country brought out an optical pyrometer using a new photometric method.

<sup>a</sup> This instrument is made by the Siemens and Halske Aktiengesellschaft, 94 Markgrafenstrasse, Berlin.

<sup>b</sup> This instrument is made by the Morse Thermo-Gage Company, Trumansburg, N. Y.

<sup>c</sup> Holborn and Kurlbaum: Ber. d. K. Akad. d. Wiss., Berlin, p. 712; 1901. Ann. d. Phys., 10; p. 225; 1903.

<sup>d</sup> Morse: American Machinist, 1903; U. S. Letters Patent 696878, 696916; 1902.

If a sufficient current is sent through the filament of an electric lamp, the filament glows red at first, and as the current is increased, the filament, getting hotter and hotter, becomes orange, yellow, and white, just as any progressively heated body. If, now, this filament is interposed between the eye and an incandescent object, the current through the lamp may be adjusted until a portion of the filament is of the same color and brightness as the object. When this occurs this part of the filament becomes invisible against the bright background and the current through the lamp then becomes a measure of the tem-

NOTE.—As a result of recent litigation the Holborn-Kurlbaum Pyrometer has been withdrawn from sale.

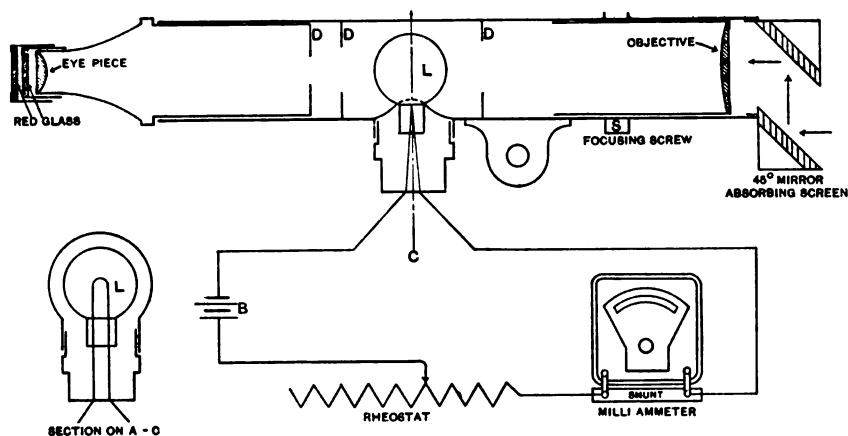


FIG. 10.—Holborn-Kurlbaum optical pyrometer.

telescope provided with suitable stops  $D, D, D$ , and a focussing screw,  $S$ , for the objective. The lamp circuit is completed through a 2-cell storage battery  $B$ , a rheostat, and a milli-ammeter.

The determination of a temperature consists in focussing the instrument upon the incandescent object, thus bringing its image into the plane  $AC$ , and adjusting the current by means of the rheostat until the tip of the lamp filament disappears against the bright background, when, from the reading of the milli-ammeter, the temperature can be found from a previous calibration of the lamp giving the relation between the currents through it and the corresponding temperatures of a radiating black body. The calibration will evidently be an independent one for each lamp used.

As the temperature of the filament increases, the effect of irradiation and too great brightness become blinding, and the photometric comparison is rendered possible at these temperatures by the introduction of one or more monochromatic red glasses before the eyepiece, giving as well all the advantages of photometry of a single color. Below 800° C. the measurements are more easily made without any red glass, as the filament itself is then red and the lowest temperatures are of course reached with the least interposition possible of absorbing media. The lower limit of the instrument is very nearly 600° C. Two red glasses are required for temperatures above 1200° C., and for very high temperatures it is necessary, in order to avoid overheating the lamp filament by the current, to put absorbing glasses or mirrors before the objective, and they also require calibration.

The coefficient of absorption of the absorbing glasses may be calculated by making use of Wien's law, supposing it to hold for the red glass used in the eyepiece. If  $K$  is the coefficient of absorption and  $T_1$ ,  $T_2$  are the apparent black-body temperatures (absolute) given by the pyrometer, sighting first without and then with the absorbing glasses, Wien's equation gives:

$$\log_{10} K = \log_{10} \frac{J_1}{J_2} = \frac{c_2}{\lambda} \log_{10} \epsilon \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

where  $c_2 = 14500$  and  $\lambda$  is the effective wave length for the red glass used. For very high temperatures, although this formula will give a consistent scale when  $K$  has been determined, the values obtained are in error by amounts depending upon the lack of monochromatism of the red glass used and upon the consequent varying absorbing power of the absorption glasses.

The eye is particularly sensitive in recognizing equality of brightness of two surfaces, one in front of the other, and this pyrometer therefore provides a very delicate means of judging temperatures. The precision attainable with this pyrometer is illustrated by the following series of observations, which are indicative of the ordinary performance of the instrument:

| Temperature from<br>H. & K. pyrometer. | Temperature from<br>thermocouple. | Temperature from<br>H. & K. pyrometer. | Temperature from<br>thermocouple. |
|----------------------------------------|-----------------------------------|----------------------------------------|-----------------------------------|
| 1347° C.                               | 1347° C.                          | 632° C.                                | 634° C.                           |
| 1351                                   | 1347                              | 634                                    | 633                               |
| 1343                                   | 1343                              | 633                                    | 633                               |
| 1338                                   | 1342                              | 633                                    | 632                               |
| 1342                                   | 1342                              |                                        |                                   |

Different observers do not differ by any appreciable amount in their readings, and at low temperatures the same values are obtained whether a red glass is used or not.

The relation between current and temperature is sufficiently well expressed by a quadratic formula of the form:

$$C = a + bt + ct^2$$

That this formula gives satisfactory results is shown by observations of Holborn and Kurlbaum, for example, with a lamp satisfying the equation.

$$C \cdot 10^3 = 170.0 + 0.1600t + 0.0001333t^2$$

| C amp. | t obs.  | t calc. | $\Delta t$ |
|--------|---------|---------|------------|
| 0.340  | 686° C. | 679° C. | -7° C.     |
| .375   | 778     | 778     | 0          |
| .402   | 844     | 850     | +6         |
| .477   | 1026    | 1032    | +6         |
| .552   | 1196    | 1196    | 0          |
| .631   | 1354    | 1354    | 0          |
| .712   | 1504    | 1504    | 0          |

The question whether or not the temperatures indicated by the lamp will repeat themselves for continued burning or aging is a vital one for the permanence of a calibration and hence for the practical usefulness of the pyrometer. Holborn and Kurlbaum, as well as ourselves, have made a thorough study of this possible source of error.

Lamps which have not been aged or burned for some time at a temperature considerably above that at which they will ordinarily be used undergo marked changes and are unreliable, but if properly aged they reach a steady condition, as indicated by the following table of results

obtained by Holborn and Kurlbaum on three lamps. The current is given in each case for a temperature of  $1,100^{\circ}\text{C}$ .

| Lamp number .....                                       | 1     | 2     | 3     |
|---------------------------------------------------------|-------|-------|-------|
| After 20 hours' burning at $1900^{\circ}\text{C}$ ..... | 0.608 | 0.592 | 0.589 |
| After 5 hours' burning at $1900^{\circ}\text{C}$ .....  | .613  | .592  | .592  |
| After 5 hours' burning at $1900^{\circ}\text{C}$ .....  | .621  | .597  | .597  |
| After 5 hours' burning at $1900^{\circ}\text{C}$ .....  | .622  | .599  | .600  |
| After 20 hours' burning at $1500^{\circ}\text{C}$ ..... | .622  | .599  | .601  |

The plot below shows the results we have obtained by calibrating a lamp, when new, after twenty hours' aging at  $1800^{\circ}\text{C}$ . and again after forty more hours' aging at  $1800^{\circ}\text{C}$ . A number of other lamps were calibrated with similar results. It will be seen that if a lamp

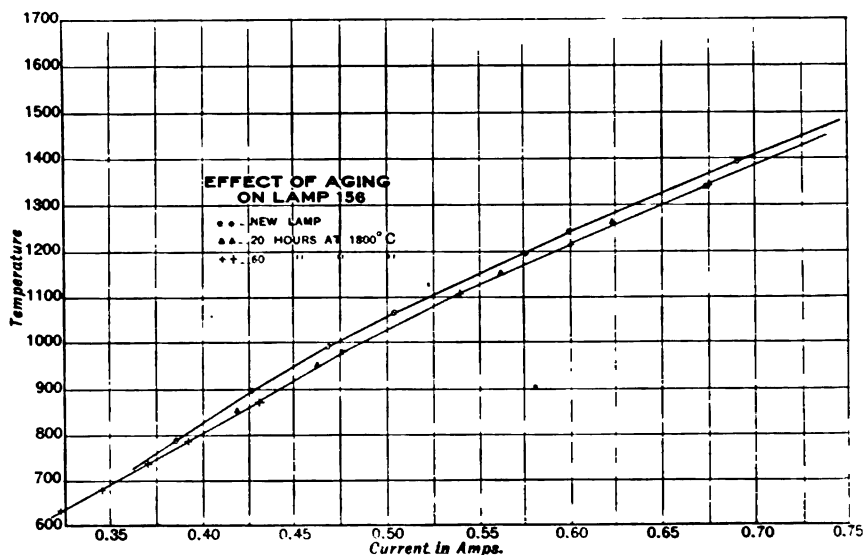


FIG. 11.—Aging of Holborn-Kurlbaum lamp.

is not aged its indications may change by as much as  $25^{\circ}\text{C}$ . with time, but after twenty hours' heating at  $1800^{\circ}\text{C}$  it will undergo no further appreciable changes over a period of time corresponding to many months as used intermittently in the shop if not heated above  $1500^{\circ}\text{C}$ . This state of permanence is sufficient to satisfy the most rigid requirements of practice.

## MORSE PYROMETER.

This instrument, based on exactly the same principle as the Holborn-Kurlbaum, is illustrated in fig. 12. It will only be necessary in describing it to point out the differences in construction.

Instead of a simple hairpin filament, Morse uses a spiral filament *S* in the lamp *L*, requiring a battery of twenty to forty volts. In sighting upon an incandescent body it is necessary to choose some particular spot of the spiral and try to make that spot disappear. This is fatiguing, as the spiral covers a large area and is of just sufficiently varying

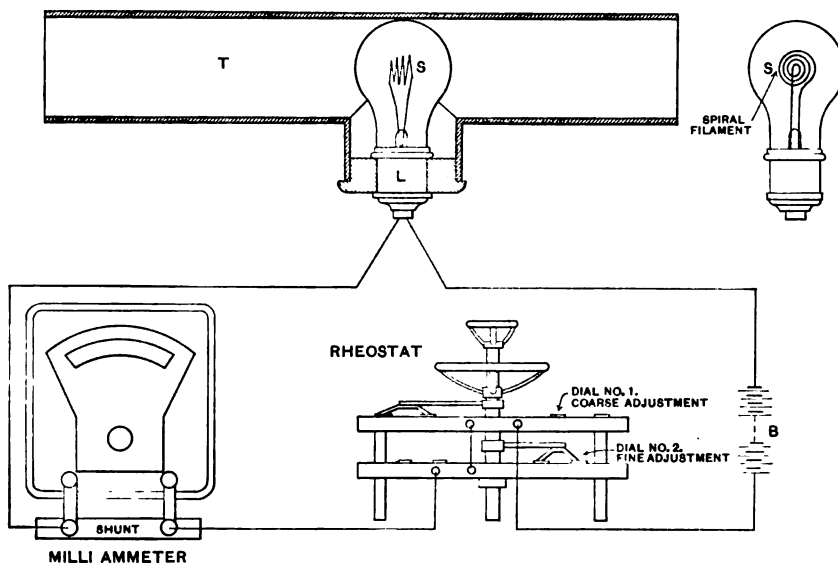


FIG. 12.—Morse thermo-gage.

color to cause the eye to wander. The instrument is not a telescope, possessing no eye-piece or objective, so that the eye has to be trained to recognize the merging of two bright surfaces at different distances. This is readily attained by practice.

The Morse instrument, called a thermo-gage, was designed for use in hardening steel, and throughout the limited temperature range required in this process, in spite of the crudeness of construction above noted, this pyrometer may be read to about  $3^{\circ}$  C. within this range, and therefore meets, in this respect, all the requirements of technical operations. Above  $1200^{\circ}$  C., however, it is very difficult and soon becomes impossible to make a satisfactory setting.

In view of the eminently satisfactory results obtained with the low-voltage lamps of Holborn and Kurlbaum, it would seem that there can be little or no gain as to permanence in the higher voltage spiral form

used by Morse. On the contrary, convenience of reading, accuracy of setting, and especially economy of installation and portability of pyrometer and battery, all are in favor of the low-voltage hairpin type.

Our tests of these spiral filament lamps show that when aged at  $1200^{\circ}$  C. they will remain constant for at least several hundred hours within the range over which they are intended to be used. The Morse instrument would also be greatly improved if provided with a simple focusing device, also with a red glass before the eye for temperatures above  $900^{\circ}$  C., and an absorbing glass before the lamp for very high temperatures.

It is interesting in this connection to note the behavior of ordinary incandescent lamps as to permanence. The accompanying plot shows the changes in candlepower of an ordinary 16-candlepower incandescent

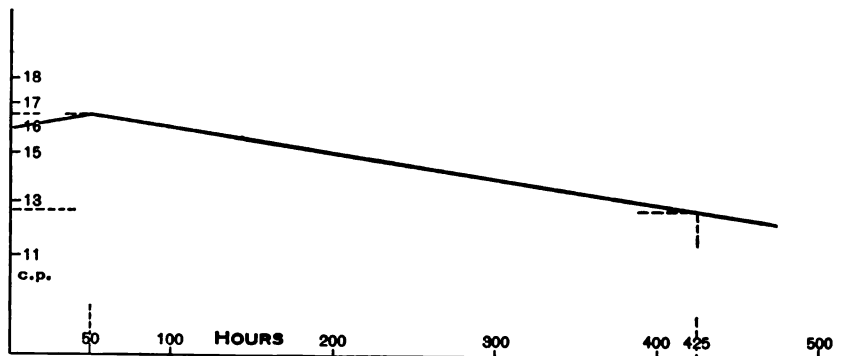


FIG. 13.—Aging of an Incandescent lamp.

lamp burning under normal conditions (3.5 watts per candlepower), or with the filament at a temperature of about  $1900^{\circ}$  C. It is seen that the brightness falls off 20 per cent in 425 hours, and that the relation between candlepower and time is a linear one with a maximum brightness after 50 hours' burning. In terms of temperature the total change in brightness is equivalent to less than  $15^{\circ}$  C., so that it is evident that an ordinary incandescent lamp run at temperatures less than  $1000^{\circ}$  C. and used pyrometrically will give a comparison standard which will not change appreciably in temperature over a period of many hundred hours.

#### FÉRY THERMOELECTRIC TELESCOPE.<sup>a</sup>

This pyrometer is the only form of instrument based on the energy of total radiation which has come into practical use for temperature

<sup>a</sup>Féry: C. R., 134, p. 977; 1902. Ann. d. Chem. et d. Phys. (7), 28, p. 428; 1903. J. de Phys., 8, p. 701; 1904. This instrument is made by M. Pellin, 21 Rue de l'Odeon, Paris.



measurements. As in the case of the photometric pyrometers, the limitations as to the realization of a black body apply here also.

Radiation from an incandescent body is focussed upon a very minute and sensitive thermocouple and raises its temperature. The electromotive force thus generated at the junction actuates a sensitive potential galvanometer in series with the couple, so that we have here a radiation pyrometer which is direct reading by means of a pointer over a scale and could, therefore, readily be made a recording instrument.

The difficulty in construction of such an instrument is in realizing a material for the lens which is transparent for all radiations, so that the pyrometer may be calibrated directly in terms of the Stefan-Boltzmann law. This is effected by the use of a fluorite lens which for temperatures above  $900^{\circ}$  C. satisfies the conditions of not altering appreciably the radiations transmitted through it; that is to say, the ratio of the radiation absorbed to the radiation transmitted is constant.

At low temperatures a large proportion of the energy exists in the form of long wave lengths, and as fluorite has an absorption band in the infra-red (near  $6\mu$ ) it will absorb a considerable portion of the radiation, and therefore the Stefan-Boltzmann law can no longer be assumed. With a sufficiently sensitive galvanometer this pyrometer could, however, still be calibrated for low temperatures.

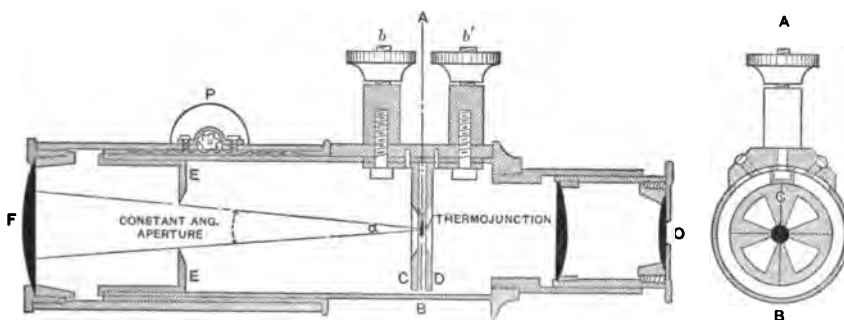


FIG. 14.—Féry thermoelectric telescope.

Fig. 14 illustrates the construction of the instrument where  $F$  is the fluorite lens,  $P$  a rack and pinion for focussing the radiations upon the thermo-junction of iron-constantan protected from extraneous rays by the screens  $C$ ,  $D$ , shown also in section at  $AB$ . The wires of the thermo-couple are of very small dimensions (a few thousandths of a millimeter) and are soldered to a silver disk. The leads are brought out to the insulated binding posts  $b$ ,  $b'$ . The arrangement adopted reduces extraneous thermal currents to a minimum. The circuit is completed through a sensitive galvanometer provided with a scale. A diaphragm

fixed in size and position,  $EE$ , gives a cone of rays of constant angular aperture independent of the focusing.

In making a temperature measurement it is necessary to focus the image of the incandescent object upon the thermo-junction by means of the eyepiece  $O$ , and care must be taken that this image is of greater size than the junction. This adjustment once made, the pyrometer functions indefinitely while sighted upon the same object, readings of the galvanometer scale giving temperatures directly from the calibration.

The precision attainable with this form of instrument, over the range in which it may be controlled with the thermoelectric pyrometer, is shown from data obtained by Féry, assuming the Stefan-Boltzmann law to hold in the form

$$CE = d = 7.66 T^4 \times 10^{-12}$$

where  $E$  is the total energy of radiation,  $d$  the galvanometer deflection, and  $T$  the absolute temperature.

| $d$  | Temperature from thermocouple. | Temperature from Stefan's law. | $\Delta$ in degrees. | Error in percentage. |
|------|--------------------------------|--------------------------------|----------------------|----------------------|
| 11.0 | 844°                           | 860°                           | +16°                 | 1.85                 |
| 14.0 | 914                            | 925                            | +11                  | .84                  |
| 17.7 | 990                            | 990                            | 0                    | .0                   |
| 21.5 | 1054                           | 1060                           | + 6                  | .60                  |
| 26.0 | 1120                           | 1120                           | $\pm$ 0              | .0                   |
| 32.2 | 1192                           | 1190                           | - 2                  | .17                  |
| 38.7 | 1260                           | 1250                           | -10                  | .80                  |
| 45.7 | 1328                           | 1320                           | - 8                  | .60                  |
| 52.5 | 1385                           | 1380                           | - 5                  | .36                  |
| 62.2 | 1458                           | 1450                           | - 8                  | .50                  |

It is evident, furthermore, that if the galvanometer has a uniform scale and the temperature  $T_1$  is known corresponding to any one scale reading  $R_1$ , the temperature  $T_2$  for any other reading  $R_2$  may be found from the relation

$$T_2 = T_1 \sqrt[4]{\frac{R_2}{R_1}}$$

which also shows that errors in the galvanometer readings are divided by 4 when reduced to temperatures. For very high temperatures deflections off the scale of the galvanometer will be obtained. Féry

overcomes this difficulty by substituting a smaller diaphragm before the objective when the radiation is reduced in the ratio of the areas of the apertures. Shunting the galvanometer will also accomplish the same end and this latter method is probably capable of more accuracy.

The laboratory form of apparatus described above is not well suited for use in technical practice, and fluorite is hard to get of sufficient size. The industrial pyrometer is made by substituting for the fluorite lens a much larger one of glass, and for the delicate galvanometer one of the same type and sensibility as is used in thermoelectric work; the resulting instrument is robust and sufficiently sensitive for all practical uses and, as made, has a range from  $800^{\circ}$  C. to  $1600^{\circ}$  C., although the upper limit could readily be extended in the manner indicated above.

The indications of the industrial form of this pyrometer will not obey Stefan's law, but the instrument may readily be calibrated by direct comparison, either with a thermo-couple or with a laboratory form of Féry's instrument, and the temperatures engraved on the scale of the pyrometer galvanometer.

Both types of instrument might be constructed so as to reach lower temperatures ( $650^{\circ}$  C.) by means of more sensitive galvanometers, so that this pyrometer could then be used to advantage in steel hardening or other processes requiring relatively low temperatures. Féry<sup>a</sup> has recently brought out a pyrometer in which the energy of radiation is focussed on the thermojunction by means of a concave mirror, thus extending the lower limit to  $500^{\circ}$  C or thereabouts. The mirror instrument is also the more sensitive.

##### 5. COMPARISON OF VARIOUS TYPES OF OPTICAL PYROMETERS.

No attempt will here be made to answer the question so often asked, "Which is the best pyrometer?" as it is evident from what precedes that a single answer to this question is impossible. We shall, however, briefly summarize the relative behavior in respect to range, calibration, precision, and permanence of indications of the instruments just described.

As was pointed out, the Mesuré and Nouel pyrometric telescope can not be considered an instrument of precision, although it is serviceable for the approximate control of temperatures above  $800^{\circ}$  C. ( $1470^{\circ}$  F.), and answers the requirements of many furnace operations sufficiently well. Its great advantage in practical use is that it is encumbered by no accessories whatever.

The Mesuré and Nouel instrument aside, all of the pyrometers dis-

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<sup>a</sup> Féry: J. de Phys., 8, p. 701; 1904.

cussed above are capable of an accuracy of 1 per cent in temperature measurement in the ranges for which they are adapted for use. The ease with which this accuracy is attained is not, however, the same in all cases. With the Wanner pyrometer, for instance, this accuracy is not reached unless a milliammeter and rheostat are included in the lamp circuit, and with the Holborn-Kurlbaum and Morse instruments, although this accuracy is readily secured, it must be shown that the lamp used is not undergoing change with continued use in order to maintain this accuracy.

In distinction to the accuracy or absolute reproduction of a given temperature is the precision or relative reliability of results. It is possible to control a given temperature or measure temperature differences much more exactly than reproduce absolutely any temperature. For relative temperature measurements the precision attainable is better than 1 per cent, except possibly for the Féry absorption pyrometer, and may be even as good as 2° at 1500° C. with the Holborn-Kurlbaum instrument.

These different pyrometers as furnished by the makers do not have the same temperature ranges. For reading the lowest attainable temperatures measurable optically, about 600° C. (1100° F.), the instruments available are the Le Chatelier, the Holborn-Kurlbaum, and the Morse and the Féry mirror telescope. The Wanner pyrometer and the Féry thermoelectric telescope have their lower limit at about 900° C. (1650° F.).

Regarding the upper limit, the Morse thermo-gage is difficult to use above 1160° C. (2000° F.) and the industrial form of the Féry thermo-electric telescope as now made reaches 1600° C. (2900° F.), while for all the other instruments the upper limit is indefinitely high or greater than temperatures attainable in industrial practice.

For very high temperatures, above 1500° C., in the region where temperatures can only be determined by extrapolation in terms of some radiation law, the most reliable results will be given by those pyrometers obeying one of these laws the most exactly. Thus instruments calibrated in terms of Wien's law but making use of a red glass which is not monochromatic, as the Le Chatelier, the Féry absorption, and also the Holborn-Kurlbaum pyrometer in the region for which the absorption glasses have to be used, will depart from the true temperature scale more than the Wanner, which, using strictly monochromatic light, will obey Wien's law if the reading circle is correctly graduated and set, although the former instruments will each give an arbitrary temperature scale consistent with itself. The Féry thermo-

electric telescope with fluorite lens calibrated directly in terms of Stefan's law should agree with the photometric pyrometers, as Wien's and Stefan's laws have been found experimentally in accord at the very high temperature of  $2000^{\circ}\text{C}.$ <sup>a</sup>

Regarding the ease of calibration and its control, the Wanner pyrometer and the Féry thermoelectric telescope with fluorite lens are perhaps the simplest, requiring only a single known temperature. Two temperatures are all that are required to calibrate the Le Chatelier and Féry absorption pyrometers, and although several points are necessary for the complete graduation of a Holborn-Kurlbaum or Morse pyrometer, two temperatures are all that is necessary if the instrument is to be used in a small temperature range. The user of an instrument, however, if he has a calibration table does not have to concern himself with this matter unless he has reason to suppose the instrument to be out of order.

The production of a recording optical pyrometer has not been announced as yet by any maker, but it would be a simple matter to render either form of the Féry thermoelectric telescope recording since the indications are given by a pointer over a dial. The other types of pyrometer probably could not by any simple means be made into recording instruments.

#### SPECIAL PROBLEMS IN OPTICAL PYROMETRY.

*Departure from black body radiation.*—Inasmuch as optical pyrometers are calibrated in terms of the radiation emitted by a black body, the temperature as determined by them will depend, as we have seen, on the nature of the incandescent body observed. It is, therefore, of interest to determine the amount by which such indications differ from true temperatures. With this object in view we have determined the *black body temperatures* of a number of substances at well-known temperatures, as given by the melting points of pure metals and salts.

*Radiation from platinum.*—Among the substances studied was polished platinum, because it deviates farthest from a black body, and thus gives an idea of the maximum difference to be expected. In these experiments for determinations up to  $1500^{\circ}\text{C}.$  use was made of a Joly meldometer<sup>b</sup> and a Holborn-Kurlbaum optical pyrometer. For experiments at the melting point of platinum a platinum strip about 5 cm long and 4 mm wide was cut narrow at the center, so that the optical pyrometer could be focused on the exact spot at which the

<sup>a</sup>Lummer and Pringsheim: Verh. d. Deutsch. Phys. Ges., (5) 1, p. 3; 1903.

<sup>b</sup>Joly: Proc. Roy. Irish Acad., 2, p. 38; 1891.

strip melted. The electric current through the strip was slowly raised until the strip fused, and the reading of the optical pyrometer at this instant gives the black body temperature corresponding to the melting point of platinum ( $1780^{\circ}\text{C.}$ ) for the particular color of light used. The results of these experiments are given in the following table, for red ( $\lambda=0.651\ \mu$ ), green ( $\lambda=0.550\ \mu$ ), and blue ( $\lambda=0.474\ \mu$ ) light.

*The black body temperature of platinum at its melting point.*

| Date.                           | Number<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black<br>body tem-<br>perature<br>for red<br>light.<br>$\lambda=0.651\mu$ | Date.                           | Number<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black<br>body tem-<br>perature<br>for green<br>light.<br>$\lambda=0.550\mu$ | Date.                           | Number<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black<br>body tem-<br>perature<br>for blue<br>light.<br>$\lambda=0.474\mu$ |
|---------------------------------|-----------------------------------------------------|---------------------------------------------------------------------------|---------------------------------|-----------------------------------------------------|-----------------------------------------------------------------------------|---------------------------------|-----------------------------------------------------|----------------------------------------------------------------------------|
| 1904.                           |                                                     |                                                                           | 1904.                           |                                                     |                                                                             | 1904.                           |                                                     |                                                                            |
| 4/13                            | 156                                                 | 1545°C.                                                                   | 4/18                            | 156                                                 | 1570°C.                                                                     | 4/18                            | 156                                                 | 1618°C.                                                                    |
| 4/13                            | 156                                                 | 1543                                                                      | 4/18                            | 156                                                 | 1585                                                                        | 4/18                            | 156                                                 | 1602                                                                       |
| 4/13                            | 156                                                 | 1550                                                                      | 4/18                            | 156                                                 | 1578                                                                        | 4/18                            | 156                                                 | 1606                                                                       |
| 4/13                            | 156                                                 | 1530                                                                      | 4/18                            | 145                                                 | 1577°C.                                                                     | 4/18                            | 145                                                 | 1625°C.                                                                    |
| 4/13                            | 156                                                 | 1520                                                                      | 4/18                            | 145                                                 | 1577                                                                        | 4/18                            | 145                                                 | 1605                                                                       |
| 4/13                            | 156                                                 | 1526                                                                      | 4/18                            | 145                                                 | 1561                                                                        | 4/18                            | 145                                                 | 1607                                                                       |
| 4/13                            | 156                                                 | 1541                                                                      | 4/18                            | 145                                                 | 1577                                                                        | 4/18                            | 145                                                 | 1609                                                                       |
| 4/14                            | 156                                                 | 1528                                                                      | 4/18                            | 145                                                 | 1577                                                                        | 4/18                            | 145                                                 | 1621                                                                       |
| 4/14                            | 156                                                 | 1547                                                                      | 4/18                            | 132                                                 | 1591°C.                                                                     |                                 |                                                     |                                                                            |
| Mean.                           | 156                                                 | 1537°C.                                                                   | 4/18                            | 132                                                 | 1587                                                                        | Final mean..... 1612°C.         |                                                     |                                                                            |
| 4/14                            | 145                                                 | 1550°C.                                                                   | 4/18                            | 132                                                 | 1576                                                                        |                                 |                                                     |                                                                            |
| 4/14                            | 145                                                 | 1560                                                                      | 4/18                            | 132                                                 | 1570                                                                        |                                 |                                                     |                                                                            |
| Mean.                           | 145                                                 | 1555°C.                                                                   | Final mean..... 1577°C.         |                                                     |                                                                             |                                 |                                                     |                                                                            |
| 4/14                            | 132                                                 | 1540°C.                                                                   |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| 4/14                            | 132                                                 | 1550                                                                      |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| 4/14                            | 132                                                 | 1533                                                                      |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| 4/14                            | 132                                                 | 1542                                                                      |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| 4/14                            | 132                                                 | 1548                                                                      |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| Mean.                           | 132                                                 | 1543°C.                                                                   |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| Final mean ..... 1541°C.        |                                                     |                                                                           |                                 |                                                     |                                                                             |                                 |                                                     |                                                                            |
| $\Delta=239^{\circ}\text{C.}^a$ |                                                     |                                                                           | $\Delta=203^{\circ}\text{C.}^a$ |                                                     |                                                                             | $\Delta=168^{\circ}\text{C.}^a$ |                                                     |                                                                            |

$^a\Delta=T-S$ , where  $T$ =abs. temperature and  $S$ =black body temperature.

### *Radiation from platinum.*

| Red $\lambda = 0.651 \mu$ .       |                                            |                                                     |                                 | Green $\lambda = 0.550 \mu$ . |                                                     |                                 | Blue $\lambda = 0.474 \mu$ .              |                                                     |                                 |
|-----------------------------------|--------------------------------------------|-----------------------------------------------------|---------------------------------|-------------------------------|-----------------------------------------------------|---------------------------------|-------------------------------------------|-----------------------------------------------------|---------------------------------|
| Tempera-<br>ture centi-<br>grade. | Date.                                      | No. of<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black body<br>tempera-<br>ture. | Date.                         | No. of<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black<br>body tem-<br>perature. | Date.                                     | No. of<br>lamp<br>used in<br>H-K<br>pyrom-<br>eter. | Black<br>body tem-<br>perature. |
| 1500° C.<br>(M. P. of Pd.)        | 1904.<br>4/12                              | 145                                                 | 1384° C.                        | 1904.<br>4/18                 | 156                                                 | 1402° C.                        | 1904.<br>4/18                             | 156                                                 | 1434° C.                        |
|                                   | "                                          | "                                                   | 1368                            | "                             | "                                                   | 1402                            | "                                         | "                                                   | 1423                            |
|                                   | "                                          | 132                                                 | 1384                            | "                             | 145                                                 | 1410                            | "                                         | 145                                                 | 1430                            |
|                                   | "                                          | "                                                   | 1370                            | "                             | "                                                   | 1398                            | "                                         | "                                                   | 1419                            |
|                                   | "                                          | 156                                                 | 1368                            | Mean =                        | 1403° C.<br>$\Delta = 97^\circ \text{C.}$           | Mean =                          | 1426° C.<br>$\Delta = 74^\circ \text{C.}$ |                                                     |                                 |
|                                   | 4/14                                       | 145                                                 | 1378                            |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 156                                                 | 1383                            |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | 4/11                                       | 132                                                 | 1365                            |                               |                                                     |                                 |                                           |                                                     |                                 |
| "                                 | 145                                        | 1365                                                |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| Mean =                            | 1374° C.<br>$\Delta = 126^\circ \text{C.}$ |                                                     |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| 1316° C.                          | 4/15                                       | 132                                                 | 1208° C.                        | 4/15                          | 132                                                 | 1233° C.                        | 4/15                                      | 132                                                 | 1269° C.                        |
|                                   | "                                          | 156                                                 | 1210                            | "                             | 156                                                 | 1233                            | "                                         | 156                                                 | 1244                            |
|                                   | Mean =                                     | 1209° C.<br>$\Delta = 106^\circ \text{C.}$          |                                 | Mean =                        | 1233° C.<br>$\Delta = 82^\circ \text{C.}$           |                                 | Mean =                                    | 1256° C.<br>$\Delta = 59^\circ \text{C.}$           |                                 |
|                                   |                                            |                                                     |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| 1215° C.                          | 4/15                                       | 132                                                 | 1122° C.                        | 4/15                          | 132                                                 | 1135° C.                        |                                           |                                                     |                                 |
|                                   | "                                          | 156                                                 | 1117                            | "                             | 156                                                 | 1138                            |                                           |                                                     |                                 |
|                                   | Mean =                                     | 1119° C.<br>$\Delta = 96^\circ \text{C.}$           |                                 | Mean =                        | 1136° C.<br>$\Delta = 79^\circ \text{C.}$           |                                 |                                           |                                                     |                                 |
|                                   |                                            |                                                     |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| 1064° C.<br>M. P. of Au.          | 4/11                                       | 156                                                 | 969° C.                         | 4/14                          | 132                                                 | 987° C.                         |                                           |                                                     |                                 |
|                                   | "                                          | 145                                                 | 967                             | "                             | 156                                                 | 985                             |                                           |                                                     |                                 |
|                                   | 3/23                                       | 156                                                 | 978                             | "                             | "                                                   | 985                             |                                           |                                                     |                                 |
|                                   | 4/14                                       | 132                                                 | 978                             | 4/15                          | "                                                   | 987                             |                                           |                                                     |                                 |
|                                   | "                                          | 145                                                 | 973                             | "                             | "                                                   | 982                             |                                           |                                                     |                                 |
|                                   | "                                          | 156                                                 | 980                             | Mean =                        | 985° C.<br>$\Delta = 79^\circ \text{C.}$            |                                 |                                           |                                                     |                                 |
|                                   | 3/23                                       | { With<br>Wanner<br>pyrom-<br>eter.                 | 970                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          |                                                     | 971                             |                               |                                                     |                                 |                                           |                                                     |                                 |
| "                                 |                                            | 974                                                 |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| Mean =                            | 978° C.<br>$\Delta = 91^\circ \text{C.}$   |                                                     |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| 782 C.<br>M. P. of NaCl           | 4/11                                       | 145                                                 | 714° C.                         |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 132                                                 | 720                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | Mean =                                     | 717° C.<br>$\Delta = 65^\circ \text{C.}$            |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   |                                            |                                                     |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |
| 723 C.<br>(M. P. of KBr.)         | 4/11                                       | 145                                                 | 660° C.                         |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 132                                                 | 665                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 156                                                 | 662                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 132                                                 | 670                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 156                                                 | 670                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | "                                          | 145                                                 | 671                             |                               |                                                     |                                 |                                           |                                                     |                                 |
|                                   | Mean =                                     | 666° C.<br>$\Delta = 67^\circ \text{C.}$            |                                 |                               |                                                     |                                 |                                           |                                                     |                                 |

specimen of the metal or salt on the strip and slowly increasing the heating current until the specimen was observed to melt, when the length of the strip was determined by the electric contact micrometer screw and the current regulated to maintain this length (and therefore the temperature) constant throughout a series of observations with the optical pyrometer. A second determination of the melting point was always made with the meldometer at the end of every series of observations.

The black body temperatures of polished platinum at  $1500^{\circ}\text{C.}$ ,  $1064^{\circ}\text{C.}$ ,  $782^{\circ}\text{C.}$ , and  $723^{\circ}\text{C.}$ , as fixed by the melting points of Pd,<sup>a</sup> Au, NaCl, and KBr, and at  $1315^{\circ}\text{C.}$  and  $1215^{\circ}\text{C.}$ , as fixed by the observed length of the meldometer strip, are given in the table.

The results of all the experiments on platinum are best shown graphically in fig. 15, where for comparison we have added the results obtained by Holborn and Kurlbaum<sup>b</sup> for red light by a somewhat different method.

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<sup>a</sup> The melting point of Pd is uncertain, and  $1525^{\circ}\text{C.}$  is perhaps a better value than the one we have taken ( $1500^{\circ}\text{C.}$ ). The value for this point as obtained by Holborn and Wien is  $1587^{\circ}\text{C.}$  (Wied. Ann. **56**, 1895) by extrapolation of the thermoelectric relations, but is certainly too high as they determined the gold point  $8^{\circ}$  too high. Violle found  $1500^{\circ}\text{C.}$  by the specific heat method, which also gave the Pt fusing point as  $1780^{\circ}$ . Callendar (Phil. Mag., **48**, 1899) with Eumorfopoulos found  $1530^{\circ}\text{C.}$  and  $1550^{\circ}\text{C.}$  by the expansion and resistance of platinum respectively. Taking the Pd point as  $1525^{\circ}\text{C.}$  gives in fig. 11 a value of  $\Delta=150^{\circ}\text{C.}$  from our observations so that the T vs. T-S curve would then have no abrupt turn as the platinum point is approached.

<sup>b</sup> Holborn and Kurlbaum: Ann. d. Phys., **10**, p. 225; 1903.



The black body temperature of the melting point of platinum for red light as found by these observers is  $1545^{\circ}\text{C}$ . ( $\Delta=235^{\circ}$ ) which is in most satisfactory agreement with the results found by us, which is  $1541^{\circ}$  ( $\Delta=239^{\circ}$ ), obtained by the same method of experiment.

Our results would seem to indicate that the values of  $\Delta$  increase somewhat less rapidly with rise in temperature than is indicated by the results of Holborn and Kurlbaum. These differences may arise from several causes. The maldometer strip is probably at a slightly

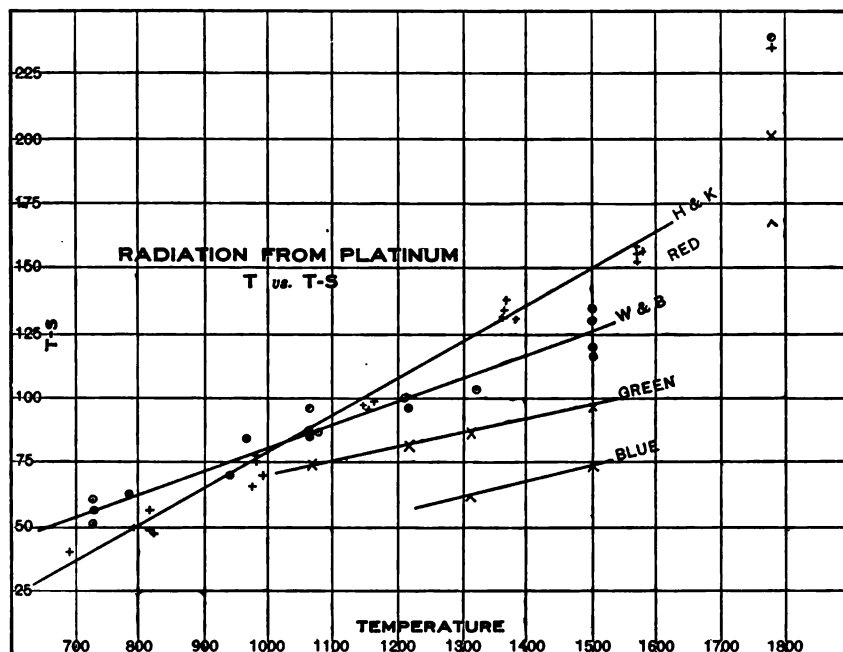


FIG. 15.—Radiation from platinum.

higher temperature than indicated by the melting point calibration and by an amount which is probably greater at high temperatures. This would tend to make the value of  $\Delta$  small at high temperatures. In the experiments of Holborn and Kurlbaum the platinum was in the form of a cylinder surrounding a thermocouple. It is thus possible that the couple was at a slightly higher temperature than the outer radiating surface of the platinum, which would tend to increase the values of  $\Delta$ , especially at high temperatures.

In order to pass through the platinum point ( $\Delta = 239^\circ$ , red light) a very sudden bend in the curves is necessary (but see note on p. 246). No evidence is yet at hand to explain whether this can in any way be associated with a decreased radiating power of a substance as it approaches a change of state.

It will also be seen, as has been previously shown, that platinum approaches more nearly to black-body radiation for green and blue than for red light.

*Radiation from other substances.*—Experiments were made on a number of other substances, including clay, copper and iron oxides, and unglazed porcelain to determine the departure of these bodies, at various temperatures, from black-body radiation. Another method of experiment was here necessary, as it is not possible to accurately fix the temperature of oxidizable substances by the maldometer method. The method used will be understood from the following diagrammatic sketch, fig. 16.

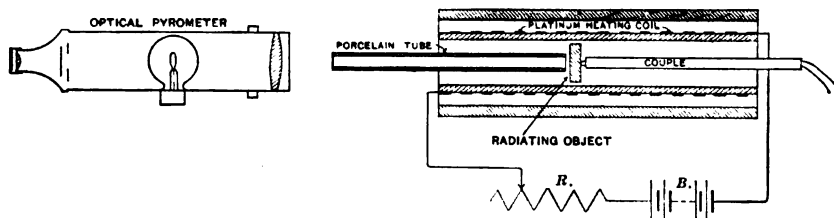


FIG. 16.—Furnace method of studying radiation.

The radiating substance is placed at the center of an electric furnace. The radiations emitted by the substance under these conditions is a very close approximation to black-body radiation, and its temperature as measured by the optical pyrometer will be the true temperature. As a further check on the optical pyrometer a thermocouple in contact with the radiating substance was always read. A porcelain tube, open at both ends and blackened on the interior, was then thrust into the furnace, and the radiation coming out of this cold tube, which is the radiation of that particular substance, was observed with the optical pyrometer. The temperature measured in this way is the equivalent black-body temperature of the substance for the particular color of light used. The tube was then quickly withdrawn and another measurement taken of the true temperature. There was always a slight cooling due to insertion of the tube, which was allowed for by taking the observations in the order indicated above. The results of these experiments giving the departure from black-body radiation are given on page 250.

The experiments at the melting point of copper were carried out in a manner similar to those at the melting point of platinum. A copper strip cut narrow at the center was observed with an optical pyrometer at the instant it was fused by an electric current. The temperature of melting was assumed to be  $1065^{\circ}\text{C.}$ , corresponding to the melting point of copper in an oxidizing atmosphere, and the corresponding black-body temperature was given by the optical pyrometer.

In order to compare the *meldometer method* with the *furnace method* we determined the departure of platinum from black-body radiation by both methods. The agreement is even better than we have any right to expect; thus, at  $950^{\circ}\text{C.}$ , the furnace method gave  $74^{\circ}\text{C.}$  and the meldometer method (by interpolation)  $75^{\circ}\text{C.}$  as the difference between the true temperature and the black-body temperature of platinum.

The interesting range of iron and steel in which hardening and annealing operations are carried out is from about  $600^{\circ}\text{C.}$  to  $950^{\circ}\text{C.}$ , and, as will be seen from the table, the indications of an optical pyrometer in this range, if calibrated in the usual way in terms of black-body radiation, will be low by about  $20^{\circ}$  at the lower temperature and about  $40^{\circ}$  at the upper temperature.

The radiation from the fine-grained crucible seemed to be at least as black as that from iron, which would hardly be expected. When a fresh piece of thin Russian iron was inserted into the furnace and measurements were immediately taken with an optical pyrometer, it was observed that the initial readings were some  $20^{\circ}$  higher at  $1000^{\circ}\text{C.}$  than when the oxidation of the surface was complete, which takes two or three minutes.

*Departure from black-body radiation.*

| Temp. °C.   | Platinum.                                                                                                          | Iron oxide.                                                                                            | Fine-grain<br>Battersea<br>crucible.                                                             | Copper<br>oxide.                                                                                                   | Unglazed<br>porcelain.                                                                           |
|-------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| 730° .....  |                                                                                                                    |                                                                                                        |                                                                                                  |                                                                                                                    | $\left. \begin{array}{c} 31^{\circ} \\ 35 \\ 25 \\ 30 \end{array} \right\} 80^{\circ} \text{C.}$ |
| 750° .....  |                                                                                                                    |                                                                                                        |                                                                                                  | $\left. \begin{array}{c} 22^{\circ} \\ 28 \\ 25 \end{array} \right\} 25^{\circ} \text{C.}$                         |                                                                                                  |
| 770° .....  |                                                                                                                    | $\left. \begin{array}{c} 20^{\circ} \\ 26 \end{array} \right\} 28^{\circ} \text{C.}$                   | $\left. \begin{array}{c} 11^{\circ} \\ 16 \end{array} \right\} 14^{\circ} \text{C.}$             |                                                                                                                    |                                                                                                  |
| 790° .....  |                                                                                                                    | $\left. \begin{array}{c} 20^{\circ} \\ 21 \\ 22 \\ 26 \\ 23 \end{array} \right\} 28^{\circ} \text{C.}$ |                                                                                                  |                                                                                                                    |                                                                                                  |
| 950° .....  | $\left. \begin{array}{c} 76^{\circ} \\ 71 \\ 77 \\ 81 \\ 82 \\ 60 \\ 67 \end{array} \right\} 74^{\circ} \text{C.}$ |                                                                                                        |                                                                                                  |                                                                                                                    |                                                                                                  |
| 980° .....  |                                                                                                                    | $\left. \begin{array}{c} 43^{\circ} \\ 43 \\ 46 \\ 53 \\ 45 \end{array} \right\} 46^{\circ} \text{C.}$ |                                                                                                  |                                                                                                                    |                                                                                                  |
| 1050° ..... |                                                                                                                    |                                                                                                        | $\left. \begin{array}{c} 50^{\circ} \\ 58 \\ 57 \\ 52 \end{array} \right\} 54^{\circ} \text{C.}$ |                                                                                                                    |                                                                                                  |
| 1065° ..... |                                                                                                                    |                                                                                                        |                                                                                                  | $\left. \begin{array}{c} 37^{\circ} \\ 50 \\ 33 \\ 37 \\ 43 \\ 43 \\ 30 \end{array} \right\} 89^{\circ} \text{C.}$ |                                                                                                  |

*Polarized light from incandescent surfaces.*—Arago was among the first to observe that the light emitted by incandescent surfaces was partially polarized, the amount of polarized light varying widely for different substances, and increasing with the angle at which the surface was viewed, being *nil* in a direction normal to the surface. Prevostaye and Desain, and Magnus, showed that the infra red waves are also polarized in the same plane as the light waves. Arago attributed the polarization to refraction, near the surface, of the light coming from molecules below the surface.

Millikan,<sup>a</sup> who has done the most complete work in this field, has pointed out that this view of Arago fails to explain why the light coming from platinum near grazing emergence is practically completely polarized, for such light must to a large extent be surface light. His experiments indicate that the polarization is a phenomenon of refraction, and that all the light suffers refraction at the surface. Millikan has examined the amount of polarization in the light emitted by a large number of substances. The phenomenon is most marked in platinum, silver, and gold, and is very feeble in the readily oxidizable substances, such as iron, and in such substances as glass and porcelain. It is quite marked for iron in the molten state.

Our experiments have been made mainly with a view to determining the amount by which this phenomenon could influence the indications of optical pyrometers that make use of polarizing devices to adjust to equality the light from the incandescent body observed and from the standard comparison light.

In the experiments on platinum the temperature of the horizontal strip of a Joly meldometer, maintained constant by regulating the electric heating current, was measured with a Wanner optical pyrometer, which employs polarizing devices. To eliminate the effect of the sag of the strip the readings were taken with the pyrometer in four positions at right angles. For each temperature the measurements were made at different angles with the normal to the surface. In rotating the pyrometer the portion of the radiating strip viewed was slightly changed, but an examination of the strip showed that the heating was very uniform, the variations not exceeding 4° over a

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<sup>a</sup> Millikan: Phys. Rev., 3, pp. 81, 177; 1895.

distance of 2 cm. The results of the measurements on platinum are given in the following table:

| PLATINUM AT 1450° C.                           |                                                                     |                                |                                                             |
|------------------------------------------------|---------------------------------------------------------------------|--------------------------------|-------------------------------------------------------------|
| Angle at which incandescent surface is viewed. | Position of slit of pyrometer with reference to incandescent strip. | Mean reading Wanner pyrometer. | Maximum $\Delta$ for two positions of slit at right angles. |
| 30°                                            | $\uparrow\downarrow$                                                | 63                             | 3.3 divs. $\odot$ 34°                                       |
| 0°                                             | $\leftrightarrow$                                                   | 64.6                           |                                                             |
| 30°                                            | $\rightarrow$                                                       | 66.3                           |                                                             |
| 50°                                            | $\uparrow\downarrow$                                                | 61.9                           | 7.6 divs. $\odot$ 80°                                       |
| 0°                                             | $\leftrightarrow$                                                   | 66.0                           |                                                             |
| 50°                                            | $\rightarrow$                                                       | 69.5                           |                                                             |
| 70°                                            | $\uparrow\downarrow$                                                | 60.8                           | 8.4 divs. $\odot$ 90°                                       |
| 0°                                             | $\leftrightarrow$                                                   | 64.9                           |                                                             |
| 70°                                            | $\rightarrow$                                                       | 69.2                           |                                                             |
| PLATINUM AT 1150° C.                           |                                                                     |                                |                                                             |
| 50°                                            | $\uparrow\downarrow$                                                | 23.0                           | 2.8 divs. $\odot$ 22°                                       |
| 0°                                             | $\leftrightarrow$                                                   | 24.5                           |                                                             |
| 50°                                            | $\rightarrow$                                                       | 25.8                           |                                                             |

From the above table it will be seen that if the temperature of a platinum surface is measured with a polarizing pyrometer large errors may result if the effect of polarized light is neglected. Thus, if the surface be viewed at an angle of 50° with the normal, two readings may be obtained differing from one another by 80° C. at 1450° C., i. e., the reading may differ from the actual temperature by 40° or thereabouts.

As the light emitted in a direction perpendicular to the surface is not polarized, this source of error can always be avoided by viewing the surface normally. For most substances on which optical pyrometers are used in industrial operations, the amount of polarized light is very feeble compared with platinum, so that its effect is entirely negligible. In some experiments where the incandescent surface of iron was viewed at an angle of 75° the effect was less than 5° C. In any case,

the effect may be eliminated by taking readings in four azimuths at intervals of  $90^\circ$ .

*Effect of diffuse and reflected light.*—The ordinary diffuse daylight or light from other sources which is not reflected directly into the pyrometer exerts a comparatively negligible effect on the indications of these instruments, not exceeding  $5^\circ$  C. for a change from a darkened room to bright daylight.

The magnitude of the effect of directly reflected light is very variable, depending upon the reflecting power of the incandescent body observed, and on the temperature, size, and location of the disturbing sources. With a good reflecting surface like polished platinum, which remains a good reflector at high temperatures, the effect may be very great, while with a substance like iron (oxide) it is very much less for the same surrounding sources of light. Again, the greater the difference in temperature between the object viewed and the disturbing light the greater will be the effect.

Some of the effects noted with platinum were as follows: A platinum strip at  $800^\circ$  C. had its temperature apparently increased  $15^\circ$  C. by direct reflected light from an incandescent lamp,  $120^\circ$  C. by a large gas flame, and with the strip mounted within a ring of gas flames an apparent rise of over  $300^\circ$  C. was observed.

With a sheet of iron at  $725^\circ$  C., within the ring and with the gas flames burning low, a change of  $35^\circ$  C. was noted in the indications of the pyrometer, while with the flames burning brightly no measurements could be taken. When the iron was at  $1100^\circ$  C., however, turning off and on the flames made practically no difference in the temperature readings.

The arrangement with the ring burner approximates a number of industrial operations, where heating is done by radiation from flames. The above results would seem to show that even for poor reflectors, such as iron, clays, etc., considerable errors may be introduced at temperatures below  $800^\circ$ , and that the error becomes small above  $1100^\circ$ .

This source of error may be very nearly eliminated by viewing the object through a tube which cuts off most of the light from surrounding flames. Thus in the instance cited above for iron the error of  $35^\circ$  C. was reduced to  $5^\circ$  C. by this means.

*The measurement of very high temperatures.*—The temperatures that have been discussed thus far are within the range controlled by thermocouples, calibrated to agree with the gas scale to about  $1150^\circ$  C., which marks at present the upper limit of satisfactory gas thermometry. The thermocouple scale is then extrapolated for  $500^\circ$  or  $600^\circ$  more.

Already there are many operations, such as those carried out in the Moissan furnace, the Goldschmidt thermite process, the production of carbides and metallurgical products in electric furnaces, and many pyrochemical reactions that involve temperatures of  $2000^{\circ}$  or over. It therefore becomes necessary to establish at least some tentative scale that can be used at these extreme temperatures.

Attempts are being made by Nernst and others to estimate these high temperatures by means of chemical phenomena taking place at high temperatures, but this work is still in a preliminary state. For this purpose, therefore, recourse must be had alone to the extrapolation of the laws of radiation which have been verified throughout the range of measureable temperatures.

Lummer and Pringsheim<sup>a</sup> have recently taken a single set of observations on an electrically heated carbon tube in an atmosphere of nitrogen, using three radiation methods: Photometric (Wien's law), spectrophotometric ( $\lambda_m$ ,  $T=A$ ), and total radiation (Stefan's law), the results agreeing to  $20^{\circ}$  at  $2300^{\circ}$  C. absolute.

From our own work it would seem that the radiation laws are still in agreement at the temperature of the arc.<sup>b</sup> Our measurements have given as the black body temperature of the hottest part of the positive crater  $3690^{\circ}$ ,  $3680^{\circ}$ , and  $3720^{\circ}$  absolute, as determined with the Holborn-Kurlbaum, Wanner, and Le Chatelier pyrometers, based on the extrapolation of Wien's law. Féry<sup>c</sup> gets for this temperature  $3760^{\circ}$  by a method based on Stefan's law.

On the basis of these experiments it would seem that the several laws of radiation are in quite satisfactory agreement at the highest attainable temperatures, and thus serve to define the same scale of temperatures.

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<sup>a</sup> Lummer and Pringsheim: *Verh. d. Deutsch. Phys. Ges.*, (5) 1, p. 3; 1903.

<sup>b</sup> Waidner and Burgess: *Bulletin No. 1*, Bureau of Standards; 1904.

<sup>c</sup> Féry: *Comptes Rendus*, 134, p. 977; 1902.



# ON THE THEORY OF THE MATTHEWS AND THE RUSSELL-LÉONARD PHOTOMETERS FOR THE MEASUREMENT OF MEAN SPHERICAL AND MEAN HEMISPHERICAL INTENSITIES.

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By EDWARD P. HYDE.

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## I. INTRODUCTION.

Much attention has been given in the last few years to the development of photometers designed for the measurement of mean spherical and mean hemispherical intensities of light sources. Among other instruments of this type that have appeared may be mentioned particularly the two instruments<sup>a</sup> designed by Matthews, and the more recent instrument<sup>b</sup> of Léonard, based on theory first given by Russell.<sup>c</sup> In each of these photometers mirrors are employed in order to project upon the photometer screen light emitted by the source at various inclinations to the vertical axis, but in each instrument a different method is used for weighting the light from each mirror, so that in the summation the light from each mirror shall be diminished in proportion to the area of the zone in which the mirror lies and for which it is supposed to give the mean value.

In the Matthews integrating photometer for incandescent lamps the light from each mirror falls upon the screen at an angle of incidence,  $90^\circ - \theta$ ,<sup>d</sup> which is the same as the angle between the horizontal direction and the line joining the mirror in question with the light source. The area of a zone of latitude  $90^\circ - \theta$  is proportional to  $\sin \theta$ , and the intensity of illumination produced by light incident at angle  $90^\circ - \theta$  is

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<sup>a</sup> Trans. Amer. Inst. of Elec. Eng., 18, p. 677, 1901; ~~20~~<sup>19</sup> p. 1465, 1902.

<sup>b</sup> L'Éclairage Électrique, 40, p. 128, 1904.

<sup>c</sup> Jour. Inst. of Elec. Eng., 32, p. 631, 1903.

<sup>d</sup> In order to make the use of " $\theta$ " in this connection consistent with its subsequent use in the paper, it is necessary to denote the angle of incidence by " $90^\circ - \theta$ " instead of by " $\theta$ ."

cut down in the ratio of  $1:\sin \theta$ ; therefore, the light from each mirror produces an illumination of the screen proportional to the area of the zone in which the mirror lies, and hence receives its proper weight in the summation. In the integrating photometer for arc lamps, however, the light from each mirror is incident upon the screen at the same angle, but between the screen and the circle of mirrors is interposed a circular glass disk divided into as many sectors as there are mirrors, and having each sector so smoked that its coefficient of transmission is proportional to the cosine of the angle,  $90^\circ - \theta$ , between the horizontal direction in the plane of the mirrors and the line joining the light-source with the mirror to which the sector in question corresponds. In this way the light from each mirror is cut down in the ratio of  $1:\sin \theta$ , and therefore receives its proper weight in the summation for the mean spherical intensity.

It is to be especially noted that in the theory of each of the above instruments the surface of the unit sphere is divided into an integral number of zones, not of equal area, but of equal arc, so that the area of each zone is proportional to  $\sin \theta$ , and to each zone a mirror is assigned. Suppose, now, that instead of dividing the surface of the sphere into zones of equal arc we divide it into zones of equal area, and in each zone place a mirror in such a position that it gives the mean value of the intensity for the zone in which it lies. Under these circumstances the light from each mirror, since it stands in every case for a zone of the same area, is to be given the same weight in the summation for mean spherical intensity, and consequently no further weighting is necessary.

This principle is made use of in the instrument of Léonard, though it was Russell who first suggested that in the determination of mean spherical and mean hemispherical candle-power by a number of readings at intervals in the vertical plane, the readings be made, not at equal angular intervals, as is ordinarily done, but at intervals corresponding to the middle points of successive zones of equal area, so that the unweighted arithmetical mean of the various readings may be taken as the mean spherical intensity of the source.

It was the original intention of this paper to present a more complete theoretical discussion of the Matthews integrating photometers, particularly of the instrument intended for incandescent lamps and sources of like intensity, as it was in connection with the design of such an instrument for the Bureau of Standards that this investigation was undertaken. Since, however, the Russell-Léonard instrument may be treated in an entirely similar manner, we shall conclude the paper with a brief theoretical discussion of this new type of photo-

meter, as applied both to arc lamps and to incandescent lamps and sources of like intensity.

In the theoretical discussion of the Matthews integrating photometer for incandescent lamps, as given by Professor Matthews before the American Institute of Electrical Engineers, the assumption is implicitly made that eleven pairs of mirrors placed every  $15^\circ$  (the two end pairs at  $0^\circ$  and  $180^\circ$  being omitted) will yield a value of mean spherical candle-power which will agree with the true integral solution to within a negligibly small error. Earlier, in describing his integrating photometer for arc lamps, Professor Matthews makes a definite statement to this effect, and adduces as evidence values computed from the hypothetical intensity curve

$$I_\theta = I_o \sin \theta$$

Thus, starting with the two equations for the mean spherical candle-power, the exact integral equation

$$I_{ms} = \frac{1}{2} \int_0^\pi I_\theta \sin \theta d\theta$$

and the approximate equation

$$I_{ms} = \frac{\pi}{2n} \sum_0^\pi I_\theta \sin \theta$$

he computes first the true mean spherical candle-power for the distribution  $I_\theta = I_o \sin \theta$  by integration, as follows:

$$\begin{aligned} I_{ms} &= \frac{1}{2} \int_0^\pi I_\theta \sin \theta d\theta = \frac{I_o}{2} \int_0^\pi \sin^2 \theta d\theta \\ &= .785 I_o = 78.5 \text{ when } I_o = 100. \end{aligned}$$

He then computes the approximate value from the expression

$$I_{ms} = \frac{\pi}{2n} \sum_0^\pi I_\theta \sin \theta = \frac{\pi I_o}{2n} \sum_0^\pi \sin^2 \theta$$

for  $n=12$  and  $\theta=0^\circ, 15^\circ, 30^\circ, \dots, 180^\circ$ , obtaining as before

$$I_{ms} = .785 I_o = 78.5.$$

As a result of this agreement between the integral and the summational values, the conclusion is drawn that a mirror every  $15^\circ$  is entirely sufficient to give an approximate value which will differ from the true value by a negligibly small error. Now, although it is perfectly allowable to apply results obtained from a hypothetical intensity

curve to actual intensity curves agreeing more or less closely with it, it can not be assumed that because the integral and summational values agree quite closely for one form of intensity curve that the same agreement can be expected in the cases of intensity curves of different forms. In fact, as will be shown later, there is an error of over 2 per cent entering when we apply the above summational expression to the intensity curve

$$I_{\theta} = I_0 \cos \theta.$$

In the second place it is interesting to note that the hypothetical intensity curve which happened to be chosen in order to show that eleven mirrors are sufficient to give a summation very close to the integral value, will show equally well that any number of mirrors whatever, arranged at equal angular intervals beginning at  $\theta=0^{\circ}$  and ending at  $\theta=\pi$  will give absolutely the true result. This follows from the equation

$$\sum_{m=1}^{m=n} \sin^2 \frac{m\pi}{n} = \frac{n}{2}$$

which can be proved as follows:

In order to get rid of the square, put

$$\sin^2 \frac{m\pi}{n} = \frac{1 - \cos \frac{2m\pi}{n}}{2} = \frac{1}{2} - \frac{1}{2} \cos \frac{2m\pi}{n}$$

Then the sum

$$\sum_{m=1}^{m=n} \sin^2 \frac{m\pi}{n} = \frac{n}{2} - \frac{1}{2} \sum_{m=1}^{m=n} \cos \frac{2m\pi}{n}$$

For convenience, put  $\phi = \frac{2\pi}{n}$  so that

$$\cos \frac{2m\pi}{n} = \cos m\phi$$

We shall now evaluate

$$\sum_{m=1}^{m=n} \cos m\phi$$

Introducing the complex expression

$$\cos m\phi + i \sin m\phi = e^{im\phi}$$

we have

$$\sum_{m=1}^{m=n} \cos m\phi = \text{real part of } \sum_{m=1}^{m=n} e^{im\phi}$$

Now the sum of the geometric series

$$\begin{aligned}\sum_{m=1}^{m=n} e^{im\phi} &= e^{i\phi} \frac{1 - e^{in\phi}}{1 - e^{i\phi}} \\ &= (\cos \phi + i \sin \phi) \frac{1 - \cos n\phi - i \sin n\phi}{1 - \cos \phi - i \sin \phi}\end{aligned}$$

the real part of which reduces to

$$\frac{\cos \phi + \cos n\phi - \cos (n+1)\phi - 1}{2(1 - \cos \phi)}$$

If, now, we put for  $\phi$  its value,  $\frac{2}{n}\pi$ , we obtain

$$\text{real part of } \sum_{m=1}^{m=n} e^{im\phi} = \sum_{m=1}^{m=n} \cos \frac{2m\pi}{n} = 0$$

Therefore,

$$\sum_{m=1}^{m=n} \sin^2 \frac{m\pi}{n} = \frac{n}{2} - \frac{1}{2} \sum_{m=1}^{m=n} \cos \frac{2m\pi}{n} = \frac{n}{2}$$

From this it follows that

$$I_{ms} = \frac{\pi}{2n} \sum_{m=1}^{m=n} I_o \sin^2 \frac{m\pi}{n} = \frac{\pi}{2n} \times \frac{n}{2} I_o = .785 I_o = 78.5$$

Hence we see that if we should divide the sphere into but two zones, placing a single mirror-pair at  $\theta = 90^\circ$ , we would still obtain exactly the true integral value for an intensity curve of the form  $I_\theta = I_o \sin \theta$ , so that no definite information with regard to the necessary number of mirrors can be obtained from the consideration of this intensity curve alone.

## II. GENERAL THEORY.

We shall now pass on to a more complete discussion of the problem of the number of mirrors and their positions on the arc, with particular reference rather to the integrating photometer for incandescent lamps than to the instrument intended especially for arc lamps, since, as stated above, it was in connection with the design of an instrument of the former type for the Bureau of Standards that these computations were made.

Returning to the two equations for the integral and the approximate summational values of the mean spherical intensity,—

$$(1) \quad I_{ms} = \frac{1}{2} \int_0^\pi I_\theta \sin \theta \, d\theta$$

$$(2) \quad I_{ms} = \frac{\pi}{2n} \sum_0^{\pi} I_{\theta} \sin \theta,$$

to which may be added the corresponding expressions for mean hemispherical intensity,—

$$(3) \quad I_{mhs} = \int_0^{\pi} \frac{1}{2} I_{\theta} \sin \theta \, d\theta$$

$$(4) \quad I_{mhs} = \frac{\pi}{n} \sum_0^{\frac{\pi}{2}} I_{\theta} \sin \theta$$

we note that the underlying assumption in passing from the integral to the summational expression is that for each zone,  $\frac{\pi}{n}$ ,

$$(5) \quad I_{\theta} \sin \theta = \text{constant}$$

or, what amounts to the same thing, that  $\theta_n$  has been so chosen for each zone that  $I_{\theta_n} \sin \theta_n$  gives the mean value of  $I_{\theta} \sin \theta$  for that zone.

Suppose, then, that from a previous decision with regard to the general dimensions of the instrument to be constructed the number of mirrors is determined, the problem consists in so arranging the mirrors that each mirror produces an illumination of the screen that in a way gives the mean for the respective zone,  $\frac{\pi}{n}$ , in which it lies. Thus we

divide the semicircumference into  $n$  equal arcs,  $\frac{\pi}{n}$ , and to each arc

we assign a mirror. This mirror should be placed in its arc in such a position that, so far as possible, it will produce on the screen the mean illumination of the arc. Of course the arrangement of the mirrors would in general be different for each different intensity curve, but we seek to determine which arrangement of mirrors will most nearly satisfy all the more ordinary forms of intensity curves with which we have to deal in the photometry of incandescent lamps and sources of like intensity, such as the Nernst lamp. Obviously, in general it would be undesirable to place the mirrors at the divisions between successive zones. This, however, is what the arrangement in the Matthews instrument as ordinarily constructed would amount to if we attempted to use it for the determination of mean hemispherical intensities, as the horizontal mirrors would be giving the illumination of the zone  $\theta = 90^\circ$  to  $\theta = 90^\circ - \frac{\pi}{n}$ . When used for mean spherical deter-

minations it would seem that the same objection could be urged, because of the two end mirrors at  $\theta=0^\circ$  and  $\theta=\pi$ ; but in this case we can assume that each mirror gives the mean value of the zone of which it lies in the center, in which case the illumination produced by each end mirror would have to be divided by 2, since it represents only a half-zone. But since at this incidence ( $90^\circ$ ) there is no illumination at all produced by these end mirrors in the hypothetical case of a point source, no error is introduced by the incorrect weighting of the end mirrors, for in all cases the effect is zero. (In fact these end mirrors are left out entirely.)

Let us now take several simple, hypothetical curves which are more or less typical of the actual intensity curves of the more ordinary sources, and let us determine for each the distribution of mirrors which will give the true mean spherical value for that case, supposing the semicircumference divided into 20 zones, as that is the number of mirror-pairs determined upon for the instrument under construction at the Bureau of Standards. Let us then apply the true distribution for each individual curve to the other hypothetical intensity curves under consideration, to see how closely it satisfies them; and, finally, let us see how nearly an equal distribution of mirrors satisfies each case, taking both of the equal distributions (for 20 mirror-pairs)  $0^\circ, 9^\circ, 18^\circ \dots 180^\circ$ , and  $4\frac{1}{2}^\circ, 13\frac{1}{2}^\circ, 22\frac{1}{2}^\circ \dots 175\frac{1}{2}^\circ$ .

It may be well to call attention here to the advantages gained by dividing the semicircumference into an even number of arcs, and by avoiding such a use of fractions of an arc as is made in the Matthews instrument unless we assume the mirrors of that instrument to be at the divisions between the consecutive arcs, as was pointed out above. If we divide the semicircumference into an even number of arcs and begin counting whole arcs at  $\theta=0^\circ$  we shall then have a whole number of arcs between  $\theta=0^\circ$  and  $\theta=90^\circ$ . The advantage of this is that mean hemispherical as well as mean spherical candle-power readings can be made, if in each arc a mirror is so placed as to give the mean value for that arc. Hence, since we have divided the semicircumference into 20 whole arcs, and have assigned a mirror to each arc, every distribution of mirrors which we shall find in the following investigation will be equally applicable to both mean spherical and mean hemispherical determinations. The equal distribution  $4\frac{1}{2}^\circ, 13\frac{1}{2}^\circ \dots 175\frac{1}{2}^\circ$  also will be applicable to either mean spherical or mean hemispherical measurements, but the equal distribution  $0^\circ, 9^\circ \dots 180^\circ$ , having one mirror at  $\theta=90^\circ$ , will only be applicable to mean spherical measurements for the reasons given above.

It may be urged as a practical objection to omitting the mirror at

$\theta=90^\circ$  that the adaptability of the instrument to horizontal measurements is thus destroyed. There seems, however, to be no reason why an extra mirror-pair could not be placed in the horizontal position, keeping it covered during mean spherical determinations. In fact, this would seem preferable to the constant removal and use of the horizontal mirror-pair of the spherical system.

To return to the consideration of the different intensity curves, if the limited number of mirrors is to give the true integral value for any intensity curve, we must assume that each mirror gives the mean value of  $I_\theta \sin \theta$  for the arc in which it is placed. We must, therefore, in order to get the true distribution of mirrors, divide the semicircumference into  $n$  equal arcs and then find the mean value of  $I_\theta \sin \theta$  for each arc. If the equation of the intensity curve is

$$(6) \quad I_\theta = f(\theta)$$

then the mean value of  $I_\theta \sin \theta$  for an arc,  $\frac{\pi}{n}$ , is

$$(7) \quad \begin{aligned} \text{mean } [I_\theta \sin \theta] &= \frac{\int I_\theta \sin \theta \, d\theta}{\int d\theta} = \frac{\int f(\theta) \sin \theta \, d\theta}{\frac{\pi}{n}} \\ &= \frac{n}{\pi} \int_{(m-1)\frac{\pi}{n}}^{m\frac{\pi}{n}} f(\theta) \sin \theta \, d\theta \end{aligned}$$

where  $m$  indicates the order of the zone beginning at  $\theta=0^\circ$ .

If now we were to take the average of all of these means we should get the same result as the true total integral mean, which is

$$(8) \quad \text{mean } [I_\theta \sin \theta] = \frac{\int_0^\pi I_\theta \sin \theta \, d\theta}{\int_0^\pi d\theta} = \frac{1}{\pi} \int_0^\pi f(\theta) \sin \theta \, d\theta$$

In order, now, to find the angular positions of the mirrors corresponding to the solution, we must evaluate the integral of equation (7) for each zone and then solve for  $\theta_m$  in the equation

$$(9) \quad I_{\theta_m} \sin \theta_m = f(\theta_m) \sin \theta_m = \frac{n}{\pi} \int_{(m-1)\frac{\pi}{n}}^{m\frac{\pi}{n}} f(\theta) \sin \theta \, d\theta$$

If we placed the mirrors at the angles found, then for that particular intensity curve we would get the true mean spherical intensity.



Let us now consider several typical cases:

CASE I.

$$(10) \quad I_{\theta} = \text{constant} = 1$$

This curve corresponds approximately to the intensity curve of a "flattened oval" or of a "spiral" filament. The integral solution is [eq. (8)]

$$\begin{aligned} \text{mean value of } I_{\theta} \sin \theta &= \frac{1}{\pi} \int_0^{\pi} f(\theta) \sin \theta \, d\theta \\ &= \frac{1}{\pi} \int_0^{\pi} \sin \theta \, d\theta \\ (11) \quad &= \frac{2}{\pi} \left[ \cos \theta \right]_{\frac{\pi}{2}}^0 = \frac{2}{\pi} = 0.6366 \end{aligned}$$

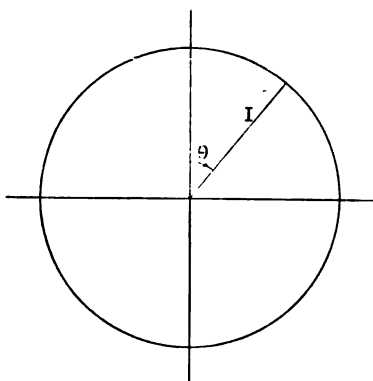


FIG. 1.

For 20 mirrors the angular distribution would be [eq. (9)]

$$\begin{aligned} I_{\theta_m} \sin \theta_m &= \sin \theta_m = \frac{n}{\pi} \int_{(m-1)\frac{\pi}{n}}^{m\frac{\pi}{n}} f(\theta) \sin \theta \, d\theta = \frac{20}{\pi} \int_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}} \sin \theta \, d\theta \\ (12) \quad &= \frac{20}{\pi} \left[ \cos \theta \right]_{m\frac{\pi}{20}}^{(m-1)\frac{\pi}{20}} \end{aligned}$$

from which

$$(13) \quad \theta_m = \sin^{-1} \frac{20}{\pi} \left[ \cos \theta \right]_{m\frac{\pi}{20}}^{(m-1)\frac{\pi}{20}}$$

The solution to this case is given in column III of Table I, which is self-explanatory. In columns I and II are given the two equiangular

distributions. In columns III, V, and VII are given the angular distributions of mirrors satisfying cases I, II, and III, and in columns IV, VI, and VIII are given the angles through which the mirrors must be shifted from the middle points of the arcs in order to give the true results for cases I, II, and III, respectively. Values of angles in the quadrant  $\theta=90^\circ$  to  $\theta=180^\circ$  are merely the supplements of those given in the table, and are hence omitted.

## CASE II.

$$(14) \quad I_\theta = 2a \sin \theta = \sin \theta$$

This curve corresponds approximately to the intensity curve of a "hairpin" or "horseshoe" filament. The integral solution [eq. (8)] is

$$\begin{aligned} \text{mean } [I_\theta \sin \theta] &= \frac{1}{\pi} \int_0^\pi f(\theta) \sin \theta \, d\theta = \frac{1}{\pi} \int_0^\pi \sin^2 \theta \, d\theta \\ &= \frac{1}{\pi} \left[ \theta - \sin \theta \cos \theta \right]_0^\pi = \frac{1}{\pi} \times \frac{\pi}{2} = 0.5000 \end{aligned}$$

For 20 mirrors the angular distribution would be [eq. (9)]

$$(15) \quad I_\theta \sin \theta_m = \sin^2 \theta_m = \frac{20}{\pi} \int_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}} \sin^2 \theta \, d\theta = \frac{10}{\pi} \left[ \theta - \sin \theta \cos \theta \right]_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}}$$

from which

$$(16) \quad \theta_m = \sin^{-1} \left\{ \frac{10}{\pi} \left[ \theta - \sin \theta \cos \theta \right]_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}} \right\}^{\frac{1}{2}}$$

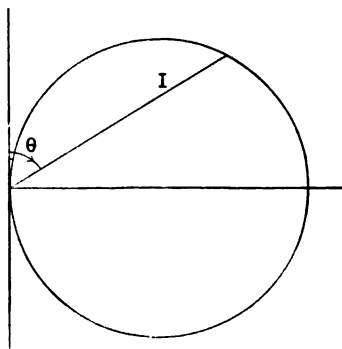


FIG. 2.

The solution to this case is given in column V of Table I.

### CASE III.

$$(17) \quad I_\theta = 2a \cos \theta = \cos \theta$$

This curve corresponds approximately to the intensity curve of a Nernst or of a "meridian" lamp.

The integral solution [eq. (8)] is

$$(18) \quad \text{mean } [I_\theta \sin \theta] = \frac{1}{\pi} \int_0^\pi f(\theta) \sin \theta d\theta = \frac{1}{\pi} \int_0^\pi \cos \theta \sin \theta d\theta = \frac{1}{\pi} \left[ \sin^2 \theta \right]_0^\pi \\ = \frac{1}{\pi} = 0.3183$$

For 20 mirrors the angular distribution would be [eq. (9)]

$$(19) \quad I_{\theta_m} \sin \theta_m = \sin \theta_m \cos \theta_m = \frac{1}{2} \sin 2\theta_m = \frac{20}{\pi} \int_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}} \sin \theta \cos \theta d\theta \\ = \frac{10}{\pi} \left[ \sin^2 \theta \right]_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}}$$

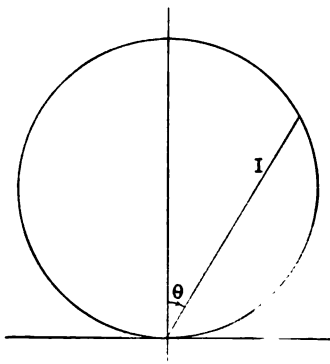


FIG. 3.

from which

$$(20) \quad \theta_m = \frac{1}{2} \sin^{-1} \frac{20}{\pi} \left[ \sin^2 \theta \right]_{(m-1)\frac{\pi}{20}}^{m\frac{\pi}{20}}$$

The solution to this case is given in column VII of Table I.

Having determined the actual distributions of mirrors for cases I, II, and III, the distribution for each case was tried in the other two cases, and the two equal distributions were also tried in each of the three cases.

TABLE I.—Various distributions of mirrors.

| Equiangular distributions. | Distribution satisfying case I. ( $I_\theta=1$ ) | Column III minus column II. | Distribution satisfying case II. ( $I_\theta=\sin \theta$ ) | Column V minus column II. | Distribution satisfying case III. ( $I_\theta=\cos \theta$ ) | Column VII minus column II. |
|----------------------------|--------------------------------------------------|-----------------------------|-------------------------------------------------------------|---------------------------|--------------------------------------------------------------|-----------------------------|
| 0° 4½°                     | 4° 30'                                           | — 0'                        | 5° 10'                                                      | +40'                      | 4° 29'                                                       | — 1'                        |
| 9° 13½°                    | 13° 29'                                          | — 1'                        | 13° 44'                                                     | +14'                      | 13° 26'                                                      | — 4'                        |
| 18° 22½°                   | 22° 29'                                          | — 1'                        | 22° 37'                                                     | + 7'                      | 22° 23'                                                      | — 7'                        |
| 27° 31½°                   | 31° 28'                                          | — 2'                        | 31° 33'                                                     | + 3'                      | 31° 16'                                                      | —14'                        |
| 36° 40½°                   | 40° 27'                                          | — 3'                        | 40° 31'                                                     | + 1'                      | 39° 49'                                                      | —41'                        |
| 45° 49½°                   | 49° 26'                                          | — 4'                        | 49° 29'                                                     | — 1'                      | 50° 12'                                                      | — 42'                       |
| 54° 58½°                   | 58° 24'                                          | — 6'                        | 58° 26'                                                     | — 4'                      | 58° 44'                                                      | —14'                        |
| 63° 67½°                   | 67° 21'                                          | — 9'                        | 67° 23'                                                     | — 7'                      | 67° 37'                                                      | — 7'                        |
| 72° 76½°                   | 76° 16'                                          | —14'                        | 76° 16'                                                     | —14'                      | 76° 34'                                                      | — 4'                        |
| 81° 85½°                   | 84° 48'                                          | —42'                        | 84° 50'                                                     | —40'                      | 85° 31'                                                      | + 1'                        |
| 90°                        |                                                  |                             |                                                             |                           |                                                              |                             |

TABLE II.—Results obtained by applying the various distributions of mirrors to each of the three cases.

|             | Intensity curves considered.              | Integral solutions. | Equiangular distributions of mirrors. |                  | Solution of case I. | Solution of case II. | Solution of case III. | Application of equiangular distribution 0°, 15°, 30°, etc. (Matthews's case). |
|-------------|-------------------------------------------|---------------------|---------------------------------------|------------------|---------------------|----------------------|-----------------------|-------------------------------------------------------------------------------|
|             |                                           |                     | 0°, 9°, 90°.                          | 4½°, 13½°, 85½°. |                     |                      |                       |                                                                               |
| Case I....  | $I_\theta=\text{constant}=1$ ....         | .6366               | .6353                                 | .6373            | .6366               | .6388                | .6368                 | .6330                                                                         |
|             |                                           |                     | $\Delta=13$                           | $\Delta=7$       | $\Delta=0$          | $\Delta=22$          | $\Delta=2$            | $\Delta=36$                                                                   |
| Case II.... | $I_\theta=2a \sin \theta=\sin \theta$ ... | .5000               | .5000                                 | .5000            | .4990               | .5000                | .5000                 | .5000                                                                         |
|             |                                           |                     | $\Delta=0$                            | $\Delta=0$       | $\Delta=10$         | $\Delta=0$           | $\Delta=0$            | $\Delta=0$                                                                    |
| Case III..  | $I_\theta=2a \cos \theta=\cos \theta$ ... | .3183               | .3157                                 | .3196            | .3213               | .3230                | .3183                 | .3110                                                                         |
|             |                                           |                     | $\Delta=26$                           | $\Delta=13$      | $\Delta=30$         | $\Delta=47$          | $\Delta=0$            | $\Delta=73$                                                                   |

In Table II are given the results of applying the five different distributions to each of the three intensity curves considered. Column

II contains the equations of the intensity curves of the different cases; column III contains the true integral values; columns IV and V give the results of applying the two equal distributions to each of the three cases; columns VI, VII, and VIII give the results of applying the distributions of cases I, II, and III to each of the three cases; and, finally, column IX gives the results of applying the equiangular arrangement of eleven mirrors to each of the three cases, in order to see how accurately the Matthews photometer, as ordinarily constructed, gives the mean spherical value. In each column the differences,  $\Delta$ , between the values found and the true integral values are given.

From a consideration of Table II it seems clear that the distribution of mirrors corresponding to case III satisfies all the cases most completely, the greatest error being well under 0.1 per cent. The equal distribution  $4\frac{1}{2}^\circ$ ,  $13\frac{1}{2}^\circ$ , . . . .  $175\frac{1}{2}^\circ$ , however, is very satisfactory, the greatest error being about  $\frac{1}{2}$  per cent. The equal distribution of eleven mirrors every  $15^\circ$  ( $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ , . . . .  $180^\circ$ ), may give errors amounting to over 2 per cent.

It is impossible, of course, to take into consideration all the actual forms of intensity curves existing, but it would seem that a distribution of mirrors which will satisfy the three typical curves considered would also satisfy any of the ordinary forms of curves with which we are familiar in the photometry of such sources as incandescent lamps. For example, the empirical curve of the "downward light" lamp coincides with the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{(8.2a)^2} = 1$$

so closely that it is scarcely possible to detect any difference between the two curves platted to the scale of  $a=4$  cm. Now the true integral value for the above ellipse is 1.722, whereas the value obtained by applying the distribution of case III comes out 1.723.

Thus we see that twenty mirror-pairs properly distributed are entirely sufficient theoretically to give mean spherical, and at the same time mean hemispherical candle-power values to within an error well under that which could be detected in making a setting under the most favorable conditions.

### III. GENERAL THEORY OF THE RUSSELL-LÉONARD PHOTOMETER.

As before, let

$$(21) \quad I_\theta = f(\theta)$$

be the equation of the intensity curve in any vertical plane through the source, assuming the surface of distribution of intensity to be one

of revolution around the vertical axis, which is taken as the axis of the lamp in every case.

Then the flux across any zone,  $d\theta$ , of the unit sphere is,

$$(22) \quad L_s = 2\pi I_\theta \sin \theta d\theta$$

the total flux is,

$$(23) \quad L_s = \int_0^\pi 2\pi I_\theta \sin \theta d\theta$$

and hence the mean spherical intensity is,

$$(24) \quad I_{ms} = \frac{L_s}{4\pi} = \frac{1}{2} \int_0^\pi I_\theta \sin \theta d\theta$$

Similarly, the mean hemispherical intensity is,

$$(25) \quad \begin{aligned} I_{mhs} &= \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} 2\pi I_\theta \sin \theta d\theta \\ &= \int_0^{\frac{\pi}{2}} I_\theta \sin \theta d\theta \end{aligned}$$

or, in general, the mean intensity for any zone  $\theta_{m-1}$  to  $\theta_m$  is,

$$(26) \quad \begin{aligned} I_{ms} &= \frac{\int_{\theta_{m-1}}^{\theta_m} 2\pi I_\theta \sin \theta d\theta}{\int_{\theta_{m-1}}^{\theta_m} 2\pi \sin \theta d\theta} = \frac{\int_{\theta_{m-1}}^{\theta_m} I_\theta \sin \theta d\theta}{\left[ \cos \theta \right]_{\theta_{m-1}}^{\theta_m}} \\ &= \frac{1}{\cos \theta_{m-1} - \cos \theta_m} \int_{\theta_{m-1}}^{\theta_m} I_\theta \sin \theta d\theta \end{aligned}$$

If, now, we divide the surface of the sphere into  $n$  zones of equal area, the mean spherical intensity is given by the equation

$$(27) \quad I_{ms} = \frac{1}{n} \sum_{m=1}^{m=n} I_{ms} = \frac{1}{n} \sum_{m=1}^n \frac{1}{\cos \theta_{m-1} - \cos \theta_m} \int_{\theta_{m-1}}^{\theta_m} I_\theta \sin \theta d\theta$$

in which  $m$  indicates the order of the zone, counting from  $\theta=0^\circ$ , and the limits of integration for the several partial integrals are the angles corresponding to the circles of division between the successive zones.

Suppose, now, that we place a mirror in each of the  $n$  zones and let the light from these mirrors fall directly upon the photometer screen. It is evident that the illumination produced on the screen is  $n$  times that which would be produced by a source having an intensity in the direction of the screen equal to the mean spherical intensity of the lamp under test, assuming that the summational value differs from the true integral value of the mean spherical intensity by a negligibly small error. This error, however, depends greatly of course, as we saw above, upon the form of the intensity curve, but just as in the theory of the Matthews's instrument, for any one simple form of curve assumed, it is not difficult to determine the position of each mirror in its respective zone, so that the intensity given by the mirror is exactly equal to the mean intensity of the zone in which it lies.

Thus, if we know the intensity curve

$$I_{\theta} = f(\theta)$$

we determine  $\theta_m^1$  for each zone, so that

$$(28) \quad I_{mz} = f(\theta_m^1) = \frac{1}{\cos \theta_{m-1} - \cos \theta_m} \int_{\theta_{m-1}}^{\theta_m} f(\theta) \sin \theta \, d\theta$$

Then the intensity at each angle,  $\theta_m^1$ , will be the mean intensity for the respective zone.

If, now, we place a mirror at each angle,  $\theta_m^1$ , as determined from the previous equation, assuming the form of intensity curve, then the mean value of the several intensities as given by the various mirrors will exactly equal the mean spherical intensity of the source, provided, of course, that the source has the same form of intensity curve as was assumed in the calculation of the various angles,  $\theta_m^1$ .

#### IV. APPLICATION OF THE RUSSELL-LÉONARD PHOTOMETER TO ARC LAMPS.

As the Russell-Léonard photometer is of the same general form as the Matthews arc lamp instrument, with the important point of difference that in the former instrument the objectionable smoked disk of the Matthews photometer is eliminated, we shall briefly consider first the distribution of mirrors which must be made in the Russell-Léonard photometer, in order that in the photometry of arc lamps it may yield a summational result agreeing as nearly as possible with the true integral value.

We must first decide upon the dimensions of the instrument to be designed, the size and number of the mirrors, etc., which will depend to a great extent upon the accuracy that we desire to obtain. We then

divide the sphere into the number of zones decided upon, determine the limiting angles between the successive zones, and with the use of eq. (28) calculate the angle,  $\theta_m$ , for each zone corresponding to the mean value of the intensity for that zone, upon the assumption of the polar equation of the intensity curve of the arc. Though this equation is not the same for different kinds of arc lamps, or even for different vertical planes of the same arc at any one time, yet we can assume a hypothetical curve which will correspond more or less closely with the different curves met with, since all the curves for arc lamps are of the same general type, with a maximum at about  $\theta=45^\circ$  or  $50^\circ$  and falling off rapidly toward  $\theta=0^\circ$  and  $\theta=90^\circ$ . If the arc is surrounded by a globe that scatters the light, the curve approaches more nearly to a circle with the source at the center, and since any arrangement of mirrors within the arcs of equal area will necessarily give absolutely the true result for a source having the same intensity in all directions, a distribution of mirrors which will satisfy any definite intensity curve will not yield very large errors if that curve is made to approach a circle with the source at the center. Hence, if we were to take the typical curve for a direct-current open arc, and determine the mirror arrangement satisfying this case, the errors that would arise on using the instrument for the inclosed arc would be relatively small; and, moreover, since the instrument is symmetrical with respect to the horizontal direction, it would be equally applicable to the alternating-current arc, whose intensity curve is approximately the same as that of the direct-current lamp, except that the distribution in the lower hemisphere is repeated in the upper one.

Let us then, as a first approximation, assume that the intensity curve is an ellipse with the major axis inclined at  $45^\circ$  to the vertical, and with one end of the major axis coincident with the source (fig. 4).

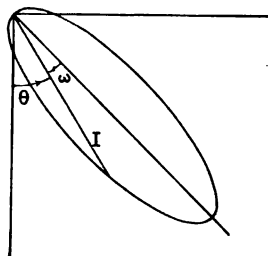


FIG. 4.

The equation, expressed in rectangular coordinates, is

$$(29) \quad \frac{(x-a)^2}{a^2} + \frac{y^2}{b^2} = 1$$



Now the mean intensity for any zone,  $\theta_{m-1} - \theta_m$ , is given by eq. (26),

$$I_{mz} = \frac{1}{\cos \theta_{m-1} - \cos \theta_m} \int_{\theta_{m-1}}^{\theta_m} I_{\theta} \sin \theta \, d\theta$$

If we make  $I_{\theta}$  equal to the radius vector of the ellipse given by eq. (29), we get, on transforming to the variable,  $\omega$ ,

$$(30) \quad I_{mz} = \frac{ab^2 \sqrt{2}}{\cos(45^\circ - \omega_{m-1}) - \cos(45^\circ - \omega_m)} \int_{\omega_{m-1}}^{\omega_m} \frac{\cos^2 \omega - \sin \omega \cos \omega}{b^2 \cos^2 \omega + a^2 \sin^2 \omega} d\omega$$

which on integration becomes

$$(31) \quad I_{mz} = \frac{ab^2 \sqrt{2}}{[a^2 - b^2] [\cos(45^\circ - \omega_{m-1}) - \cos(45^\circ - \omega_m)]} \left[ \frac{a}{b} \tan^{-1} \frac{a}{b} \tan \omega - \omega - \frac{1}{2} \log \left\{ (a^2 - b^2) \sin^2 \omega + b^2 \right\} \right]_{\omega_{m-1}}^{\omega_m}$$

If, now, we determine upon eight zones, the limiting values of  $\theta$  are obtained by evaluation of the equations

$$(32) \quad \begin{aligned} \theta_0 &= \cos^{-1} 1 = 0^\circ \\ \theta_1 &= \cos^{-1} \frac{3}{4} = 41^\circ 25' \\ \theta_2 &= \cos^{-1} \frac{1}{2} = 60^\circ \\ \theta_3 &= \cos^{-1} \frac{1}{4} = 75^\circ 31' \\ \theta_4 &= \cos^{-1} 0 = 90^\circ \end{aligned}$$

from which the limiting values of  $\omega$  can be calculated from the equation

$$(33) \quad \omega = 45^\circ - \theta$$

Moreover for eight zones

$$(34) \quad \cos \theta_{m-1} - \cos \theta_m = \cos(45^\circ - \omega_{m-1}) - \cos(45^\circ - \omega_m) = \frac{1}{4}$$

If, now, we take the semimajor axis equal to unity, and the semi-minor axis equal to one-third,—a ratio quite closely in accord with observation—we obtain

$$(35) \quad I_{mz} = \frac{\sqrt{2}}{2} \left[ 3 \tan^{-1} 3 \tan \omega - \omega - \frac{1}{2} \log \frac{1}{9} \left\{ 8 \sin^2 \omega + 1 \right\} \right]_{\omega_{m-1}}^{\omega_m}$$

If we evaluate this expression for each of the eight zones, or, since there is perfect symmetry with respect to the horizontal axis, for each

of four zones, and then substitute successively the values obtained in the equation

$$(36) \quad \theta_{mz} = 45^\circ - \cos^{-1} \frac{1}{8} \frac{I_{mz}}{I_{mz}} \left( \sqrt{72 I_{mz}^2 + 1} - 1 \right)$$

which is easily deduced from the equation of the ellipse, putting  $I_{mz}$  for the radius vector, we get as the angles to which the mean radii vectores of the several zones of equal area correspond

$$(37) \quad \begin{aligned} \theta_1' &= 28^\circ 51' \\ \theta_2' &= 52^\circ 33' \\ \theta_3' &= 67^\circ 23' \\ \theta_4' &= 82^\circ 20' \end{aligned}$$

If, now, we were to place a mirror at each of these angles, both above and below the horizontal axis, but only on one side of the vertical diameter, then for a source having an intensity curve identical with the assumed ellipse, the mean value of the intensity from the eight mirrors, obtained by dividing the sum of the intensities from the mirrors by eight, would exactly equal the mean spherical intensity of the source, provided, of course, that the surface of distribution of intensity is a figure of revolution around the vertical axis. This is never the case, and since, in general, when the maximum curve is on one side of the arc the minimum curve is on the other side, by placing eight other mirrors on the other side of the vertical diameter at the same angles as those already located, the mean intensity in the complete vertical plane is obtained, which will be more nearly equal to the mean spherical value than the value obtained from mirrors on one side of the vertical diameter only. In order to obtain mean spherical values of any considerable accuracy the mean of a great number of determinations of the mean vertical intensity must be taken.

In a similar manner mean hemispherical determinations can be made if the surface of the sphere is divided into an even number of zones, as was pointed out above in the discussion of the Matthews instrument.

The typical curve of the open arc does not correspond exactly with such an ellipse as assumed in the preceding discussion, but the errors in arc-light photometry are in other respects necessarily so large that the difference between the true mean value for a vertical plane and that given by the mirrors arranged to satisfy the ellipse distribution will in general be much smaller than the other errors incident to the measurement. If a closer agreement between the integral and summational values of the arc is desired without increasing the number of mirrors, it would be necessary to investigate in detail the typical curve

of the arc, and from this curve to compute the angles corresponding to the true distribution of mirrors.

In order to determine the magnitude of the error that would arise if the form of intensity curve differed to any extent from an ellipse, the distribution of mirrors satisfying the ellipse was applied to the two curves,

$$(38) \quad I_{\theta} = \sin \theta$$

and

$$(39) \quad I_{\theta} = \cos \theta$$

The true integral means of these are, respectively, [eq. (24)]

$$(40) \quad I_{ms} = \frac{1}{2} \int_0^{\pi} I_{\theta} \sin \theta \, d\theta = \int_0^{\frac{\pi}{2}} \sin^2 \theta \, d\theta = .7854$$

$$(41) \quad I_{ms} = \frac{1}{2} \int_0^{\pi} I_{\theta} \sin \theta \, d\theta = \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta \, d\theta = .5000$$

whereas the values given by one-fourth of the summations of the sines and cosines, respectively, of the angles of eq. (37) are

$$\begin{aligned} I_{ms} &= .7976 \\ I_{ms} &= .5005 \end{aligned}$$

which are in error by 1.6 per cent and 0.1 per cent, respectively.

#### V. APPLICATION OF THE RUSSELL-LÉONARD PHOTOMETER TO INCANDESCENT LAMPS.

In the application of the Russell-Léonard photometer to incandescent lamps the problem is just the same as in the application to arc lamps, except that here we seek the distribution of mirrors that will satisfy the three intensity curves

$$\begin{array}{ll} \text{Case I} & I_{\theta} = \text{constant} \\ \text{Case II} & I_{\theta} = \sin \theta \\ \text{Case III} & I_{\theta} = \cos \theta \end{array}$$

Any distribution of mirrors which will satisfy these three cases will yield very small errors when applied to any actual curves. Moreover, since any distribution of mirrors in zones of equal area will satisfy case I, we simply have to determine the distribution most nearly satisfying cases II and III.

The true integral values for cases II and III, respectively, are [eqs. (40) and (41)]

$$\begin{aligned} I_{ms} &= .7854 \\ I_{ms} &= .5000 \end{aligned}$$

If we divide the surface of the sphere into twelve zones instead of eight, the distribution of mirrors satisfying case II, as determined by evaluation of eq. (28) for each of the twelve zones, after putting for  $f(\theta)$  its value  $\sin \theta$ , gives an error of 1.0 per cent when applied to case III. Similarly the true distribution for case III gives an error of 0.7 per cent when applied to case II. If, however, we take a distribution of mirrors intermediate between the two true distributions, i. e., if we take

$$\begin{aligned} \theta_1 &= 22^\circ 47' \\ \theta_2 &= 41^\circ 14' \\ \theta_3 &= 54^\circ 15' \\ \theta_4 &= 65^\circ 14' \\ \theta_5 &= 75^\circ 20' \\ \theta_6 &= 85^\circ 8' \end{aligned} \quad (42)$$

the error is divided between the two cases, so that the summational values for cases II and III are, respectively,

$$\begin{aligned} I_{ms} &= .7883 \\ I_{ms} &= .5025 \end{aligned}$$

the corresponding percentage errors being 0.4 per cent and 0.5 per cent.

We have divided the surface of the sphere into 12 zones in this case, as compared with 8 in dealing with arc lamps, because in the first place a higher accuracy is obtainable in the photometry of incandescent lamps, and, secondly, if the incandescent lamp is rotated, 12 zones demand only 12 mirrors which can be arranged around the whole circle, whereas 8 zones demand 16 mirrors in the arc-light photometer,—8 mirrors on each side of the vertical diameter. Thus, for incandescent lamp work, 12 mirrors, each 6 inches broad, can be arranged in a circle of 3 feet diameter, and yield a result accurate to within about 0.5 per cent for lamps of various intensity curves, whereas the Matthews instrument, as at present constructed, gives an error of over 2 per cent (Table II) when applied to the lamp having the intensity curve  $I_\theta = \cos \theta$ .

It has not been the intention, in this discussion of the Russell-Léonard photometer, to make the treatment exhaustive, but rather to suggest the method which can be applied to locate the mirrors in the zones after the uses to which the instrument is to be put and the consequent general dimensions of the instrument have been determined.

## THE TESTING OF CLINICAL THERMOMETERS.

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By C. W. WAIDNER and L. A. FISCHER.

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Soon after the organization of the Bureau of Standards work preparatory to the testing of clinical thermometers was taken up.<sup>a</sup> At the same time all of the prominent manufacturers of clinical thermometers were requested to submit samples of their product in order that the Bureau could determine what accuracy might be expected and what defects were most common.

The design and construction of the apparatus required for this purpose was undertaken with two main objects in view: first, that the highest accuracy should be obtained, and second, that the tests could be made so rapidly that the fees for testing would be reduced to a minimum. The apparatus finally adopted has been in use for some time, and the methods of testing have been subjected to exhaustive trials. As both the apparatus and methods used by the Bureau have been the subject of numerous inquiries, it is deemed advisable to publish a full description for the use of those interested and also an account of the experiments upon which the regulations finally adopted are based.

The first tests made showed that many of the clinical thermometers had errors as large as  $0.5^{\circ}$  or  $0.6^{\circ}$  F., and in some cases even larger errors were found. Moreover, a study of groups of thermometers suggested that the standards used were in error. The manufacturers were accordingly requested to submit their standards to the Bureau for examination. Without exception this request was complied with, and, as suspected, the conclusion that the standards were in error was confirmed.

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<sup>a</sup>The work was begun in the section of weights and measures under the direction of Mr. Fischer, and was subsequently transferred to the section of thermometry, upon its organization, under the direction of Dr. Waidner.

## CLINICAL STANDARD THERMOMETERS.

Some of the standards submitted were found to be radically defective in construction. Some were so constructed that when used in the water bath for the pointing of ordinary clinical thermometers the stems projected above the surface of the bath by amounts corresponding to 60° F. and even more. Since changes of 20° F. or more occur in the temperature of rooms where clinical thermometers are graduated, and since the temperature of the exposed portion of the stem is largely controlled by the temperature of the room, there is here presented a possible source of error of about 0.1° F. Another defect common to most of the clinical standards submitted was the absence of the ice point (32° F.) from the scale. The presence of this point is important, since it enables one immediately to detect any change in the indications of a thermometer by determining the ice point in a mixture of finely crushed pure ice and distilled water. If any change is detected in this point the reading of the thermometer may then be corrected. It is true that most of the standards used by manufacturers are quite old, and hence changes in these thermometers would probably be small. Nevertheless changes might occur on account of the development of minute air bubbles or other causes, which, as before stated, would be readily detected by a determination of the ice point.

A form of clinical standard well adapted for the pointing or testing of clinical thermometers is shown in fig. 1.

Immediately above the ice point is an enlargement of the bore, which makes it possible to obtain on a short stem the ice point and also the range of temperature required in clinical thermometry. Above the enlargement there are graduations corresponding to 0° .1 and covering the range 90° to 110° F. In ordinary use these standard thermometers are immersed in the water bath to about the 93° mark, so that not more than 17° is emergent from the bath, and hence the variations in the temperature of the room may be neglected. The corrections to these thermometers are carefully determined by comparing them with the primary standards of the Bureau.

These latter are made of the usual thermometric glasses, namely, French hard glass or Jena 16<sup>III</sup> normal glass, and are artificially aged, before final filling, by exposing them to a temperature of about 850° F. for at least 60 hours, after which they are allowed to cool slowly. When treated in this manner subsequent time changes are extremely small. The graduations on these thermometers are very fine, whereas some of the standards submitted to the Bureau for examination were only graduated into 0° .2 F., and the graduation lines were nearly as wide

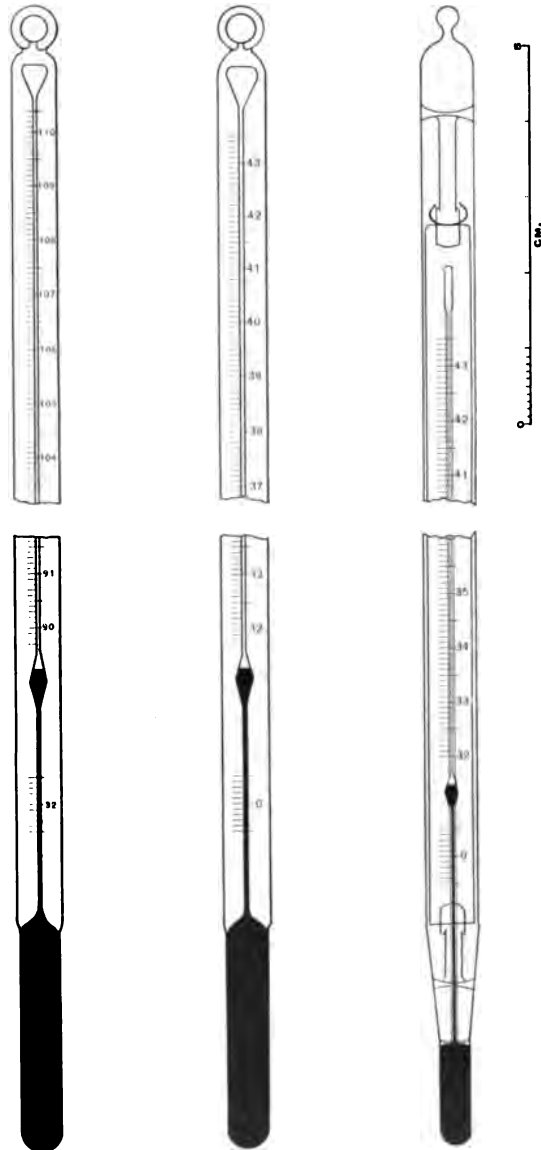


FIG. 1. CLINICAL STANDARDS.





as half an interval. Under such conditions it is easily possible to make an error of several hundredths of a degree in reading the thermometer. It might seem at first sight that errors of this magnitude are negligible in clinical thermometry, but they are sufficient to make the corrections to large numbers of clinical thermometers  $0^{\circ}.1$ , while if the correct temperature of the bath had been known when the clinical thermometers were pointed the corrections resulting from the tests of the Bureau would have been given as  $0^{\circ}.0$ . To illustrate, if the observed correction is  $0^{\circ}.05$  or less it is given as  $0^{\circ}.0$ , while if  $0^{\circ}.06$  had been observed the correction would have been given as  $0^{\circ}.1$ .

To facilitate the use of standards of proper construction and of a uniform scale of temperature the Bureau has loaned a few of these specially constructed standards to a number of manufacturers for brief periods. The Bureau has also pointed a number of standards made and submitted by the manufacturers for this purpose. The result of this work has been greatly to reduce the errors of clinical thermometers, those now submitted for test having, on the average, errors less than one-half as great as those first submitted.

#### SCALE OF TEMPERATURE.

The advantages resulting from the universal use of one and the same scale of temperature are almost self-evident, but notwithstanding this fact a number of slightly different scales were found in use by manufacturers, all of whom supposed their standards to be correct. Since frequent inquiries are made as to the scale of temperature used by the Bureau, the question will be briefly considered here.

The interval covering the ordinary range of temperature is fixed by the temperature of melting ice and by the temperature of steam under normal pressure.<sup>a</sup> These two temperatures are universally used to determine the so-called fixed points of thermometers of various kinds.

It is well known that the indications of mercury thermometers constructed of different kinds of glass may differ very materially from one another, after making all necessary corrections to their readings for errors in the fixed points, and for nonuniformity in the caliber of the tubes. These differences, due to the fact that different glasses have different coefficients of expansion, may amount to more than  $0^{\circ}.2$  F. Thermometers made up from the same pot of glass, or even from the same piece of tubing that have been worked differently in the blast flame, will often show very appreciable differences in their temperature scales. For these reasons mercury in glass thermometers are not suited for establishing a standard scale of temperature.

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<sup>a</sup>The pressure of 760 mm of mercury at  $0^{\circ}$  C. and at sea level and latitude  $45^{\circ}$ .

The scale that has now come into almost universal use in all accurate scientific and technical work is what is known as the International Hydrogen Scale, based upon the researches of Chappuis carried out at the International Bureau of Weights and Measures. This scale is defined as follows in a resolution of the International Committee on Weights and Measures adopted October 15, 1887:

The International Committee on Weights and Measures adopts as the standard thermometric scale for the International Service of Weights and Measures the centigrade scale of the hydrogen thermometer, having as fixed points the temperature of melting ice ( $0^{\circ}$ ), and of the vapor of distilled water boiling ( $100^{\circ}$ ) at standard atmospheric pressure; the hydrogen being taken at an initial manometric pressure of 1 m. of mercury; that is to say,  $\frac{1000}{760} = 1.3158$  times the standard atmospheric pressure.

On this scale of temperature, therefore,  $1^{\circ}$  is measured by  $\frac{1}{760}$  of the change of pressure, between the two fixed points, of a confined mass of hydrogen gas whose volume is kept constant, and whose initial pressure (at  $0^{\circ}$  C.) is equivalent to 1 meter of mercury (at  $0^{\circ}$  C., and at sea level, lat.  $45^{\circ}$ ).

#### TYPES OF CLINICAL THERMOMETERS.

Clinical thermometers are made of many different forms. By way of illustration it is only necessary to describe a few of the leading types. Nearly all clinical thermometers are self-registering; that is to say, they register the highest temperature to which they have been exposed. Various devices for accomplishing this object are in use, but the one almost universally used in the United States and in Great Britain, and also extensively in Europe, is shown in (a), fig. 2.

It consists of a contraction or trap in the capillary tube of the thermometer, which permits the mercury to expand through the contraction in fine globules, but does not allow it to return unless the mercury is forced back. This contraction must be very skilfully made, for if the globules of mercury that pass are large the readings will increase by a series of large jumps and the thermometer will not register accurately. If the contraction is too large ("loose"), the thermometer will not hold its index at all. On the other hand, if the contraction is too small ("tight") it is difficult to force the mercury below the contraction (throw back the index) when a new reading is desired. These defects cause the rejection of a considerable number of thermometers submitted for test. The contractions are made by heating the glass tube with a blowpipe and allowing the walls to collapse. This operation introduces strains in the glass, which sometimes cause small slivers of glass to break off in the vicinity of the contraction, often after long

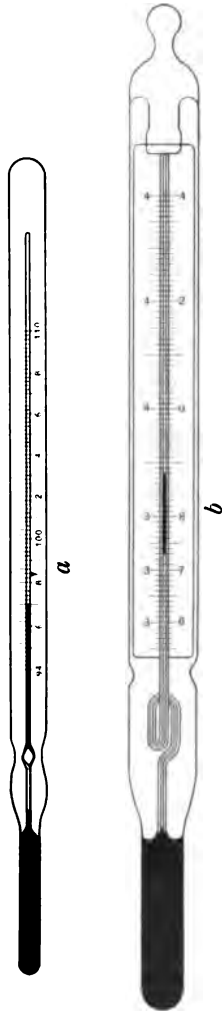


FIG. 2.  
TYPES OF CLINICAL THERMOMETERS.

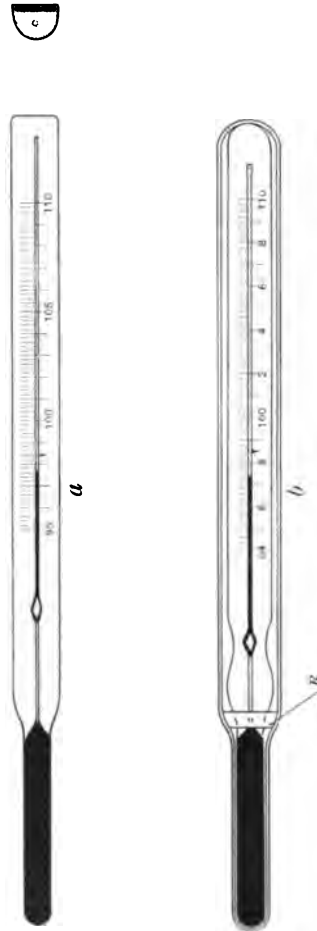


FIG. 3.  
TYPES OF ASEPTIC THERMOMETERS.



intervals of time, frequently rendering the thermometer untrustworthy. These defects are referred to because of the prevailing belief that if a thermometer has once been tested and has received a certificate, it will always remain reliable as long as it continues to register.

In some of the larger forms of clinical thermometers widely used in Europe other devices are employed. The device shown in (*b*) fig. 2 is quite common. It consists of a small air bubble in the column of mercury, which pushes before it a short column (index) of mercury when the temperature is raised, but which leaves the index in position when the highest temperature has been reached. The capillary stem is bent in a loop to prevent the air bubble from receding into the bulb when the index is thrown back. Another device frequently used is a small glass rod attached to the bottom of the bulb, which runs through the center of the bulb and terminates in the bottom of the capillary stem. As the mercury expands it forces its way through the narrow annular opening between the rod and the capillary, but when the mercury contracts the mercury above the trap can not retreat, and hence the highest temperature is recorded.

The form of thermometer shown in (*a*), fig. 2, usually has the graduations on the outside of the glass tube, and is made with what is called a magnifying lens front and a white enamel back. The fine thread of mercury is thus seen greatly magnified against the white background.

In the form of thermometer shown in (*b*), fig. 2, known as the "Einschluss" or inclosed scale type, the scale is ruled on a strip of milk glass, which is placed directly back of the fine capillary stem of the thermometer, both being inclosed in a glass tube. The outer glass tube is always fused to the bulb or the capillary tube, but different methods are used to fasten the scale in position. In some the scales are fastened with sealing wax in a groove cut in a cork that closes the top of the outer glass tube. In the better forms the scale is fused to the top of the outer glass tube, as shown in (*b*), fig. 2.

It is claimed by some that the graduations of thermometers of type (*a*), fig. 2, which are in part filled with some black coloring matter, will retain disease germs, which can not be as readily removed as in the case of the inclosed scale type (*b*), fig. 2, when the thermometer is placed in the antiseptic liquid. The latter are therefore sometimes called "aseptic" thermometers. Whether this criticism is valid would have to be determined by the bacteriologist.

Two other types of aseptic thermometers, shown in (*a*) and (*b*), fig. 3, are also found on the market.

In type (*a*), fig. 3, the outside of the stem of the thermometer is

perfectly smooth; the scale, graduated on mica, is sealed in the rectangular slot in the stem, immediately behind the capillary bore. In the type (*b*), fig. 3, a thermometer of the form shown in (*a*), fig. 2, is inclosed in a neatly fitting glass envelope, displacement being prevented by the rubber ring *R*.

Another advantage claimed for the "aseptic" type of thermometers is that the black coloring matter in the graduations is not exposed to the action of the antiseptic fluid, so that its scale always remains legible.

It should be added that if there is any advantage in the aseptic form of thermometer, from a bacteriological standpoint, it is often lost by the addition of metallic caps to the thermometers, which must be far more dangerous germ traps than outside graduations, because the metal caps are seldom exposed to the antiseptic fluid.

All of the types illustrated are sometimes modified to adapt them to special purposes. Thus the bulb is sometimes made in the form of a flat spiral for determining local surface temperatures, and in others it has a special form to suit the organ whose temperature is to be measured, as, for example, the eye.

#### TIME REQUIRED TO ATTAIN THE PROPER TEMPERATURE.

The question of the so-called "time of action" of clinical thermometers frequently gives rise to misunderstandings between the manufacturer and user. The time required by a thermometer to attain its final temperature depends largely upon how it is used, and hence to say that a thermometer is a half, one, or two minute thermometer means nothing definite. Owing to the demand for a short "time of action" the manufacturers have been compelled to use extremely thin wall tubing for the bulbs, and also to reduce to a minimum the amount of mercury used in the thermometers. Owing to the latter they have also been compelled to use for the stem very fine capillary bore tubing. The general effect has been to produce somewhat frail thermometers, inconvenient to use because of the difficulty of throwing back the index; hence it seems probable that the effort in this direction has been carried too far. To increase the exposed surface of the bulb the latter is sometimes made double and are then called "twin bulb" thermometers.

To get the best results the thermometer should be kept under and in good contact with the tongue, and care should also be taken by the user not to inhale air through the mouth. The time may be slightly reduced

by taking care not to press the thermometer against the tongue hard enough to impede circulation. When properly used, thermometers of type (a), fig. 2, require from less than a half minute to two minutes. In a water bath thermometers of this type will generally take up the temperature of the bath in five seconds or less. The larger clinical thermometers of type (b), fig. 2, or the form illustrated in (b), fig. 3, necessarily have a longer time of action.

#### AGING OF CLINICAL THERMOMETERS.

One of the important and most difficult questions that had to be settled before regulations concerning the certification could be adopted was that of aging. Thermometers not aged, or "green," as they are called, might pass the most rigid test requirements, and yet, if made of certain kinds of glass, they might in the course of two or three months develop errors of  $0^{\circ}.6$  F. or more. To protect the certificate of the Bureau and at the same time meet the requirements of all those interested, several courses were open, each of which was both favored and criticised when submitted to the interested parties for their opinion.

One suggestion was that the thermometers be deposited with the Bureau for at least four months. They could then be withdrawn by the maker for final inscription and returned to the Bureau for test. The Bureau would then be satisfied that the thermometer had been duly aged for a period that would probably render any subsequent time changes negligible. This procedure is open to objections, chief among which are the uncertainty of the demand, the necessity for double transportation, and the twofold process of engraving.

Another suggestion was to test them as soon as possible, hold them for a period of from four to six weeks, and again test them. Since a considerable part of the time change in green thermometers occurs within the first month or two, the two tests would determine whether the thermometers had been sufficiently aged. The principal objection to this procedure is the necessity of two tests, with consequent increase in the cost of testing.

In order to determine definitely the magnitude of the errors that might result from time changes, a series of experiments were undertaken with a large number of green clinical thermometers, made of different kinds of glass and by different manufacturers. The results of these experiments, based on many hundreds of observations, are

given in the following table for different periods from the date the thermometers were made:

| Glass.                            | Average increase in reading of clinical thermometers. |                         |                          |
|-----------------------------------|-------------------------------------------------------|-------------------------|--------------------------|
|                                   | At the end of 1 month.                                | At the end of 2 months. | At the end of 14 months. |
| Jena 16 <sup>III</sup> glass..... | 0°.04 F.                                              | 0°.06 F.                | 0°.11 F.                 |
| Soft glass .....                  |                                                       | 0°.30 F.                | 0°.68 F.                 |

These experiments show that if soft glass is used the time changes are by no means negligible. On the other hand, when the better grades of thermometer glass are used, such as Jena normal or French hard glass, the total change in fourteen months is only about 0°.1 F., and about one-half of this total change takes place in the first two months. Other experiments have shown that time changes after fourteen months are insignificant. It is therefore evident that if clinical thermometers are made of the proper glass three or four months are sufficient to age them so that subsequent changes will be negligible.

When glass is heated to a certain temperature it expands quite rapidly to the volume corresponding to that temperature. If cooled again to its initial temperature it contracts rather quickly to nearly, but not quite, its original volume. To completely recover takes days, and with some kinds of glass weeks and months. Consequently the strains due to working the glass in the high temperature of the blowpipe disappear slowly, the disappearance being made manifest by a rise in the indications of the thermometers as the glass contracts. If the thermometer is made by making the bulb out of glass tubing, by closing one end and fusing the other end to the stem, the strains, and consequently the time changes, are less than if the bulb is blown on the stem.

The common practice in the United States is to make the bulbs of the hard thermometer glasses, and the stems of softer enamel back and lens front tubing. As the volume of the mercury in the bulb is many times greater than that in the stem, the changes in the stem are negligible, and hence thermometers constructed in this manner are just as satisfactory as those made entirely of hard glass.

In view of these circumstances and of the foregoing experiments the Bureau decided to waive all requirements as to aging, at least for the present, and instead the statement is made in the certificate furnished by the Bureau that the indications are liable to change unless the thermometer has been suitably aged.



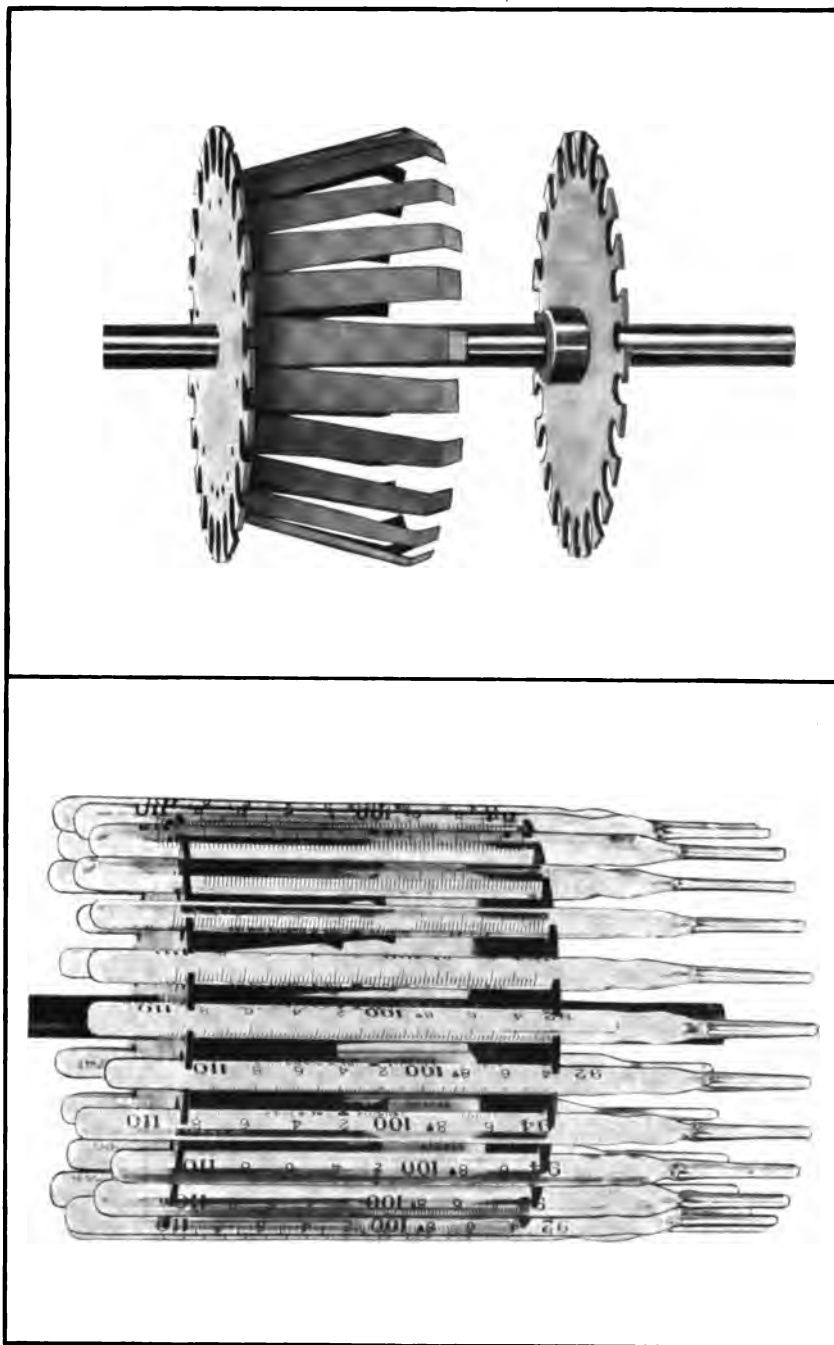


FIG. 4. HOLDERS FOR THERMOMETERS.



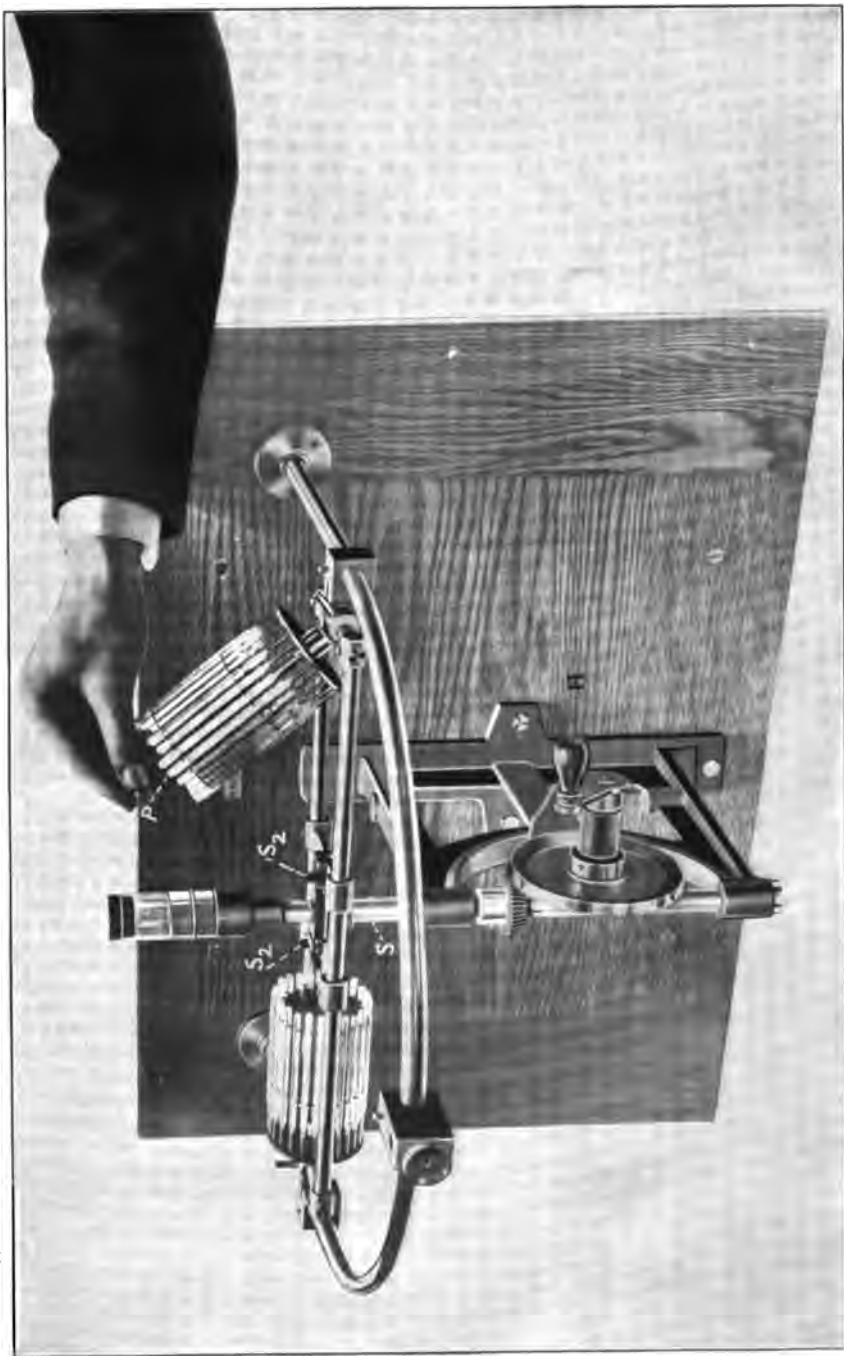


FIG. 5. WHIRLING MACHINE.



## REQUIREMENTS FOR CERTIFICATION.

Before being tested for accuracy clinical thermometers are examined for defects of construction, such as presence of air bubbles or moisture in the mercury or in the capillary bore, cracks in the glass and defective graduations; the operation of the registering device ("index") is also tested to see that the index is neither too easy nor too difficult to throw back. If this test is passed the thermometer is then compared with two standard thermometers of the Bureau at the four test points,  $96^{\circ}$ ,  $100^{\circ}$ ,  $104^{\circ}$ , and  $108^{\circ}$ , two independent comparisons being made at each point. If the two tests at any point differ by more than  $0^{\circ}.15$  F., or if the mean of the two tests give a correction in excess of  $0^{\circ}.3$  F., the thermometer is rejected. Furthermore, errors in the intervals between test points must not exceed  $0^{\circ}.3$  F.<sup>a</sup>

Thermometers submitted for test must have a clear space of at least 2 cm. ( $\frac{3}{4}$  inch) near the stop of the stem for etching thereon the identification marks of the Bureau.

## METHOD OF TESTING.

## PRELIMINARY TEST.

Those thermometers which successfully pass the examination for such defects as are evident on careful inspection are mounted in lots of 24 in the small holders shown in fig. 4. The thermometers are held in position in the holders by the pressure of springs made of phosphor bronze.

The holders have identification numbers, and to each holder is assigned a record sheet similar to that shown in fig. 9. The sheets are numbered to correspond with the holders, and the thermometers are entered on this sheet in the same order that they are mounted in the holder, the manufacturer's number being entered in the second column.

The next step is to test the registering device. Where large numbers of thermometers have to be tested, the usual method of throwing back the index by hand is impracticable, on account of the time and effort required. This test is conveniently and rapidly made by means of the "whirling machine" shown in fig. 5.

Two of the holders loaded with thermometers are mounted in this machine, by pulling back the lock spring  $S_2$ , raising the shaft  $P$ , and slipping on the holder, as shown in the figure, and then dropping the

<sup>a</sup> For example, if the correction at  $96^{\circ}$  is  $+0^{\circ}.3$  and at  $100^{\circ}$   $-0^{\circ}.1$ , the error in the interval would be  $0^{\circ}.4$  F., and the thermometer would be rejected.

shaft back into a horizontal position where it is held by the lock spring. By whirling the thermometers the mercury index is driven back into the bulb. The machine is operated by turning the handle *H*, which acts through bevel gears on the shaft *S*, on top of which is a glass tube filled with oil or glycerin. When the shaft *S* is rotated the surface of the liquid assumes a parabolic form, the center being depressed by an amount dependent upon the speed of rotation. This is therefore a very simple and convenient speed indicator.

The speed of rotation has been determined corresponding to what is considered the maximum muscular effort that should be required to throw back the index. The test then simply consists in whirling the machine until this limiting speed has been reached. The thermometers are then rapidly examined in succession by turning the holders on the shaft *P*. Such as have failed to fall below 95° F. are removed from the holder and placed among those rejected. This completes the preliminary test.

#### DETERMINATIONS OF CORRECTIONS.

The thermometers that have fulfilled the preliminary test are next compared at 96°, 100°, 104°, 108° F., or, if centigrade thermometers, at 35°, 37°, 39°, 42° C. in a well-stirred water bath with two standard thermometers similar to those already described.

#### COMPARISON TANK.

The tank shown in fig. 6 is a double-wall vessel made of seamless brass tubing. The water in the inner tank is thoroughly stirred by a small propeller *K* which is driven at a high speed by the small direct-connected electric motor *M*. This propeller forces the water downward through the central tube and upward in the space between the inner and outer tube, as indicated by the arrows. The stirring is so thorough that the temperature of the bath is constant throughout to within 0°.01. The inner tank is covered with a very fine layer of sheet mica, on which are wound two heating coils, *C*<sub>1</sub> and *C*<sub>2</sub>, of cotton-covered constantan wire shellaced in place. Each coil has a resistance of about 80 ohms, and its terminals are connected to binding posts on the bottom of the tank. The coils are connected to a 110-volt circuit through the rheostat *R*, which serves to regulate the current in the heating coils. If rapid heating is desired, as when raising the temperature of the bath up to the first test point, or from one test point to another, the two coils are put in parallel by throwing the double-pole double-throw switch *S*<sub>2</sub> to the side marked "Fast." When a constant temperature or a slowly-rising one is desired, the

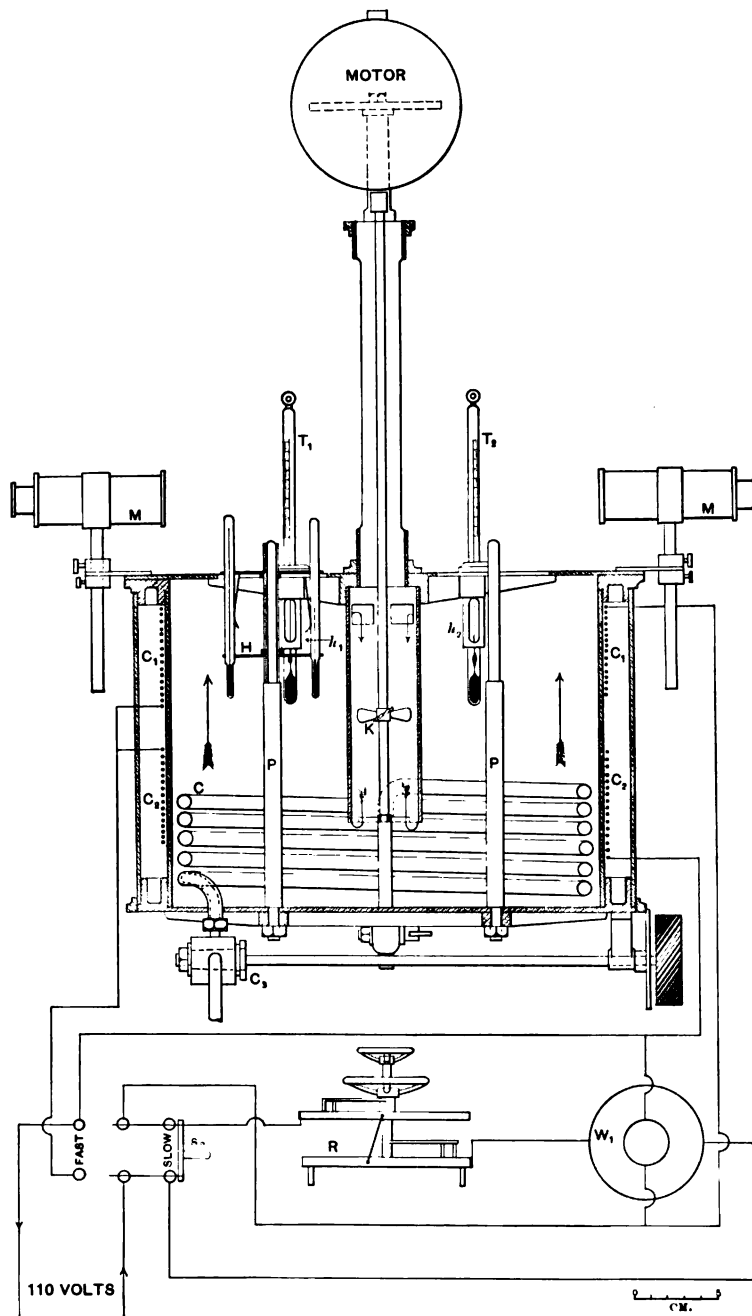


FIG. 6. COMPARISON TANK.





two coils are placed in series by throwing the switch to the side marked "Slow." A Weston wattmeter  $W^1$  serves to measure the electrical energy required in heating the coils. When the switch is thrown to the side marked "Fast" the wattmeter and rheostat are cut out.

From a series of preliminary experiments, a table has been made out giving the amount of electrical energy required to maintain constant temperatures at the several test points for different room temperatures. The amount of energy necessary is so small, never exceeding 75 watts, that its cost is trifling.

The rheostat  $R$  has two dials of 24 steps each, the step on one corresponding to nearly the entire turn on the other.

Within the tank is a small coil  $C$  of seamless copper tubing through which hot or cold water may be circulated to also control the temperature. This water is introduced through the two-way mixing cock  $C_s$  connected to hot and cold water tanks, and the amount of each may be easily controlled by turning the knurled hard-rubber head, shown on the front of the tank. The circulation of cold water enables the temperature of the bath to be lowered and is a necessary adjunct to the electrical heating. The hot water has been added because it helps to bring the temperature quickly to the different test points. The hot and cold water circulation, with the use of a sensitive stop cock, is sufficient for all purposes, but on account of its convenience the electric current regulation is always used. From the bottom of the tank, which is made with four stiffening webs of brass, there project upward four brass pins  $P$  on which a like number of loaded holders are slipped. Hence, 96 thermometers may be immersed at one time.

Two standards,  $T_1$  and  $T_2$ , pass through short brass tubes in the cover of the tank. Inside of the short tubes are springs which keep the thermometers vertical and at any desired depth. On top of the tank are mounted two small reading microscopes  $MM$ , through which the thermometers are read with accuracy and ease.<sup>a</sup> The microscope shown in the illustration, which has proven to be entirely satisfactory, is known as the Bruecke dissecting lens, and was made by the Bausch and Lomb Optical Company, of Rochester, N. Y.

A complete testing apparatus is shown in fig. 7. The whirling machine is shown at the right end of the table, the comparison tank near the center; the hot and cold water tanks, with necessary supply and overflow connections, are mounted on a cast-iron support about 75 cm in height. The hot water tank contains an electric heating coil wound in two sections inside a thin-walled copper tube. The

---

<sup>a</sup> One microscope is sufficient, since it may be quickly turned from one standard to the other.

copper tube is wound in a spiral of 15 cm diameter. The two sections of this coil can be joined, by means of a simple switch, to give fast, medium, and slow heating, the latter being sufficient to keep the temperature 15 or 20° F. below the boiling point.

The supply of electrical energy for the comparison tank is controlled by the switches and rheostat handles shown on the front of the table, and by a wattmeter mounted inside of and read through a small plate glass window in the top of the table. On top of the table is shown a reading stand, in which the holders are placed and the thermometers read through a small microscope. There is also shown a tray with holders to illustrate the method of carrying them around. The scales of the two standard thermometers in the tank are illuminated by the incandescent lamp mounted on the flexible stand in the rear. The large drawer on the left of the table is used to store thermometers undergoing test, while the door on the right is a switchboard on which are mounted the necessary switches and the rheostat. The door is hinged and opens outward, thus rendering all the connections accessible. The above testing outfit was exhibited at the Louisiana Purchase Exposition at St. Louis, Mo.

To return to the process of testing, the temperature of the bath is quickly brought up to within a fraction of a degree of the temperature of the lowest test point (96° F.) by the hot water circulation, and then by means of the rheostat *R* and the wattmeter *W* the current is regulated to maintain the temperature at 96° F.

Four holders are then put into the bath, and by momentarily throwing the switch to the side marked "Fast," the temperature of the bath is brought to exactly 96°.00 F. The temperature can be brought with ease to within 0°.01 F. of the test point, and it rarely happens that the readings of the two standard thermometers differ by more than this amount. The readings of the two thermometers are then entered on the test record sheet (see fig. 9). This procedure is repeated until all the holders have been dipped at the first test point.

#### READING OF THE CLINICAL THERMOMETERS.

After all the thermometers have been dipped, the holders are mounted one at a time in the reading stand shown in fig. 8.

One observer reads the thermometers as they are successively brought into the field of view of the microscope by turning the knurled head of the arbor on which the holders are slipped, while another records the readings on the test record sheet. With a little practice the thermometers may be read at the rate of 20 per minute, without sacrificing accuracy, as has been demonstrated by repeated trials. The readings,



FIG. 7. CLINICAL TESTING TABLE.





FIG. 8. READING STAND.



even when made by different observers, rarely differ by more than  $0^{\circ}.03$  F.

After all the loaded holders have been read at the first test point, the bath is brought to the next test temperature,  $100^{\circ}$  F., and the clinical thermometers are again dipped and read in the manner described. This procedure is repeated for the other test points,  $104^{\circ}$  and  $108^{\circ}$  F. The holders are then mounted two at a time in the whirling machine and the mercury is made to retreat below  $95^{\circ}$ , and the entire series of observations at the four test points is repeated.

The record sheet previously referred to on page 283 is the record for one holder. The corrections at each test point as determined by the two independent tests are entered in columns 6, 10, 14, and 18, and the means of the two tests to the nearest  $0^{\circ}.1$  are entered in columns 7, 11, 15, and 19. In the test record sheet shown thermometers Nos. 5 and 17 were rejected because the indices were too difficult to throw back, No. 8 failed to hold its index ("retreater"), No. 11 because the error exceeded the allowable error ( $0^{\circ}.3$ ), and Nos. 14 and 22 because they failed to repeat their readings within  $0^{\circ}.15$  F. in the two tests. The remaining thermometers fulfilled all the test requirements and were then given B. S. serial numbers, which are entered in column 3.

#### B. S. IDENTIFICATION MARKS.

The serial numbers just referred to, preceded by the letters B. S., are then etched on the thermometers. To accomplish this the upper parts of the stems are thoroughly cleaned and dried, and then covered with a thin coating of wax by immersion in a pot of molten wax. The extreme ends are further protected by a second dipping before engraving. The wax pot used in this work has proved so convenient that a brief description will be given. It consists of a double-wall vessel wound with two heating coils connected to a lighting circuit by an ordinary fuse-plug connector. To melt the cold wax quickly the two coils are thrown in parallel, and when the wax is all melted it is kept at a constant temperature by the two coils in series.

The letters B. S. and the serial numbers are engraved on the back of the thermometers by means of an engraving machine made by Eaton and Glover. This machine, though somewhat large for this character of work, is nevertheless quite convenient. The etching is done by immersing the end of the thermometer for a short time in a bath of hydrofluoric acid.

#### CERTIFICATE.

The corrections to the thermometers are then transferred from the test record to the certificates, a facsimile of which is shown in fig. 11.

## RÉSUMÉ SHEET.

When a number of thermometers are submitted for a test a "résumé sheet" is returned with the certificates. This sheet gives a statement of the number of thermometers that have received certificates, the reasons why others were rejected, etc. A sample résumé sheet corresponding to the test record sheet shown on the preceding page is shown in fig. 10.

## RESULTS OF THE WORK.

It is gratifying to note a marked improvement in the quality of the thermometers now being submitted, especially in the product of those firms which have freely availed themselves of the facilities of the Bureau. Not the least important result has been the introduction into the clinical-thermometer industry of a uniform scale of temperature. At the present time practically all the large manufacturers have sent their standards to the Bureau for comparisons, so that not only are the standards in accord with one another, but they are also in accord with the scale of temperature used throughout the scientific and technical world. An impartial and authoritative test is of value alike to the manufacturer, the dealer, and the user of thermometers. To the first mentioned it is a constant check on his standards and product, and to the dealer and user the certificate is a guarantee of the accuracy of the thermometer.

The allowable errors for certification have been made as liberal as is consistent with a good product and with due regard to the interests of all concerned. The present limits were fixed after extensive experiments made on clinical thermometers obtained from different sources, but as the standard of product increases it may be desirable to modify these limits.

The investigation of the methods described and testing on a small scale have been in progress for the past three years. The announcement, however, that the Bureau was prepared to test thermometers was made only about eight months ago. Since then about five thousand thermometers have been submitted for test, so that ample opportunity has been afforded to test both the apparatus and the methods.

On account of the desire among users of clinical thermometers to have instruments with zero corrections the demand on the manufacturer has been unreasonably severe. As a matter of fact, a thermometer having corrections of  $0^{\circ}.1$  F. is practically as good as one with zero correction, since this order of accuracy is all that is essential. A thermometer with corrections as high as  $0^{\circ}.2$  F. may ordinarily be



Department of Commerce and Labor  
BUREAU OF STANDARDS  
Washington

B. S. TEST NO. 938 of 24 CLINICAL THERMOMETERS

Submitted by Clinical Mfg. Co. Marked The E. J. H. Drug Co.

Observers: H. D. and F. W.

Computed by F. W.

Checked by H. D. Holder No. 10 Date 4/23/04.

| Beadin Therm |       | 96.00 | 96.05 |       |     | 100.00 | 100.05 |        |     | 104.00 | 104.05 |        |     | 108.00 | 108.05 |        |     | Δ  | REMARKS |
|--------------|-------|-------|-------|-------|-----|--------|--------|--------|-----|--------|--------|--------|-----|--------|--------|--------|-----|----|---------|
| No. 15936    |       | -05   | -05   |       |     | -04    | -04    |        |     | -02    | -02    |        |     | .00    | .00    |        |     |    |         |
| Green Therm  |       | 95.99 | 95.99 |       |     | 100.00 | 100.00 |        |     | 104.01 | 104.01 |        |     | 108.04 | 108.04 |        |     | Δ  |         |
| No 1539      |       | +01   | +01   |       |     | .00    | .00    |        |     | -02    | -02    |        |     | -05    | -05    |        |     |    |         |
| COR. TEMP.   |       | 96.00 | 96.00 |       |     | 100.00 | 100.00 |        |     | 104.00 | 104.00 |        |     | 108.00 | 108.00 |        |     | Δ  |         |
| 1            | 29.10 | 37.25 | 96.01 | 96.20 | -02 | -1     | 100.00 | 100.06 | -06 | .0     | 104.04 | 104.04 | -04 | .0     | 108.06 | 108.12 | -06 | -1 |         |
| 2            | 100   | 26    | .02   | .00   | -02 | .0     | .00    | .00    | -06 | .0     | .02    | .04    | -02 | .0     | .00    | .00    | -06 | .0 |         |
| 3            | 110   | 27    | .03   | .05   | -03 | .0     | .04    | .04    | -04 | .0     | .00    | .00    | .00 | .0     | .06    | .06    | -06 | -1 |         |
| 4            | 101   | 28    | .04   | .06   | -04 | .0     | .00    | .00    | -04 | .0     | .00    | 103.96 | .00 | .0     | 107.90 | 107.92 | +10 | +1 |         |
| 5            | 108   | X     |       |       |     |        |        |        |     |        |        |        |     |        |        |        |     |    | W       |
| 6            | 107   | 29    | .10   | .05   | -10 | -1     | .10    | .10    | -10 | -1     | .06    | 104.00 | -08 | -1     | 108.14 | 108.15 | -10 | -1 |         |
| 7            | 103   | 37.10 | .02   | .14   | -08 | -1     | .10    | .08    | -08 | -1     | .07    | .00    | -07 | .0     | .03    | .00    | -08 | .0 |         |
| 8            | 109   | X     | .00   | .10   |     |        | 99.80  | 99.88  |     |        | 103.70 | 103.80 |     |        | 107.10 | 106.50 |     |    | R       |
| 9            | 130   | 31    | .10   | .04   | -10 | -1     | 100.00 | 100.00 | .00 | .0     | 104.00 | 104.05 | .00 | .0     | 90     | 107.86 | +10 | +1 |         |
| 10           | 121   | 32    | 95.95 | .00   | +05 | .0     | 99.94  | .00    | +06 | .0     | .00    | .00    | .00 | .0     | 108.00 | 108.04 | .00 | .0 |         |
| 11           | 115   | X     | .98   | .00   |     |        | 100.18 | .20    |     | .25    | .32    |        |     | .40    | .46    |        |     |    | >       |
| 12           | 114   | 33    | 96.01 | .08   | -01 | .0     | .00    | .05    | -05 | .0     | .06    | .02    | -06 | .0     | 107.80 | .00    | +14 | +1 |         |
| 13           | 117   | 34    | 95.80 | 95.77 | +20 | +2     | 99.97  | 99.84  | +03 | +1     | 103.86 | 103.90 | +14 | +1     | .83    | 107.82 | +17 | +2 |         |
| 14           | 135   | X     | 96.05 | .80   |     |        | .90    | .90    |     |        | 104.00 | .80    |     |        | .80    | .84    |     |    | a       |
| 15           | 137   | 35    | .04   | 96.02 | -04 | .0     | 100.03 | 100.00 | -03 | .0     | .00    | 104.10 | -14 | -1     | .90    | .95    | +00 | +1 |         |
| 16           | 139   | 36    | .00   | .04   | -04 | .0     | .00    | .00    | .00 | .0     | .03    | .00    | -03 | .0     | 108.00 | 108.06 | .00 | .0 |         |
| 17           | 116   | X     |       |       |     |        |        |        |     |        |        |        |     |        |        |        |     |    | W       |
| 18           | 113   | 37    | .12   | .04   | -12 | -1     | .00    | .04    | -04 | .0     | .00    | .05    | -05 | -1     | 107.92 | .00    | +08 | .0 |         |
| 19           | 119   | 38    | .00   | .00   | .00 | .0     | .00    | .00    | .00 | .0     | .04    | .16    | -04 | -1     | 108.24 | .20    | -22 | -2 |         |
| 20           | 124   | 39    | .10   | .08   | -10 | -1     | .06    | .03    | -06 | .0     | .07    | .02    | -07 | .0     | .00    | .02    | -02 | .0 |         |
| 21           | 132   | 37.40 | .05   | .10   | -05 | -1     | .07    | .06    | -07 | -1     | .00    | .02    | -02 | .0     | .00    | .04    | -04 | .0 |         |
| 22           | 141   | X     | 95.80 | .00   |     |        | 99.80  | 99.82  |     |        | 103.75 | 103.76 |     |        | .00    | .27    |     |    | a       |
| 23           | 102   | 41    | .85   | 96.84 | +14 | +2     | .82    | .80    | +18 | +2     | .84    | .92    | +16 | +2     | 107.83 | 107.85 | +17 | +2 |         |
| 24           | 106   | 42    | 96.00 | 96.04 | -04 | .0     | 100.00 | 100.00 | .00 | .0     | 104.00 | 104.00 | .00 | .0     | 108.00 | 108.04 | -04 | .0 |         |

FIG. 9. TEST RECORD SHEET.



Department of Commerce and Labor  
BUREAU OF STANDARDS

RÉSUMÉ OF B. S. TEST NO. 938.

24 OF  
CLINICAL THERMOMETERS

SUBMITTED BY

*Clinical Mfg. Co.*

| REASON FOR FAILURE TO RECEIVE<br>CERTIFICATE                                                                                                                             | IDENTIFICATION MARK ON THERMOMETER | TOTAL NO.<br>THERMS. |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------|----------------------|
| Defects in construction:<br>e. g. Presence of air bubbles in bulb,<br>cracks in glass, defects in grad-<br>uation, failure to hold index in<br>every part of scale, etc. | 29135<br>29141                     | 2                    |
| Too difficult to throw back<br>index below 95°.                                                                                                                          | 29108<br>29116                     | 2                    |
| Broken when received for test.                                                                                                                                           |                                    | 0                    |
| Broken during process of test-<br>ing.                                                                                                                                   |                                    | 0                    |
|                                                                                                                                                                          |                                    | 4                    |
| Exceed limits of allowable<br>error for certification.                                                                                                                   | 29118<br>29109                     | 2                    |
|                                                                                                                                                                          |                                    |                      |

For thermometers that failed to pass the preliminary test, for reasons specified in the first four sections of this table, a rebate is allowed at the pro rata rate corresponding to the number of thermometers submitted for test.

No. of thermometers submitted for test = 24  
 " " failed to pass preliminary test = 4  
 " " included in test = 20 @ per doz. =

DIRECTOR

FIG. 10. RÉSUMÉ SHEET.



**Department of Commerce and Labor**  
**BUREAU OF STANDARDS**

**CERTIFICATE OF EXAMINATION**  
**OF**  
**REGISTERING CLINICAL THERMOMETER**

Submitted by **Clinical Mfg. Co.**

Marked "N. B. S.

No. **3731.**"

**This certifies** that the above thermometer was found to have the following corrections at this date, compared with the official standards of this Bureau:

| THERMOMETER<br>READING | CORRECTIONS |
|------------------------|-------------|
| 96° F.                 | + 0.1       |
| 100°                   | .0          |
| 104°                   | -.1         |
| 108°                   | -.2         |

NOTE.—When the correction is + it should be added to and when — subtracted from the reading.

*Unless this thermometer has been suitably aged before testing, its indications are liable to change with time.*

Washington, D. C.

**4-23-04**

Test No. 938.

*S. W. Stratton*

Director.

Form 35. (Ed. 4, Jan. 2, 1904—3,000.)

11-592

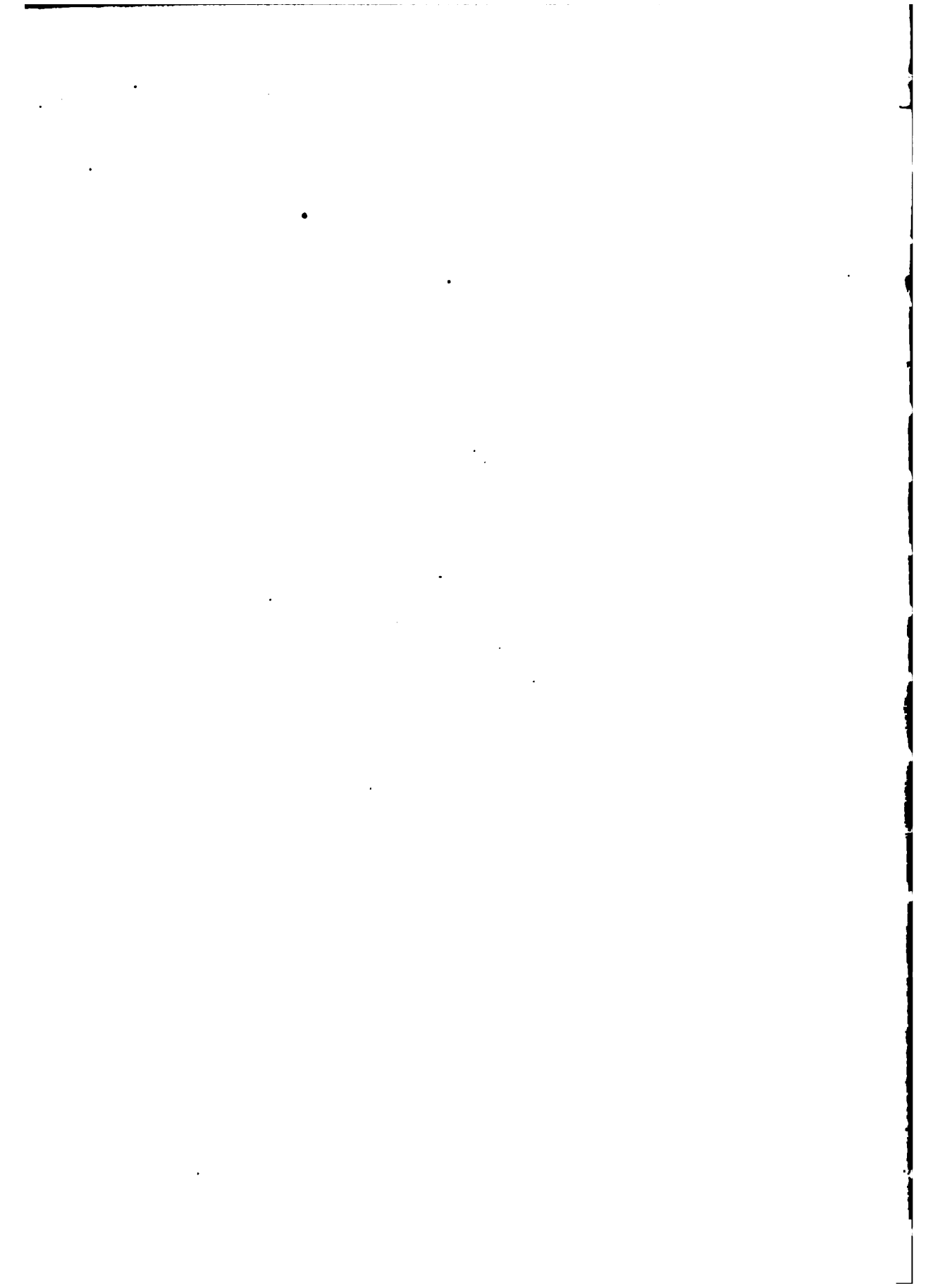
FIG. 11. CERTIFICATE.



used without regard to the corrections, and in cases where the accurate temperature is desired the corrections may be applied.

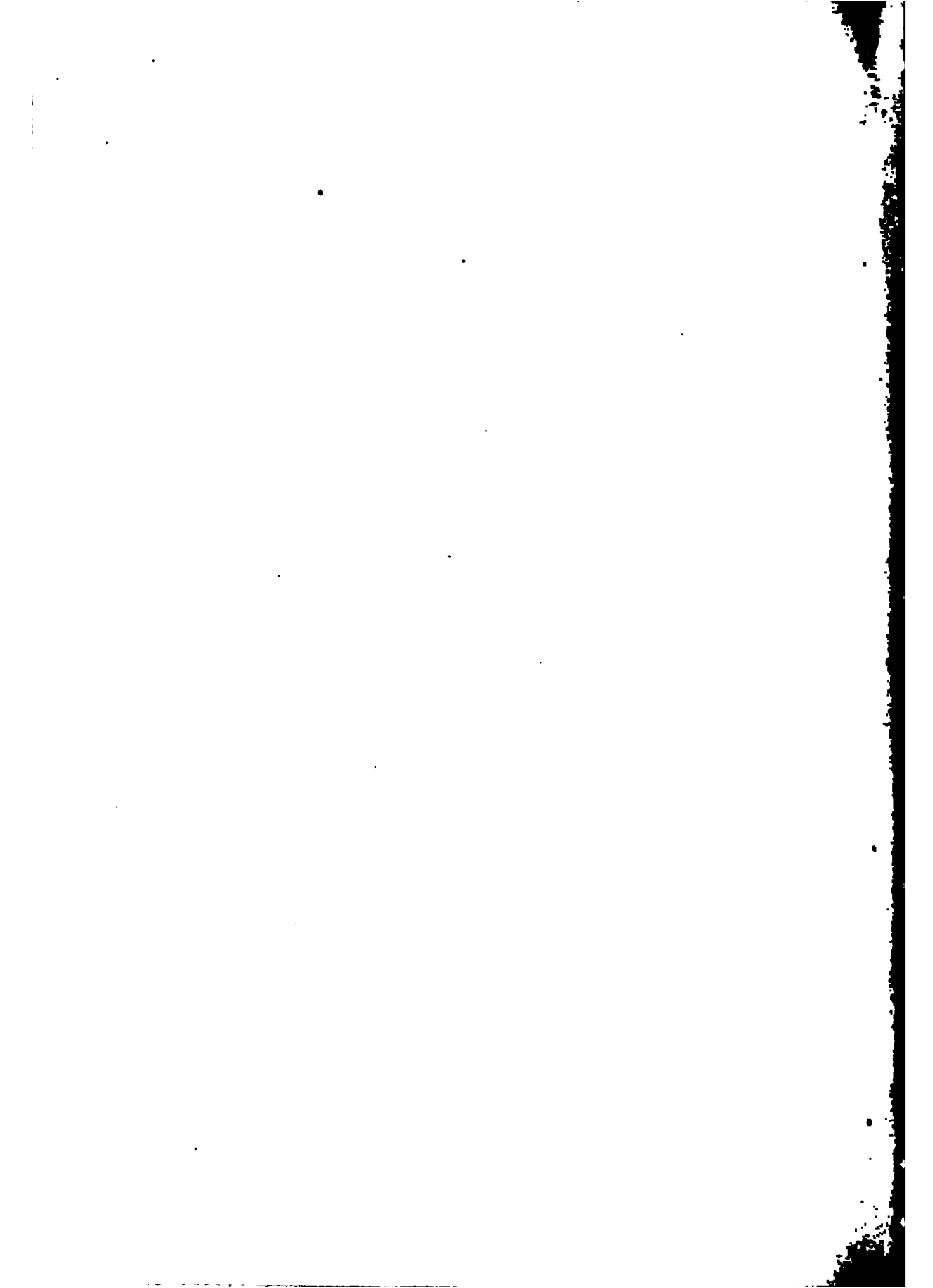
Of the clinical thermometers tested during the year ending July 1, 1904, 88.2 per cent satisfied all the requirements and received certificates, 8.5 per cent were rejected because their errors exceeded the allowable error, 1.6 per cent because the mercury index was too difficult to throw back, and 1.5 per cent for other causes, such as defects in construction, moisture, or air bubbles in mercury, etc.; 0.67 per cent were broken when received and 0.50 per cent were broken during test.

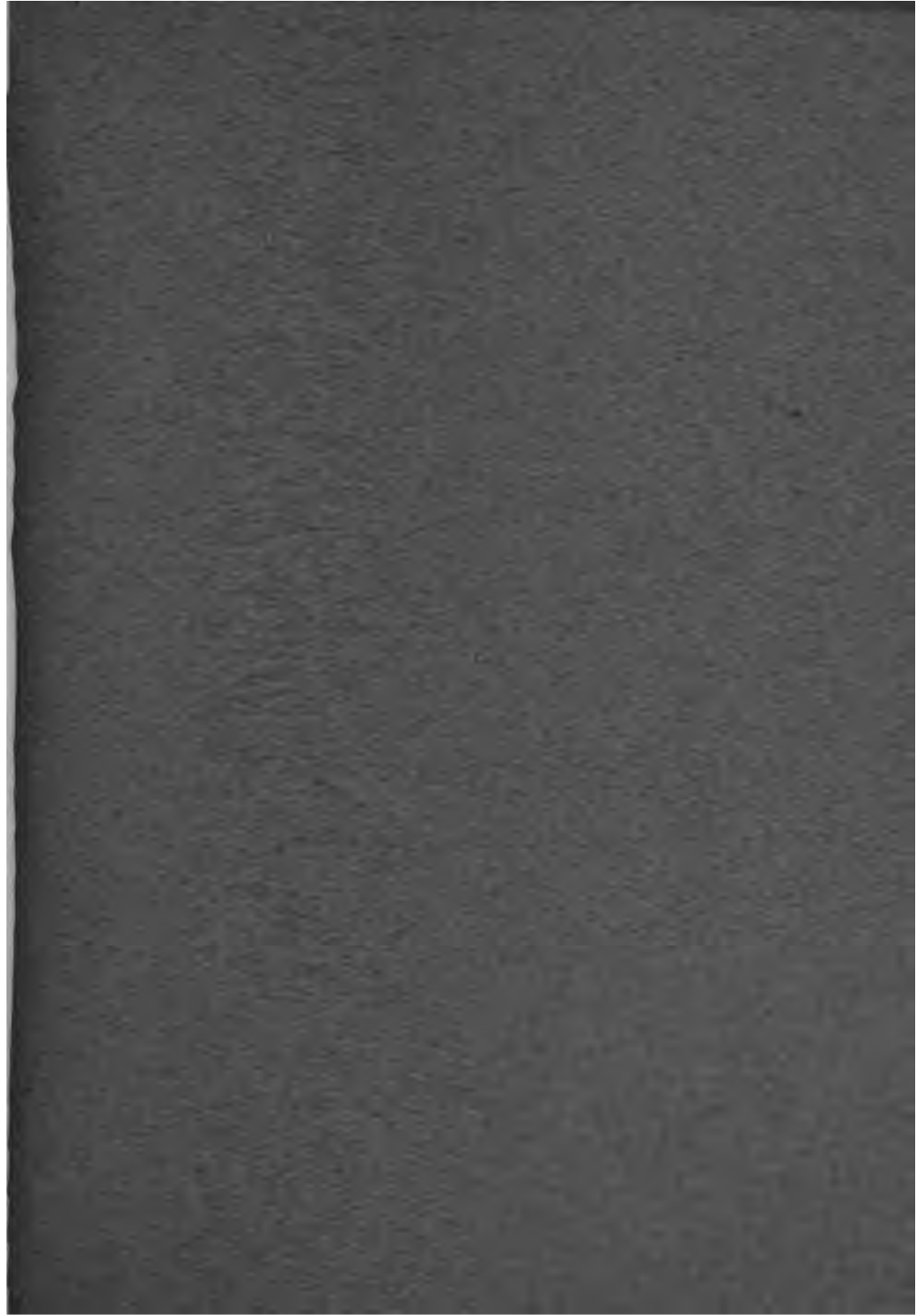
In conclusion, the authors wish to acknowledge their obligations to Dr. S. W. Stratton, Director of the Bureau of Standards, for many valuable suggestions in the design of the apparatus; to Dr. Chree, of the Kew Observatory, where clinical thermometers have been tested for many years, and to Dr. Wiebe and Dr. Hebe, of the Physikalisch-Technische Reichsanstalt, where such work is done on the largest scale, who gave the authors every facility to inspect the apparatus and methods in use at these institutions; to Mr. L. G. Hoxton and Mr. H. C. Dickinson, who were actively associated with the authors in the work. We are also pleased to acknowledge our indebtedness to the manufacturers for placing at our disposal for investigation considerable numbers of clinical thermometers, and also for much valuable advice and many suggestions. All the testing apparatus described was constructed in the instrument shop of the Bureau.











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- NO. 1. VERIFICATION OF STANDARDS AND MEASURING INSTRUMENTS.  
NO. 2. VERIFICATION OF METAL TAPES.  
NO. 3. VERIFICATION OF STANDARDS OF MASS.  
NO. 4. VERIFICATION OF STANDARDS OF CAPACITY.  
NO. 5. TESTING OF MEDICAL THERMOMETERS.  
NO. 6. VERIFICATION OF ELECTRICAL STANDARDS AND MEASURING INSTRUMENTS.  
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NO. 9. TESTING OF GLASS VOLUMETRIC APPARATUS.  
NO. 10. LEGAL WEIGHTS (IN POUNDS) PER BUSHEL OF VARIOUS COMMODITIES.

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DEPARTMENT OF COMMERCE AND LABOR

Vol. 1

BULLETIN

No. 3

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, Director

Issued November 1, 1905

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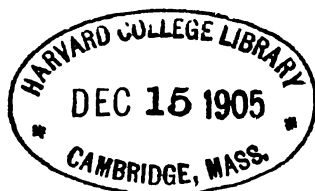


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# MEASUREMENT OF INDUCTANCE BY ANDERSON'S METHOD, USING ALTERNATING CURRENTS AND A VIBRATION GAL- VANOMETER.

By EDWARD B. ROSA and FREDERICK W. GROVER.

## 1. HISTORY OF THE METHOD.

Several modifications of Maxwell's method <sup>a</sup> of comparing an inductance with a capacity have been proposed in order to obviate the double adjustment of resistances necessary in that method. Maxwell showed

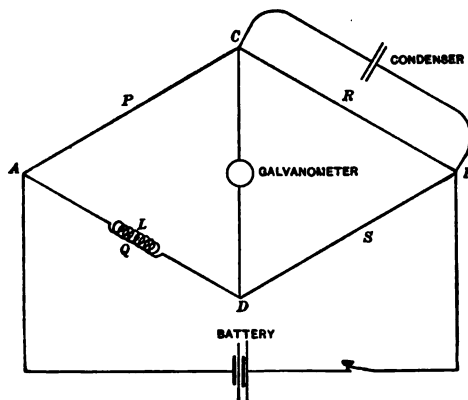


FIG. 1.—Maxwell's method.

that if (1) the bridge is balanced for steady currents and at the same time (2) the resistances are so chosen that there is no deflection of the galvanometer when the battery current is suddenly closed or broken, then

$$L = CRQ = CPS \quad (1)$$

where  $L$  is the inductance in the arm  $AD$ , the resistance of which is

<sup>a</sup> Electricity and Magnetism, § 778.

$Q$ ,  $C$  is the value of the capacity in parallel with  $R$ , and  $P$ ,  $R$ , and  $S$  are noninductive resistances.

In order to satisfy both of these conditions two of the arms of the bridge must be varied simultaneously, so that the balance for steady currents may be maintained while the balance for transient currents is sought. This is generally a tedious process, although by means of a small variable inductance in  $Q$ , in addition to the inductance to be measured, and a multiple valued condenser the process might be considerably accelerated.

In 1891 Professor Anderson proposed<sup>a</sup> an important modification of Maxwell's method, which consisted in joining the condenser to a point  $E$ , separated from  $C$  by a variable resistance  $r$ . The bridge being balanced for steady currents by varying any one of the four arms of the bridge, the balance for transient currents is then made by

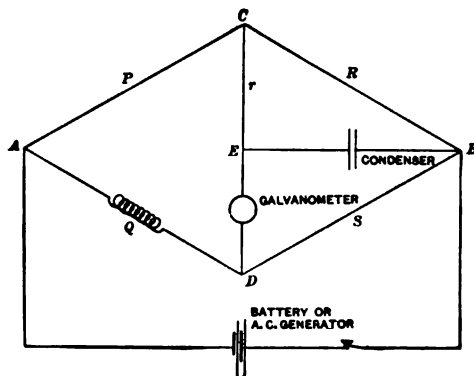


FIG. 2.—Anderson's method.

varying  $r$ , which does not disturb the balance of the bridge for steady currents. This change, which rendered the two adjustments independent, removed at once a most serious difficulty and made the method thoroughly practicable.

Anderson's demonstration for the case of transient currents gives for the value of the inductance (changing the letters to correspond to fig. 2)

$$L = C[r(Q+S) + PS] \quad (2)$$

If  $r=0$ ,  $L = CPS$ , as in Maxwell's method.

In the use of Anderson's method  $r$  may be small, so that  $CPS$  is the principal part of the expression for the inductance, or it may be larger, and the first term,  $Cr(Q+S)$ , represents the larger part of  $L$ . Thus a considerable range of values of inductance may be measured

<sup>a</sup> Phil. Mag., 81, p. 329, 1891.



without changing the arms of the bridge or the capacity of the condenser.

Stroud and Oates<sup>a</sup> have proposed another modification of Maxwell's method, which they have used with much success in measuring inductances. Instead of employing an interrupted current from a battery, as Anderson had done, they used an alternating current and an alternating-current galvanometer, the latter being essentially a d'Arsonval galvanometer, with the field magnet laminated and strongly excited by an alternating current from the generator. The galvanometer was thus made very sensitive, and to increase the sensitiveness still further the resistance  $r$  was placed outside the bridge, as shown in Fig. 3. It will be seen that this arrangement differs from Maxwell's only in separating the point  $B$  from the terminal of the condenser by the

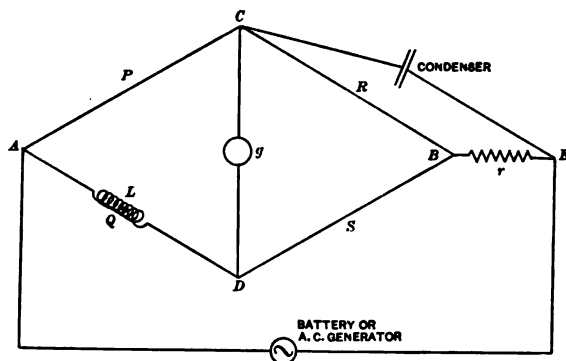


FIG. 3.—Stroud's method.

auxiliary adjustable resistance  $r$ , which in Anderson's method is in the galvanometer circuit between  $C$  and  $D$ . As the resistance  $r$  is sometimes several hundred ohms, it reduces the sensibility when in the galvanometer circuit, whereas in the arrangement of Fig. 3 the electromotive force can be increased if  $r$  is large, and so keep the same current in the bridge as when  $r$  is small, and thus maintain the sensibility.

The expression for the inductance  $L$  in Stroud's method (changing the letters to correspond with Fig. 3) is

$$L = C[r(Q + P) + PS], \quad (3)$$

which closely resembles the formula for Anderson's method, but differs in having  $Q + P$  in the first term instead of  $Q + S$ .

<sup>a</sup> Phil. Mag., 6, p. 707, 1903.

Professor Fleming has pointed out that Stroud's arrangement may be regarded as conjugate to Anderson's, the galvanometer and source of current being interchanged, Fig. 4. In this case the formula is exactly the same as for Anderson's method. If, however, Fig. 4 be rearranged so as to agree with Fig. 3, it will be found that the arms  $P$  and  $S$  are interchanged, and consequently that these letters must be interchanged in the formula for  $L$ . This changes equation (2) into equation (3).

Fleming and Clinton have employed Anderson's method for the measurement of small inductances, using a battery and a rotating commutator and galvanometer,<sup>a</sup> and later Fleming employed an interrupted current, produced by a vibrating armature, and a telephone.<sup>b</sup>

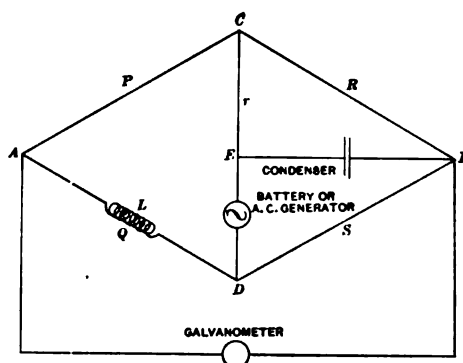


FIG. 4.—Showing Stroud's method as conjugate to Anderson's.

During the past two years we have employed Anderson's method for the measurement of both large and small inductances, using (1) a battery as a source of current and a d'Arsonval galvanometer, with a rotating commutator to interrupt and reverse simultaneously the current and galvanometer terminals; or (2), what has proved more satisfactory, an alternating current and a vibration galvanometer, the latter being tuned to the frequency of the current furnished by the generator.

## 2. ADVANTAGES OF THE METHOD.

We have found the method rapid and convenient in practice and the vibration galvanometer sufficiently sensitive to permit very accurate settings. As compared with other methods of accurately measuring inductance, it possesses striking advantages, some of which will here be specifically mentioned.

<sup>a</sup> Phil. Mag., 5, p. 493; 1903.

<sup>b</sup> Phil. Mag., 7, p. 586; 1904.

(a) All methods of measuring inductances without the use of a condenser (or other known inductance) require an accurate knowledge of the frequency of the alternating current employed. It is not difficult to determine accurately the mean frequency of an alternating current, even when the generator is inaccessible, as a counter may be employed to record on a chronograph the number of revolutions in a given time; moreover, the speed of the generator may be maintained sufficiently constant to enable good settings to be made. But to hold the speed steady enough to make settings of a high order of accuracy is difficult and requires an assistant to control the speed. With Anderson's method, even with a tuned galvanometer, *slight* changes of frequency are not detrimental, and hence the labor of taking the observations is greatly reduced.

(b) The inductance is determined in terms of a capacity, in addition to several resistances, which are also required in other methods of measuring inductance. A capacity can be measured by Maxwell's bridge method, using a commutator, with very great exactness, provided care is taken in choosing the resistances of the arms of the bridge,<sup>a</sup> and also provided the temperature of the condenser is taken and a temperature correction subsequently applied whenever necessary. The capacity of a condenser is not the same for slow charges as for rapid charges, and hence, if Anderson's method is used for transient currents, the capacity employed in the formula should correspond to the conditions of the experiment. As the successive makes and breaks of the current are likely to be irregular, the result would be that the effective capacity would vary slightly in successive trials, even with the best mica condensers. On the other hand, using an interrupted or alternating current of constant frequency, the capacity is uniform and definite, and if it is measured at the same frequency there is no uncertainty as to its value. In our experiments we employ an eight-pole generator, giving four complete cycles in each revolution. To this generator is joined the commutator which is employed in charging and discharging the condenser when measuring its capacity, the commutator having four segments, and hence charging and discharging the condenser four times in each revolution. Thus the frequency of charge and discharge of the condenser may be made exactly the same in use as when its capacity is measured. The change of capacity of a condenser with the frequency is very slight, but in measurements of the highest accuracy it is well to eliminate the slight uncertainty due to change of frequency.

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<sup>a</sup> Bulletin of the Bureau of Standards, No. 2, 1905.



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In order to maintain the frequency at the point of maximum sensibility a Maxwell bridge is employed, as when measuring the capacity of a condenser. The condenser capacity and resistances of the bridge remaining constant, any change in speed causes a deflection of the galvanometer. An adjustable carbon resistance in the armature circuit of the driving motor permits the speed to be adjusted so that the deflection is reduced to zero. The motor is driven by current from a storage battery, and hence the changes in speed are relatively small. A glance at the galvanometer scale at any time shows whether the speed is correct, and if not, it is quickly adjusted by means of the rheostat.

Fig. 5 gives the sensibility curve of the vibration galvanometer, showing two peaks of high sensibility at 110.6 and 120 vibrations per second, respectively. At a frequency of 115 the sensibility is very low—much less than it is at frequencies outside the peaks of maximum sensibility. The curve is affected by changes of temperature, and can be altered at pleasure by varying the length and tension of the suspension wire.

#### 4. THE APPARATUS.

A Rubens vibration galvanometer,<sup>a</sup> having a resistance of 200 ohms, is used. Its frequency may be varied between 100 and 200 per second, but has been used chiefly at about 110.

The several resistances are of manganin, and are all submerged in oil, to prevent heating and to enable their temperatures to be more accurately determined. The values of these resistances have been carefully measured every day that measurements of inductance have been made, when results of the highest accuracy have been sought. In series with the resistances  $r$  and  $Q$ , and forming part of them, are two slide wires which enable these resistances to be adjusted to 0.001 ohm, or even less, when necessary.

In order to eliminate as far as possible the errors due to slight changes in the arms  $P$  and  $R$  of the bridge, as well as any difference in their residual inductance and capacity, these resistances are always made equal and a commutator is employed to reverse them; a pair of readings is taken in every case, the mean of which is used in the calculation. The resistances  $Q$  and  $S$  were taken from two resistance boxes, in which the higher coils are subdivided to reduce the electrostatic capacity of the coil. We found in some of our early work that the residual capacity or inductance of noninductive resistances may be considerable; in the lower resistance coils the inductance predominates, and in the higher coils the capacity predominates. The connecting

<sup>a</sup> W. Oehmke, maker, Berlin.

Table I.—RESULTS OF MEASUREMENTS OF TWO INDUCTANCE COILS OF 1 HENRY EACH; COILS MEASURED SINGLY AND IN SERIES,  
MARCH 23, 1905.

[ $P = R = 250.002$  ohms.]

| 1   | 2      | 3       | 4                  | 5                  | 6                         | 7        | 8                           | 9                 | 10                         | 13                                 | 14                 |
|-----|--------|---------|--------------------|--------------------|---------------------------|----------|-----------------------------|-------------------|----------------------------|------------------------------------|--------------------|
| No. | Coils. | $S$     | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r + P$ | Temp.<br>of con-<br>denser. | $C$               | $L$<br>measured<br>values. | $L_0$ sums<br>of single<br>values. | $\Delta = L_0 - L$ |
|     |        | Ohms.   | Ohms.              | Ohms.              | Ohms.                     | Ohms.    | °                           | Microfa-<br>rads. | Henry.                     | Henry.                             | Henry.             |
| 1   | F      | 500.204 | 868.62             | 868.45             | 868.53                    | 1987.06  | .....                       | 1.00510           | 0.98900                    | .....                              | .....              |
| 2   | C      | "       | 883.72             | 883.56             | 868.64                    | 2017.28  | 19.25                       | 1.00510           | 1.01420                    | .....                              | .....              |
| 3   | F      | 600.268 | 702.92             | 702.80             | 702.86                    | 1655.72  | .....                       | 1.00511           | 0.98896                    | .....                              | .....              |
| 4   | C      | "       | 715.565            | 715.435            | 715.50                    | 1681.00  | 19.2                        | 1.00511           | 1.01421                    | .....                              | .....              |
| 5   | F+C    | "       | 1543.81            | 1543.56            | 1543.62                   | 3337.24  | .....                       | 1.00511           | 2.01345                    | 2.01317                            | -0.00028           |

Table II.—RESULTS OF MEASUREMENTS OF FOUR INDUCTANCE COILS OF 100 MILLIHENRYs EACH; COILS MEASURED SINGLY AND IN SERIES IN GROUPS OF TWO, MARCH 23, 1905.  
[ $P = R = 250.002$  ohms.]

| 1   | 2      | 3       | 4                  | 5                  | 6                         | 7          | 8                           | 9                 | 10                         | 13                                  | 14                 |
|-----|--------|---------|--------------------|--------------------|---------------------------|------------|-----------------------------|-------------------|----------------------------|-------------------------------------|--------------------|
| No. | Coils. | $S$     | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r + P$ . | Temp.<br>of con-<br>denser. | $C$               | $L$<br>measured<br>values. | $L_g$ =sums<br>of single<br>values. | $\Delta = L_g - L$ |
|     |        | Ohms.   | Ohms.              | Ohms.              | Ohms.                     | Ohms.      | °                           | Microfa-<br>rads. | Milli-<br>henrys.          | Milli-<br>henrys.                   | Millihenrys.       |
| 1   | A      | 250.102 | 74.000             | 73.972             | 73.999                    | 398.000    | 19.0                        | 1.00515           | 100.052                    | .....                               | .....              |
| 2   | B      | "       | 73.904             | 73.872             | 73.901                    | 397.802    | 18.95                       | 1.00515           | 100.003                    | .....                               | .....              |
| 3   | H      | "       | 73.861             | 73.822             | 73.855                    | 397.712    | 19.0                        | 1.00514           | 99.980                     | .....                               | .....              |
| 4   | C      | "       | 78.880             | 78.846             | 78.876                    | 407.754    | 19.0                        | 1.00513           | 102.503                    | .....                               | .....              |
| 5   | H + C  | 250.102 | 277.737            | 277.665            | 277.708                   | 805.418    | 19.1                        | 1.00512           | 202.488                    | 202.483                             | + 0.015            |
| 6   | A + B  | "       | 272.907            | 272.832            | 272.872                   | 795.746    | 19.1                        | 1.00512           | 200.036                    | 200.055                             | + 0.019            |
|     |        |         |                    |                    |                           |            |                             |                   |                            | Mean..                              | + 0.017            |
| 7   | A + B  | 200.068 | 372.526            | 372.417            | 372.474                   | 984.960    | .....                       | 1.00511           | 200.075                    | 200.092                             | + 0.017            |
| 8   | H + C  | "       | 378.535            | 378.434            | 378.497                   | 1006.976   | .....                       | 1.00511           | 202.495                    | 202.506                             | + 0.011            |
|     |        |         |                    |                    |                           |            |                             |                   |                            | Mean..                              | + 0.014            |
| 9   | A      | 200.068 | 123.824            | 123.775            | 123.812                   | 497.626    | .....                       | 1.00510           | 100.067                    | .....                               | .....              |
| 10  | B      | "       | 123.718            | 123.673            | 123.708                   | 497.418    | .....                       | 1.00510           | 100.025                    | .....                               | .....              |
| 11  | H      | "       | 123.664            | 123.614            | 123.650                   | 497.302    | .....                       | 1.00510           | 100.002                    | .....                               | .....              |
| 12  | C      | "       | 129.826            | 123.836            | 129.872                   | 509.746    | 19.25                       | 1.00510           | 102.504                    | .....                               | .....              |



leads have substantial terminals and the resistances of these leads, in thousandths of an ohm, is stamped on the terminals. Their values are always included in making up the corrected resistances of the bridge.

The measurements shown in Tables I and II, made March 23, illustrate the results obtained in the determination of inductances of one henry and one-tenth henry. An electromotive force of about 50 volts was employed on the bridge.

Fig. 6 shows how the commutator was connected to the bridge so as to reverse  $P$  and  $R$ , which are equal. Formula (2) in this case ( $Q=S$ ) reduces to

$$L = C S (2r + P).$$

Columns 4 and 5 give the nominal values of  $r$  in the two positions of the commutator, and column 5 the mean value, corrected from the

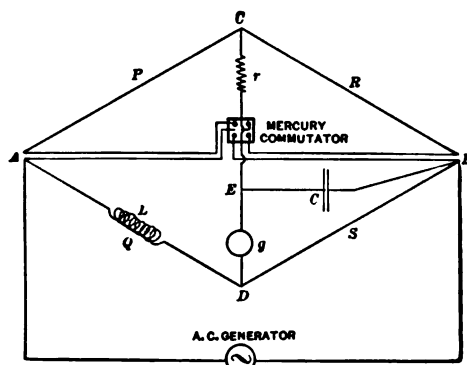


FIG. 6.—Showing commutator for interchanging two arms of the Anderson Bridge.

results of the latest comparison of the resistances with standard resistances.

Column 9 gives the capacity of the condenser  $C$ . Where two inductances are measured in series the measured sum is given in column 10, and the sum of the separate values are given in the next column. The last column gives the differences between these measured values and the sums of the separate values. While these differences are very small, averaging less than one in ten thousand, they are appreciable and always positive. This indicates that there may be some constant source of error in the bridge.

##### 5. SOURCES OF ERROR.

The results given above show that measurements of inductance of very great precision can be made by Anderson's method, provided there are no constant errors of appreciable magnitude entering into

the results. Such errors might be due to any one of the following causes:

(a) The residual inductance or capacity of the resistance  $r$  and of the arms of the bridge (not including, of course, the inductance of the coil in  $Q$  which is being measured) may introduce a constant error in  $L$ . As stated above, we have always made the arms  $P$  and  $R$  equal in value and reversed them by a commutator, in order to eliminate any difference there may be in their resistances or in their inductances or capacities. But the differences between  $Q$  and  $S$  can not thus be eliminated. The total resistance of  $Q$  is, of course, equal to  $S$ , since  $P=R$  (except for a small change due to residual inductances, to be discussed later), but part of this is in the inductive coil itself. Residual inductance in the noninductive part of  $Q$  makes the measured value of  $L$  too large, and, conversely, capacity would make it too small. The effect of inductance or capacity in  $S$  is of opposite sign to that of  $Q$ , and hence if the resistances of  $Q$  and  $S$  are *similar*, that is, made up as far as possible of the same kind of coils, then their effects will balance except for that part of  $S$  which equals the resistance of the inductive coil. In our work  $S$  was fixed in any given case, and  $Q$  was varied to secure a balance; thus  $Q$  usually contains a number of small resistances in addition to the slide wire, and these can not be counterbalanced exactly by  $S$ .

(b) The inductive coils must be removed some distance from the bridge and from each other when two or more are measured at once. This requires leads of one to three meters in length (for the larger inductances), and these leads may affect the measured value of the inductance. If they are close together, so as to be noninductive (as twisted lamp cord, for instance), they possess an appreciable capacity; and if far enough apart to be free from capacity, they possess measurable inductance. In measuring small inductance coils the capacity effect is small, and it is better to have the leads close together and as short as is safe. With large inductances the capacity of the leads is more important, and it is better to have them farther apart, to reduce it to a minimum. The inductance of the leads can then be calculated (or separately measured) and applied as a correction, if desired, or the same leads may always be employed with a given coil and considered as a part of the coil. The inductance of the wires joining the condenser to the bridge tends to reduce the capacity in the ratio of  $pl$  to  $\frac{1}{pc}$  or  $p^2lc$  to unity, where  $l$  is the small inductance of the leads; on the other hand, if these leads are close together, their capacity is added to that of the condenser. In our experiments, where the leads

were short and wide apart, both these effects were inappreciable. But if currents of high frequency are used, particularly with large capacity, the error due to the inductance of the leads may be appreciable; on the other hand, with small condenser capacity, the error due to the capacity of lead wires near together (as a twisted lamp cord) may be considerable and of opposite sign to the other. In precision measurements, therefore, care should be taken that no error is introduced in this manner.

(c) The inductive coil itself has a certain electrostatic capacity which modifies its measured inductance by an amount depending on the frequency of the current and the inductance of the coil, as well as its capacity.\* The approximate value of the expression for the measured inductance  $L'$  in terms of the true inductance  $L$  is

$$L' = L(1 + p^2 c L) \quad (4)$$

where  $c$  is the electrostatic capacity of the coil and  $p = 2\pi$  times the frequency. In practice,  $c$  is found by measuring  $L'$  at two different frequencies; it is too small to be important for the smaller values of inductance at low frequencies. One of our inductance coils, having an inductance of 1 henry, has a capacity of  $1 \times 10^{-10}$  farads. For a frequency of 112, this value makes the correction term  $p^2 c L$  in the above expression .00005, a quantity which can be detected, but which is not a large error. If, however, the frequency were ten times as great, this term would become .005, a very important correction.

The electrostatic capacity of a coil can be made relatively small by winding it in a deep channel, so that there are many layers and comparatively few turns in a layer. This, however, reduces its inductance, and in practice it is better not to depart very far from the form giving maximum inductance. The electrostatic capacity of the cord, as already pointed out, increases the value of this correction term.

(d) The capacity of the condenser, as already stated, can be determined with very great accuracy, and by taking careful account of the temperature of the condenser and its temperature coefficient, there will be very little uncertainty in the value of the capacity. The question remains, however, as to what effect the absorption in the condenser produces on the measured value of the inductance when used in Anderson's method. The effect of absorption is to cause the current to lag a little behind its phase in a perfect condenser. That is, it is in advance of the electromotive force by a little less than  $90^\circ$ . We give below a theoretical investigation of this question, and also

\* Wien: Ann. d. Phys., 44, p. 711; 1891. Dolezalek: Ann. d. Phys., 12, p. 1153; 1903.

experimental measures, wherein the phase of the condenser current is shifted back by placing a resistance in series with it. Fortunately, the error due to the slight displacement of phase produced by the small absorption in a good mica condenser, is inappreciable. The effect of slight leakage is also investigated below, and proves to be inappreciable.

#### 6. CALCULATION OF THE EFFECT OF THE RESIDUAL INDUCTANCES AND CAPACITIES OF THE ARMS OF THE BRIDGE.

Inductance in any arm of the bridge causes the current to lag, while capacity advances its phase. The angular lag due to an inductance  $l$  is  $\frac{pl}{R}$ , and the angular advance due to capacity is  $pcR$ . The latter value follows from the fact that the capacity  $c$  is in parallel with the resistance. The current through the resistance is  $\frac{E}{R}$ ; the capacity current  $90^\circ$  ahead of this is  $pcE$ . The ratio of these two currents is  $pcR$ , and this is the angle of advance of the resultant current. If  $\frac{pl}{R} = pcR$ , the current will have the same phase as though both capacity and inductance were absent. Thus, if  $l = cR^2$  the coil may be considered free from inductance; if  $l < cR^2$ , it may be considered to possess negative inductance equal to  $cR^2 - l$ .

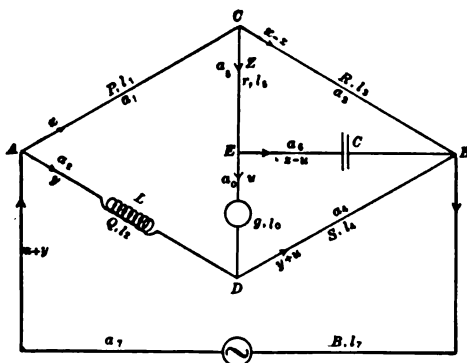


FIG. 7.—Network of Anderson Bridge, with currents, resistances, inductances, and impedances of the arms separately indicated.

The actual values of these residual positive or negative inductances vary widely, according to the length of wire on a coil, and its size, thickness of insulation, and manner of winding. High-resistance coils, made of many turns of fine wire wound in the usual noninductive manner, have relatively large capacity, which we here regard as negative inductance. In low-resistance coils of fewer turns of coarse wire the inductance predominates, and  $l$  is therefore positive.

To calculate the effect of these residual inductances, we shall form the equations for the networks of the Anderson bridge, assuming each branch to have positive or negative inductance, and solve for  $L$  the inductance of the coil to be measured. In fig. 7 the resistances, inductances, impedances, and currents in each arm of the bridge are indicated, and the positive directions of the currents are shown by arrows. Thus,

$g, P, Q, R, S, r, 0, B$  are the resistances.  
 $l_0, l_1, l_2, l_3, l_4, l_5, 0, l_7$  are the small inductances, + or -.  
 $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$  are the impedances.  
 $u, x, y, x-z, y+u, z, z-u, x+y$  are the currents.

The values of the impedances in the eight branches of the bridge are then as follows:

$$\left. \begin{aligned} a_0 &= g + i p l_0 \\ a_1 &= P + i p l_1 \\ a_2 &= Q + i p (l_2 + L) \\ a_3 &= R + i p l_3 \\ a_4 &= S + i p l_4 \\ a_5 &= r + i p l_5 \\ a_6 &= 0 + \frac{1}{i p C} \\ a_7 &= B + i p l_7 \end{aligned} \right\} \quad (5)$$

Applying Kirchhoff's law to the four circuits  $A C D$ ,  $C B E$ ,  $E B D$ ,  $A D B$ , we have the following equations:

$$\left. \begin{aligned} a_1 x + a_2 z + a_0 u - a_3 y &= 0 \\ a_2 (x-z) - a_6 (z-u) - a_5 z &= 0 \\ a_6 (z-u) - a_4 (y+u) - a_0 u &= 0 \\ a_7 (x+y) + a_3 y + a_4 (y+u) &= E \end{aligned} \right\} \quad (6)$$

Rearranging the terms of these equations, we have

$$\left. \begin{aligned} a_0 u + a_1 x - a_2 y + a_3 z &= 0 \\ a_2 u + a_3 x - (a_2 + a_5 + a_6) z &= 0 \\ (a_6 + a_4 + a_0) u + a_3 y - a_5 z &= 0 \\ a_4 u + a_7 x + (a_2 + a_4 + a_7) y &= E \end{aligned} \right\} \quad (7)$$

Solving for  $u$ , the current through the galvanometer, we obtain

$$u = \frac{1}{\Delta} \begin{vmatrix} 0 & a_1 & -a_2 & a_3 \\ 0 & a_2 & 0 & -(a_2 + a_5 + a_6) \\ 0 & 0 & a_4 & -a_5 \\ E & a_7 & (a_2 + a_4 + a_7) & 0 \end{vmatrix} = -\frac{E}{\Delta} \begin{vmatrix} a_1 & -a_2 \\ a_3 & 0 \\ 0 & a_4 \\ a_4 & -a_5 \end{vmatrix} \quad (8)$$

If  $u=0$ , we then have

$$\begin{aligned} & a_1 a_4 (a_3 + a_5 + a_6) - a_3 a_2 a_4 + a_3 a_4 a_5 = 0 \\ \text{or,} & a_1 a_3 a_4 + a_1 a_4 a_5 + a_1 a_4 a_6 + a_3 a_4 a_5 - a_3 a_3 a_6 = 0 \end{aligned} \quad (9)$$

Substituting the values of the impedances given in (5) above, we have

$$\left. \begin{aligned} & (P + ipl_1)(R + ipl_2)(S + ipl_4) + (P + ipl_1)(S + ipl_4)(r + ipl_3) \\ & + (P + ipl_1)(S + ipl_4)\frac{1}{ipC} + (R + ipl_2)(S + ipl_4)(r + ipl_3) \\ & - (Q + ip(l_3 + L))(R + ipl_2)\frac{1}{ipC} = 0 \end{aligned} \right\} \quad (10)$$

Separating the real and imaginary parts, we have first, for the real part

$$\begin{aligned} & PRS + PrS + RSr - \frac{LR}{C} + \frac{1}{C} [l_1 S - l_2 R - l_3 Q + l_4 P] \\ & - P^2 \left[ rl_1(l_1 + l_3) + S(l_1 l_3 + l_1 l_5 + l_3 l_5) + Rl_2(l_1 + l_5) + Pl_4(l_3 + l_5) \right] = 0 \end{aligned}$$

Transposing and dividing by  $\frac{R}{C}$ ,

$$\left. \begin{aligned} L = CS \left[ r \left( \frac{P+R}{R} \right) + P \right] + \frac{1}{R} [l_1 S - l_2 R - l_3 Q + l_4 P] - \\ \frac{P^2 C}{R} \left[ rl_1(l_1 + l_3) \right. \\ \left. + S(l_1 l_3 + l_1 l_5 + l_3 l_5) + Rl_2(l_1 + l_5) + Pl_4(l_3 + l_5) \right] \end{aligned} \right\} \quad (11)$$

$$\text{or, if } L_0 = CS \left[ r \left( \frac{P+R}{R} \right) + P \right],$$

$$L = L_0 + \alpha - \beta \quad (12)$$

If the small inductances  $l_1, l_2, l_3, l_4, l_5$  are all zero, as in the ideal case, the last two terms disappear and

$$L = CS \left[ r \left( \frac{P+R}{R} \right) + P \right] \quad (13)$$

and since in the Wheatstone bridge  $\frac{P}{R} = \frac{Q}{S}$  we have

$$L = C[r(Q+S) + PS],$$

which is the expression (2) given above for the inductance by Anderson's method.

The last term  $\beta$  in equation (12), having a coefficient  $\frac{P^2 C}{R}$  is negligible, unless the frequency is very high or the residual inductances excessive.

The second term  $\alpha$  gives the principal correction due to the inductances in the four arms of the bridge. It will be seen that the inductance of the resistance  $r$  enters only in  $\beta$ , which is negligible when  $l_s$  is small, and the galvanometer inductance has disappeared entirely from the equation. The second term  $\alpha$  consists of four parts, two of which are positive and two negative. The part  $\frac{1}{R} (l_1 S - l_s Q)$ , due to the two arms  $P$  and  $R$ , is eliminated in our experiments by making  $P=R$  and reversing  $P$  and  $R$ . The remainder  $\frac{1}{R} (l_1 P - l_s R) = l_1 - l_s$  (since  $P=R$ ) is due to the two remaining arms  $Q$  and  $S$ . As stated above, if these two arms consist of resistances made up of similar coils, (i. e., coils of the same resistance wound in the same manner with the same size wire) their inductances will be nearly equal. They might be exactly equal, if the inductive coil  $L$  had no resistance. It is therefore desirable to have  $l_s$  and  $l_1$  as nearly equal as possible, and then when necessary apply a correction for their difference.

If we divide the expression for  $\alpha$  in equation (11) above, by  $Q$ , remembering that  $Q = \frac{PS}{R}$  (approximately), we obtain

$$\alpha = Q \left\{ \begin{aligned} &\left[ \frac{l_1}{P} - \frac{l_s}{Q} - \frac{l_3}{R} - \frac{l_4}{S} \right] \\ &= \frac{Q}{p} \left[ \frac{pl_1}{P} - \frac{pl_s}{Q} - \frac{pl_3}{R} + \frac{pl_4}{S} \right] \\ &= \frac{Q}{p} \left[ \phi_1 - \phi_s - \phi_3 + \phi_4 \right] \end{aligned} \right\} \quad (14)$$

where  $\phi_1, \phi_s, \phi_3, \phi_4$  are the phase angles of the currents in the four arms of the bridge, due to the combined inductance and capacity of the resistances  $P, Q, R, S$ , neglecting, of course, the inductance  $L$  in  $Q$ , which is to be measured. These angles may be positive or negative. If they are all equal, or if  $\phi_1 + \phi_4 = \phi_s + \phi_3$  (algebraically) the correction term reduces to zero.

As we shall show below, these angles are appreciable in the "non-inductive" windings usual in resistance boxes, and the correction  $\alpha$  is therefore important in precision work. The resistances may, however, be so wound and adjusted as to make the angles  $\phi$  inappreciable.

The imaginary part of equation (10) above gives

$$\left. \begin{aligned} PS - RQ &= p^2 (l_1 l_4 - l_s (l_3 + L)) + p^2 C [PR l_4 + SR (l_1 + l_s) \\ &+ PS (l_3 + l_4) + r (Sl_1 + Sl_s + Pl_4 + Rl_4)] - p^4 C (l_1 l_3 + l_s l_3 + l_s l_1) l_4 = \gamma \end{aligned} \right\} (15)$$

If  $l_1, l_s, l_3, l_4, l_5$  are all zero, this reduces to  $PS - RQ = 0$ , which is the

condition for the Wheatstone bridge and the condition assumed in the ideal Anderson bridge. But when these quantities are not zero this condition does not hold and  $Q$  is not equal to  $\frac{PS}{R}$ . Consequently, an error is introduced in calculating  $L$  from the formula (2) of Anderson, unless  $Q$  is assumed equal to  $\frac{PS}{R}$ , instead of using its actual value.

The variation of  $Q$ , due to changes in one or more of the small inductances  $l$  ( $P$ ,  $R$ , and  $S$  remaining unchanged), is illustrated in some of the examples given below.

#### 7. ILLUSTRATION OF THE FORMULÆ.

In order to ascertain the order of magnitude of the corrections  $\alpha$  and  $\beta$  of formula (12), and the variation of  $PS-RQ$  from zero in (15), we shall assume the following values of the constants for cases I, II, and III:

$$\begin{aligned} L &= 1 \text{ henry,} \\ C &= 1 \text{ microfarad,} \\ P &= R = 250 \text{ ohms,} \\ S &= 500 = Q \text{ (approximately),} \\ r &= 875, \\ p^2 &= 500,000. \end{aligned}$$

INDUCTANCES IN MICROHENRYS.

| Case. | I   | II  | III  | IV     |
|-------|-----|-----|------|--------|
| $l_1$ | - 2 | +25 | - 50 | -1,500 |
| $l_2$ | + 2 | +40 | +100 | - 250  |
| $l_3$ | + 2 | +25 | + 50 | 0      |
| $l_4$ | - 2 | +50 | -100 | -5,000 |
| $l_5$ | +10 | +50 | +100 | + 100  |

Case I is supposed to represent a specially wound bridge in which all the inductances are small, but in order to make it represent the most unfavorable case two are taken positive and two negative, so as to give a maximum value to the error  $\alpha$ .

Case II represents a favorable arrangement where the coils are similar and the inductances all positive, but with larger values than those of case I, being such as might be expected in practice.

In Case III we have assumed that  $P$  is a single coil of fine wire, having the capacity effect greater than the inductance by an amount



equivalent to a negative inductance of 50 microhenrys, whereas  $R$  is made up of several coils of coarser wire, giving a smaller capacity and larger inductance, and hence  $l_3$  is taken as +50. Similarly,  $S$  is supposed to be a single coil of 500 ohms, with capacity predominating, and equivalent to a negative inductance of 100 microhenrys, while  $Q$  is the sum of several smaller coils, and  $l_4$  is therefore positive and equal to 100 microhenrys. This is perhaps an extreme case, but not an improbable one.

In case IV larger resistances are assumed, viz:

$$P=1,000 = Q \text{ (approximately).}$$

$$R=S=5,000.$$

$$r=833.3.$$

$$C=0.1 \text{ microfarad.}$$

$P$ ,  $Q$ , and  $S$  are assumed to have negative inductances, each being supposed to consist of a single coil (or mainly of large coils, as in the case of  $Q$ ), while  $R$  is supposed to be made up of smaller coils having the inductance and capacity balanced. The values of the inductances chosen for  $P$ ,  $Q$ , and  $S$  are approximately those found in noninductive resistances of such magnitudes.

Substituting the above values of the constants in equation (11) we find the following values of  $\alpha$  and  $\beta$ :

| Case. | $L_0$   | $\alpha$  | $\beta$                |
|-------|---------|-----------|------------------------|
| I     | 1 Henry | -0.000012 | + $4 \times 10^{-12}$  |
| II    | 1 "     | +0.000010 | - $1.0 \times 10^{-8}$ |
| III   | 1 "     | -0.000400 | + $1.2 \times 10^{-8}$ |
| IV    | 1 "     | -0.002250 | -34 $\times 10^{-8}$   |

These results show that the  $\beta$  term is small in comparison with  $\alpha$ , and may be neglected. The correction  $\alpha$  is as large in case I as in Case II, showing that if the several small inductances are all of the same sign, and proportional to the resistances, they cancel out, except for the necessary inequality in  $l_1$  and  $l_4$ , due to the resistance of the coil to be measured. If the inductances of the coils are adjusted to as small values as 2 microhenrys for each arm, they may be either positive or negative without making the error appreciable, as the error in case I, with  $l_1$  and  $l_4$  opposite in sign to  $l_2$  and  $l_3$  is the greatest possible for such values of  $l_1$ ,  $l_2$ ,  $l_3$ ,  $l_4$ .

The results shown in Table III illustrate the importance of the correction term  $\alpha$  for coils of smaller inductances, namely: 100, 10, 1, 0.1, and 0.01 millihenrys.

**Table III.**—SHOWING THE VALUES OF THE CORRECTION  $\alpha$  FOR VARIOUS VALUES OF INDUCTANCES AND THE CORRESPONDING VALUES OF CAPACITIES AND RESISTANCES.

| Inductance to be measured. | Capacity.     | $P=R$ | $l_1$         | $l_2$         | $Q=S$ | $l_2$         | $l_4$         | $r$   | $\alpha$      |
|----------------------------|---------------|-------|---------------|---------------|-------|---------------|---------------|-------|---------------|
| Milli-henrys.              | Micro-farads. | Ohms. | Micro-henrys. | Micro-henrys. | Ohms. | Micro-henrys. | Micro-henrys. | Ohms. | Milli-henrys. |
| 100                        | 1.0           | 250   | +2            | -2            | 250   | -2            | +2            | 75    | 0.008         |
| 10                         | 0.4           | 100   | +1            | -1.0          | 100   | -1            | +1            | 75    | 0.004         |
| 1                          | 0.1           | 50    | +0.5          | -0.5          | 100   | -1            | +1            | 25    | 0.004         |
| 0.1                        | 0.05          | 20    | +0.5          | -0.5          | 50    | -0.5          | +0.5          | 10    | 0.0045        |
| 0.01                       | 0.02          | 20    | +0.5          | -0.5          | 20    | -0.5          | +0.5          | 2.5   | 0.002         |

It will be seen that the error  $\alpha$  in the case of 100 millihenrys is scarcely appreciable, but that in the others it is appreciable and in the smaller coils it amounts to several per cent. These computed errors, as before, are the maximum values for the assumed residual inductances, since we have taken  $l_1$  and  $l_4$  of opposite sign to  $l_2$  and  $l_3$ . In practice we should therefore expect smaller errors on the average unless the values of the  $l$ 's are larger. If the coils are not wound to a minimum value of the inductance, the errors may be much larger than those above. It is therefore our practice in measuring small inductances to take them by difference, leaving  $P$ ,  $R$ , and  $S$  unchanged and altering  $Q$  to compensate for the resistance of the coil to be measured. If the inductances of the resistances of  $Q$ , or at least of the part to be replaced by the coil to be measured, are accurately known, the difference of two determinations gives the true inductance desired.

The value of  $PS - RQ$  is found from equation (15) by substituting the values of the resistances and residual inductances. In case *I* above,  $S - Q$  is only  $-0.003$  ohm, while in Case *IV* it is  $-4.55$  ohms. In other words,  $Q$  is larger by 4.55 ohms in a total of 1,000 when the bridge is exactly balanced for alternating current than it is for a direct-current balance. Consequently, if, as is sometimes done, the bridge is balanced with direct current, and then an alternating current is applied, the resistance balance no longer holds, if there are residual inductances and capacities, and  $Q$  may require a change of several ohms to secure the resistance balance, the inductive balance being

effected as we have seen above by varying  $r$ . In calculating  $L$ , however, no account need be taken of  $Q$ , as it is eliminated from the expression (13) for  $L$ . Hence there is no occasion to calculate the value of  $\gamma$  in (15).

### 8. EFFECT OF RESISTANCE IN SERIES AND IN PARALLEL WITH THE CONDENSER.

As a mica condenser is not entirely free from absorption and might also show slight leakage, it is desirable to ascertain how large an error, if any, is produced by using such a condenser instead of the ideal condenser assumed in the theory of the Anderson bridge, namely, one in which the impedance is  $\frac{1}{ipC}$ . Resistance in series with a perfect condenser produces the same phase displacement as a certain amount of absorption, and resistance in parallel with the condenser has the effect of leakage or imperfect insulation. In a good mica condenser the

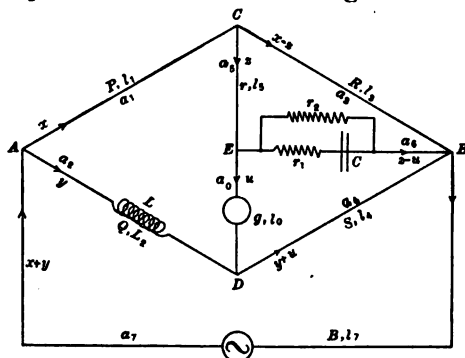


FIG. 8.—Resistance in series and in parallel with the condenser.

phase of the current with a frequency of 100 per second should differ from quadrature with the electromotive force by not more than one minute of angle, and may be as small as  $30''$ , although it may be several minutes in inferior condensers (even as high as  $30'$ ). For paper condensers the angle may be as small as  $4'$  and as large as several degrees. For a condenser of one microfarad capacity, with a frequency of 100 per second,  $30''$  of angle corresponds to a resistance in series with the condenser of 0.23 ohm, whereas  $30'$  would correspond to a resistance of 14 ohms.

In other words, such resistances in series with perfect condensers would give currents of the same phases as the imperfect condensers employed. A leakage resistance less than a thousand megohms would never occur in a good condenser. We shall now calculate (1) the effect

of introducing resistance in series with the condenser to correspond with absorption and (2) of placing a high-resistance shunt around the condenser to represent leakage. In fig. 8 these two resistances are represented by  $r_1$  and  $r_2$ . If we substitute the values of the impedances of the arms of the bridge in formula (9), we shall obtain an expression for the inductance to be measured in the same manner as before. In this case, however, we assume for convenience that the resistances  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $r$  are all free from inductance or capacity (except, of course,  $L$  in  $Q$ ), and hence

$$a_1 = P$$

$$a_2 = Q + ipL$$

$$a_3 = R$$

$$a_4 = S$$

$$a_5 = r$$

$$a_6 = r_1 + \frac{1}{ipC} \text{ in the first case,}$$

and

$$a_6 = \frac{r_2}{1 + ipCr_2} \text{ in the second case.}$$

(a) *Resistance in series with the condenser.*—Equation (9) becomes, when the above values of the impedances are substituted,

$$PRS + PSr + PS(r_1 + \frac{1}{ipC}) + RSr - (Q + ipL)(r_1 + \frac{1}{ipC})R = 0 \quad (16)$$

Separating the real and imaginary parts, we have, first, for the real part,

$$PS(R + r + r_1) + RSr - QRr_1 - \frac{L}{C}R = 0$$

$$\text{or,} \quad L = CS[r\frac{(P+R)}{R} + P] + Cr_1[\frac{PS}{R} - Q] \quad (17)$$

The first term of this expression is the same as that of (13), and is the value of  $L$  when  $r_1$  is zero.  $\frac{PS}{R} - Q$  would be zero in the ideal bridge. To find its value in this case we make use of the imaginary part of (16) above.

This imaginary part is

$$\frac{PS - RQ}{ipC} - ipLLr_1 = 0$$

$$\text{or,} \quad PS - RQ = -p^2LLr_1C.$$

Whence

$$Q = \frac{PS}{R} + p^2Lr_1C. \quad (18)$$

Substituting the value of  $\frac{PS}{R} - Q$ , derived from (18), in equation (17), we have

$$L = CS \left[ r \frac{(P+R)}{R} + P \right] - p^2 r_1^2 LC^2 \quad (19)$$

In fig. 9, which is the impedance diagram of a condenser,  $r_1$  is the series resistance, equivalent (in its effect upon the phase angle) to the absorption, and  $\theta$  is the small angle by which the current falls short of  $90^\circ$  in its phase relation to the impressed electromotive force. Hence  $\tan \theta = pCr_1$ . Substituting in (19) above,

$$L = L_0 - L \tan^2 \theta$$

$$\text{or,} \quad L = L_0 (1 - \tan^2 \theta) \quad (20)$$

since  $L_0$ , the value of  $L$  when the correction is zero, is substantially the same as  $L$ . In the best mica condensers, as stated above,  $\theta$  is about half a minute, and  $\tan \theta$  is 0.00015;  $\tan^2 \theta$  is therefore only about two parts in a hundred million. Hence the angle  $\theta$  might be ten times as large without producing an appreciable error, although in some mica condensers that we have tested the angle is large enough to produce a sensible error.

(b) *Resistance in parallel with the condenser.*—Substituting  $\frac{r_2}{1+ipCr_2}$  for  $a_2$  in equation (9) above, we get a solution for the case of resistance in parallel with the condenser. The direct substitution gives

$$PRS + PSr + PS \left( \frac{r_2}{1+ipCr_2} \right) + RSr - R(Q + ipL) \left( \frac{r_2}{1+ipCr_2} \right) = 0 \quad (21)$$

The real part of this is

$$PRS + PSr + RSr + (PS - RQ)r_2 = 0$$

$$\text{Hence,} \quad Q - \frac{PS}{R} = \frac{S}{r_2} \left[ r \frac{(P+R)}{R} + P \right]$$

$$\text{or,} \quad Q - \frac{PS}{R} = \frac{L_0}{r_2 C} \quad (22)$$

Thus, the variation in  $Q$  is inversely proportional to  $r_2$ , the shunt resistance, and to the capacity of the condenser, and directly proportional to  $L$ . The leakage resistance through a condenser is inversely proportional to the capacity, so that in general  $r_2 C$  is independent of the value of the capacity, but depends on the quality of the condenser.



FIG. 9.—Impedance diagram of an imperfect condenser.

If  $r_1=1000$  megohms and  $C=1$  microfarad,  $r_2 C=1000$  and the variation of  $Q$  is in that case very small.

The imaginary part of equation (21) above is

$$Cr_2(PRS + PSr + RSr) = Lr_2 R$$

$$\text{or,} \quad L = CS \left[ r \frac{(P+R)}{R} + P \right] = L_0 \quad (23)$$

This is equation (13), and shows that  $r_2$  has no effect whatever on the measured value of  $L$ .

#### 9. VERIFICATION OF FORMULÆ 11, 18, 19, 22, 23.

In Table IV the results are given of a series of measurements made to verify formula (11) by introducing a small inductance successively in each of the arms of the bridge; the  $\beta$  term in this formula is negligible, in comparison with  $\alpha$ , except for the case of inductance in  $r$  only, in which case  $\alpha$  is zero and the  $\beta$  term has then been computed. This is the fifth case in the table, where the observed change ( $\Delta \beta$  in this case, as  $\Delta \alpha = 0$ ) is 0.01 millihenry, whereas the calculated effect is still smaller. But the inductance coil measured has an inductance of 1 henry, and hence the observed change is only one part in a hundred thousand, a quantity barely measureable.

If we differentiate formula (12) we have, since  $L$  does not change when the inductance coil is inserted in any arm of the bridge, and  $\beta$  is assumed zero,

$$\Delta L_0 + \Delta \alpha = 0$$

or,

$$\Delta \alpha = -\Delta L_0$$

where  $\Delta \alpha$  is the change produced in the  $\alpha$  term by the insertion of the given coil in any arm of the bridge, and  $\Delta L_0$  is the change in the first term of equation (11) or (12); that is, in the computed value of  $L$  apart from the correction terms.

For example, in the first case, 0.5 millihenry was inserted in the arm  $Q$ , and hence  $l_1$  was increased by 0.5 millihenry. The second term of the  $\alpha$  correction was therefore increased by  $0.5 \times \frac{R}{R} = 0.500$  millihenry. The observed value of  $\Delta L_0$  in this case was the same. In case 3, the addition of 0.5 millihenry in  $P$  changed the first term of the  $\alpha$  correction by  $0.5 \times \frac{S}{R} = 1.000$  millihenry, whereas the measured change was 0.999 millihenry. In the fourth case the difference

Table IV.—EFFECT OF PLACING AN INDUCTANCE COIL OF 0.5 MILLIHENRY IN THE ARMS OF THE ANDERSON BRID. I.

[P=250 ohms. S=600 ohms. C=1 microfarad. L=1 henry.]

| 1   | 2                  | 3                  | 4                  | 5                | 6               | 7          | 8          | 9          | 10                          | 11                          |
|-----|--------------------|--------------------|--------------------|------------------|-----------------|------------|------------|------------|-----------------------------|-----------------------------|
| No. | Position of coil.  | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean.     | $\Delta r$      | $\Delta S$ | $\Delta R$ | $\Delta P$ | $\Delta L_o$<br>observed.   | $\Delta L_o$<br>calculated. |
| 1   | Coil in Q .....    | Ohms.<br>884.500   | Ohms.<br>884.330   | Ohms.<br>884.415 | Ohms.<br>+0.497 | Ohms.<br>0 | Ohms.<br>0 | Ohms.<br>0 | Milli-<br>henrys.<br>+0.500 | Milli-<br>henrys.<br>+0.500 |
|     | Coil removed ..... | 884.001            | 883.836            | 883.918          |                 |            |            |            |                             |                             |
| 2   | Coil in S .....    | 881.911            | 881.741            | 881.826          | -2.113          | +0.793     | 0          | 0          | -0.519                      | -0.500                      |
|     | Coil removed ..... | 884.025            | 883.853            | 883.939          |                 |            |            |            |                             |                             |
| 3   | Coil in P .....    | 881.270            | 881.084            | 881.177          | -2.790          | 0          | 0          | +0.794     | -0.989                      | -1.000                      |
|     | Coil removed ..... | 884.051            | 883.883            | 883.967          |                 |            |            |            |                             |                             |
| 4   | Coil in R .....    | 886.439            | 886.268            | 886.354          | +2.387          | 0          | +0.794     | 0          | +0.989                      | +1.000                      |
|     | Coil removed ..... | 884.051            | 883.883            | 883.967          |                 |            |            |            |                             |                             |
| 5   | Coil in r .....    | 884.051            | 883.886            | 883.969          | +0.010          | 0          | 0          | 0          | +0.010                      | +0.0002                     |
|     | Coil removed ..... | 884.040            | 883.878            | 883.959          |                 |            |            |            |                             |                             |

Table VII.—EFFECT OF PLACING RESISTANCE  $r_2$  IN PARALLEL WITH THE CONDENSER OF THE ANDERSON BRIDGE.[ $P=R=250$  ohms.  $S=250$  ohms.  $C=2$  microfarad.  $L=1$  henry.]

| 1                   | 2                  | 3                  | 4            | 5                 | 6               | 7                | 8                                                            | 9      | 10         | 11                                  |
|---------------------|--------------------|--------------------|--------------|-------------------|-----------------|------------------|--------------------------------------------------------------|--------|------------|-------------------------------------|
| $r_2$               | $r$<br>Position 1. | $r$<br>Position 2. | $r$<br>Mean. | $\Delta L_0$      | $\frac{L_0}{C}$ | $C$ calc.        | Capacity in<br>m.f. single coil<br>(coil = 100,000<br>ohms). | $Q'$   | $\Delta Q$ | $\frac{L_0}{C} \cdot \frac{1}{r_2}$ |
| Ohms.               | Ohms.              | Ohms.              | Ohms.        | Milli-<br>henrys. | $\times 10^4$   | Microfarads.     |                                                              | Ohms.  | Ohms.      | Ohms.                               |
| $\infty$            | 871.83             | 871.60             | 871.715      | —0.005            | 0.4984          | 0.00012          | 0.00048                                                      | 153.18 | +0.62      | +0.50                               |
| 1,000,000           | 871.83             | 871.59             | 871.71       | —0.006            | 0.4984          | 0.00028          | 0.00052                                                      | 153.80 | 1.23       | 1.24                                |
| 400,000             | 871.77             | 871.55             | 871.66       | —0.130            | 0.4983          | 0.00059          | 0.00059                                                      | 154.41 | 2.57       | 2.49                                |
| 200,000             | 871.71             | 871.48             | 871.596      | —0.295            | 0.4982          | 0.00124          | 0.00062                                                      | 158.07 | 4.89       | 4.98                                |
| 100,000             | 871.54             | 871.33             | 871.435      | —0.615            | 0.4980          | 0.00251          | 0.00063                                                      | 162.83 | 9.65       | 9.96                                |
| 50,000              | 871.23             | 871.00             | 871.115      | —1.250            | 0.4978          | 0.00517          | 0.00064                                                      | 172.44 | 19.28      | 19.91                               |
| 25,000              | 870.60             | 870.37             | 870.485      | —2.550            | 0.4971          |                  |                                                              | 191.52 | 38.34      | 39.76                               |
| 12,500              | 869.31             | 869.08             | 869.195      |                   |                 |                  |                                                              | 153.18 |            |                                     |
| $\infty$            | 871.86             | 871.63             | 871.745      |                   |                 |                  |                                                              |        |            |                                     |
| Coil = 10,000 ohms. |                    |                    |              |                   |                 |                  |                                                              |        |            |                                     |
| $\infty$            | 871.87             | 871.665            | 871.77       | —0.095            | 0.4984          | 0.000191         | 0.00191                                                      | 152.39 | +4.76      | +4.98                               |
| 100,000             | 871.78             | 871.57             | 871.675      | —0.105            | 0.4983          | 0.000211         | 0.00189                                                      | 157.15 | 5.22       | 5.54                                |
| 90,000              | 871.75             | 871.56             | 871.665      | —0.125            | 0.4983          | 0.000251         | 0.00176                                                      | 157.61 | 6.80       | 7.12                                |
| 70,000              | 871.74             | 871.56             | 871.65       | —0.190            | 0.4983          | 0.000382         | 0.00191                                                      | 161.90 | 9.52       | 9.97                                |
| 50,000              | 871.68             | 871.49             | 871.585      | —0.250            | 0.4983          | 0.000502         | 0.00201                                                      | 164.58 | 12.20      | 12.46                               |
| 40,000              | 871.62             | 871.43             | 871.525      | —0.300            | 0.4982          | 0.000602         | 0.00181                                                      | 168.23 | 15.86      | 16.61                               |
| 30,000              | 871.57             | 871.38             | 871.475      | —0.400            | 0.4981          | 0.000924         | 0.00185                                                      | 176.22 | 23.85      | 24.90                               |
| 20,000              | 871.42             | 871.22             | 871.32       | —0.865            | 0.4981          | 0.00174          | 0.00174                                                      | 200.00 | 47.63      | 49.80                               |
| 10,000              | 871.01             | 870.82             | 870.915      |                   | 0.4980          |                  |                                                              | 152.37 |            |                                     |
| $\infty$            | 871.89             | 871.67             | 871.78       |                   | 0.4984          |                  |                                                              |        |            |                                     |
|                     |                    |                    |              |                   |                 | Mean capacity .. | 0.00186                                                      |        |            |                                     |



succession in series with the condenser, the latter having a capacity of 2 microfarads. The changes in  $r$  were determined as before, and the changes in  $L_0$  resulting therefrom were computed, and these compared with  $p^2 r_1 C^2 L_0$  (formula 19). The changes in  $Q$  were computed from equation (18) and compared with the changes in the noninductive part of the arm  $Q$ , namely,  $Q'$ .

These measurements were made before the introduction into the bridge of the slide wire, which permits settings to 0.001 ohm, and hence were not quite as accurate as those previously given. Nevertheless the agreement between the observed and calculated values is excellent.

Table VII gives the results of measurements made to verify formulæ 22 and 23. Two series of measurements were made. In one a megohm box, consisting of 10 coils of 100,000 ohms each, was used to give resistances varying from 1,000,000 to 12,500 ohms, using various combinations of coils in series and in parallel. In the other a box of 10 coils of 10,000 ohms each was used, the coils being used in series only. In both cases the capacity of the coils produces a distinct effect on the measured value of  $L_0$ , and hence we have assumed formula (23) to be correct and have calculated the values of the capacities of the coils, which will account for the differences observed. They are given in the seventh column, and the capacity of one coil deduced from the measured capacity of the several coils is given in the eighth column. These deduced capacities per coil, averaging 0.00186 microfarad in the case of the second box, agrees quite well with an independent measurement made some months ago by a dynamometer method which gave 0.00195 microfarad for the average. It will be noticed that the changes in  $Q$  are considerable and that the observed and calculated values agree quite closely. These differences, however, can not be determined with great accuracy, as the resistance of the inductive coil (wound with copper wire) varies from time to time; hence  $Q'$  varies from this cause when  $Q$  is constant. The results, however, abundantly justify the formulæ (22 and 23).

#### 10. GRAPHICAL SOLUTION OF THE ANDERSON BRIDGE.

A graphical solution of Anderson's bridge is interesting and shows very simply some of the results derived above analytically. If the bridge is balanced and no current is flowing through the galvanometer, we may consider that the connection  $ED$  is removed. The condition of the bridge is (the same electromotive force acting on the upper half of the bridge from  $A$  to  $B$  as on the lower half) that  $E$  and  $D$  are always at the same potential.

To construct the electromotive-force diagram for the upper half of the bridge, we lay off  $CB$  (fig. 11) to represent the emf. acting on the

arm  $R$  of the bridge. The same electromotive force acts on the branches  $C E B$ , consisting of the resistance  $r$  and the condenser of impedance  $\frac{1}{pC}$ . Therefore, in a semicircle described on  $C B$  as a diameter construct a right-angled triangle  $C E B$ , the sides  $C E$  and  $B E$  being proportional to  $r$  and  $\frac{1}{pC}$ , respectively.  $C E$  is then equal to  $e_3$ , the emf. acting on  $r$ , the fifth arm of the bridge, and  $E B = e_5$ ,

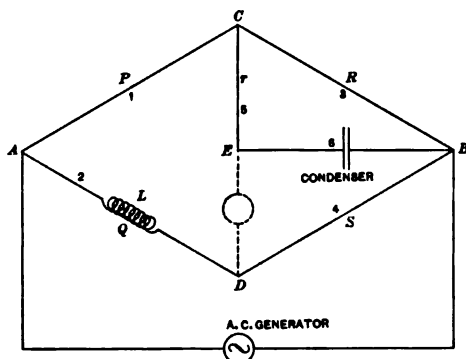


FIG. 10.—Anderson Bridge. When balanced, galvanometer may be removed.

the emf. on the condenser.  $C c'$  and  $C b'$  are the currents in  $r$  and  $R$ , respectively (calculated from the electromotive forces and resistances), and their resultant  $C b'$  equals the current in the arm  $P$ —that is,  $i_1$ . Of course  $i_1$  is also the current through the condenser. The emf. acting on the arm  $P$  is in phase with the current  $C b'$ , since  $P$  is non-inductive. Therefore, if we project  $C b'$  backward to  $A$ , so that

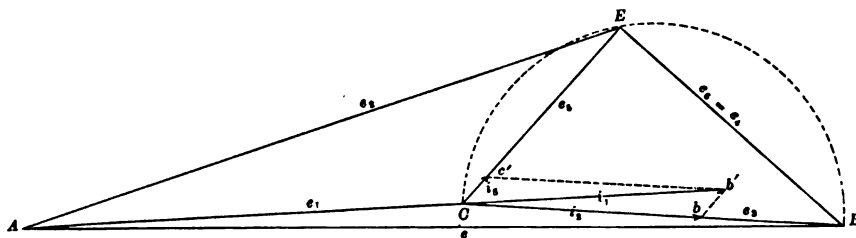


FIG. 11.—Vector diagram of Anderson Bridge.

$CA = i_1 \times P$ ,  $CA$  is the emf.  $e_1$  acting on  $P$ , and the vector sum of  $AC$  and  $CB$ , or  $AB$ , will be the total emf. on the bridge.

Since the lower half of the bridge has the same emf. acting on it and the point  $D$  has the same potential as  $E$ , it is evident that the triangle  $A E B$  is also the emf. triangle of the lower half  $A D B$ . This enables us to find graphically the inductance  $L$  in the branch  $Q$ .

Lay off  $DB$  (fig. 12) equal to  $EB$  (fig. 11), since the same emf. acts on these two branches, which terminate at the common point  $B$ , their initial ends  $E$  and  $D$  having the same potential. Draw lines  $DA$  and  $BA$  so that the triangle  $ADB$  is equal to the triangle  $AEB$  of fig. 11.  $ADB$  is then the emf. triangle of the lower half of the bridge, and  $AD$  is the emf.  $e_s$  expended on the branch  $Q$ . Of this,  $DG$ , perpendicular to  $DB$ , overcomes the reactance  $pL$ , and  $AG$ , perpendicular to  $DG$ , overcomes the resistance  $Q$ . This construction gives  $L$  when  $p$  is known.

But  $p$  need not necessarily be known, as the values of the capacity and resistances of the bridge are independent of  $p$ . The distribution

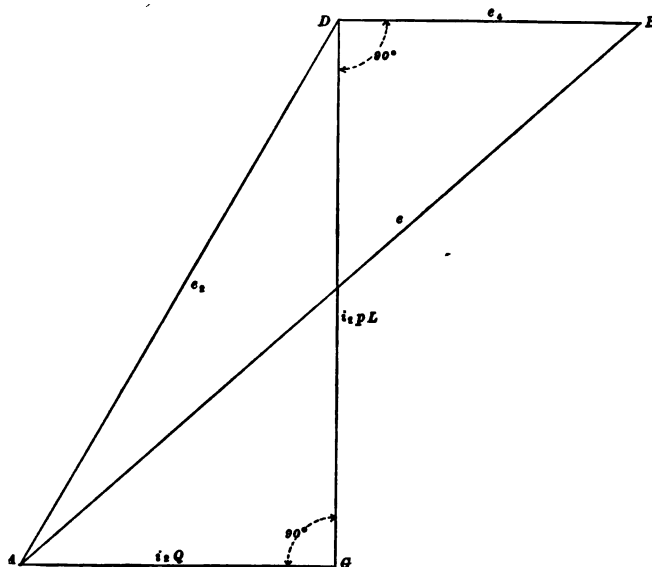


FIG. 12.—Graphical solution of Anderson Bridge.

of electromotive forces is, however, affected by the frequency, and hence the emf. triangle depends on  $p$ . If, however, any convenient value of  $p$  be assumed in constructing figures 11 and 12, the same value of  $L$  will be derived from  $DG$ ; that is,  $DG$  will always come out proportional to  $p$ .

The inductance  $L$ , derived from  $DG$ , is the total inductance of the branch  $Q$ ; hence, if that part of the resistance of  $Q$  not included in the inductance coil to be measured possesses positive or negative inductance ( $l_s$ ), this must be subtracted from the measured value ( $L_0$ ) to obtain the true inductance of the coil  $L$ .

If the branch  $S$  contains inductance  $l_s$ ,  $DBB'$  will be its voltage

triangle, and the angle  $\phi_i$  will be  $\frac{pl_i}{S}$ . The triangle  $ADB$ , which still represents the distribution of voltages, both in the upper and lower

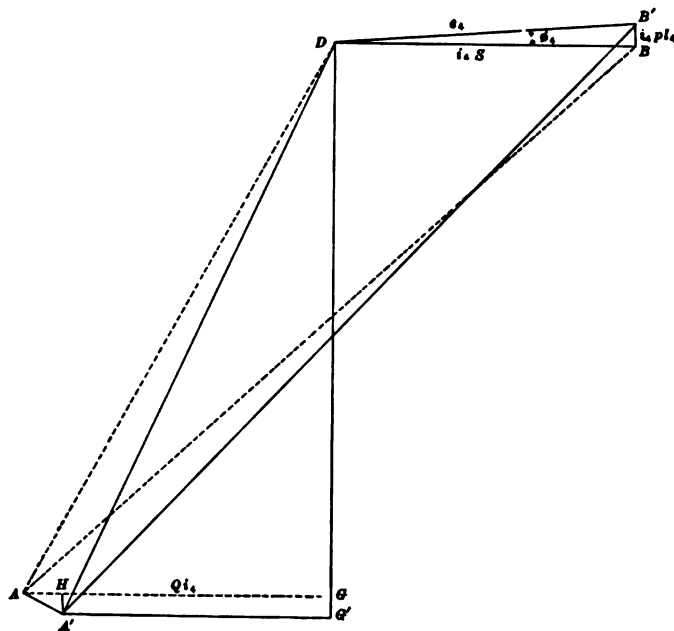


FIG. 13.—Solution when arm  $S$  has inductance  $L_i$ .

halves of the bridge, will therefore be rotated through the angle  $\phi_i$  into the position  $A'DB'$ , where

$$AA' = AD \times \phi_i$$

$$A'H = AA' \sin A'AH = AA' \sin ADG$$

$$\text{Hence, } GG' = AD \times \phi_i \times \frac{AG}{AD} = \phi_i \times Qi_i = \Delta(pL)i_i$$

$$\therefore \Delta L = \frac{Q}{p} \phi_i = \frac{l_i}{S} Q.$$

This is the value of the correction due to  $l_i$  found above analytically and given by equations (11) and (14), neglecting small quantities of the second order occurring in the  $\beta$  term. A similar construction obviously applies to the case of capacity in the resistances; that is, to negative inductance.

#### 11. RESISTANCE IN SERIES AND IN PARALLEL WITH THE CONDENSER.

Fig. 14 is the electromotive-force diagram for the Anderson bridge on which 100 volts is impressed, the frequency being such that



$$L = C S (2r + P) \quad (24)$$



Table IX.—RESULTS OF MEASUREMENTS ON A WIDE RANGE OF COILS TO TEST THE BRIDGE.

| 1   | 2           | 3                  | 4                  | 5                         | 6       | 7       | 8           | 10                         | 11                                  | 12                                    | 13                                  |
|-----|-------------|--------------------|--------------------|---------------------------|---------|---------|-------------|----------------------------|-------------------------------------|---------------------------------------|-------------------------------------|
| No. | Coils.      | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r+P$  | $P=R$   | $S$ and $C$ | $L_0$<br>milli-<br>henrys. | $L=L_0+\alpha$<br>milli-<br>henrys. | $L_0$<br>sums of<br>single<br>values. | $\Delta=L_0-L$<br>micro-<br>henrys. |
| 1   | 0.1 A       | 66.294             | 66.256             | 66.288                    | 382.570 | 250.007 | 260.216     | 100.040                    | 100.040                             | .....                                 | .....                               |
| 2   | 0.1 H       | 66.148             | 66.108             | 66.142                    | 382.277 | .....   | 1.00492 mf  | 99.964                     | 99.964                              | .....                                 | .....                               |
| 3   | A+H         | 257.447            | 257.368            | 257.416                   | 764.825 | .....   | .....       | 199.999                    | 199.998                             | 200.004                               | +6                                  |
| 4   | 0.1 A       | 66.301             | 66.266             | 66.297                    | 382.591 | .....   | 260.216     | 100.046                    | 100.039                             | .....                                 | .....                               |
| 5   | 0.1 B       | 66.220             | 66.184             | 66.215                    | 382.423 | 250.007 | 1.00492 mf  | 100.002                    | 99.998                              | .....                                 | .....                               |
| 6   | A+B         | 257.523            | 257.437            | 257.488                   | 764.969 | .....   | .....       | 200.036                    | 200.034                             | 200.037                               | +3                                  |
| 7   | 50 A        | 11.732             | 11.706             | 11.729                    | 273.451 | .....   | .....       | 50.126                     | 50.119                              | .....                                 | .....                               |
| 8   | 50 B        | 11.465             | 11.442             | 11.464                    | 272.921 | 250.007 | 260.216     | 50.029                     | 50.021                              | .....                                 | .....                               |
| 9   | 50 A+50 B   | 148.191            | 148.138            | 148.174                   | 546.341 | .....   | 0.70445 mf  | 100.149                    | 100.144                             | 100.140                               | -4                                  |
| 10  | 10 M. H. A  | 0.224              | 0.216              | 0.229                     | 100.481 | .....   | 99.084      | 10.0050                    | 10.0052                             | .....                                 | .....                               |
| 11  | 10 M. H. B  | 0.232              | 0.221              | 0.236                     | 100.485 | 100.023 | 1.00492 mf  | 10.0064                    | 10.0066                             | .....                                 | .....                               |
| 12  | A+B         | 50.482             | 50.458             | 50.483                    | 200.989 | .....   | .....       | 20.0128                    | 20.0136                             | 20.0118                               | -1.8                                |
| 13  | 5 M. H. A   | 12.928             | 12.913             | 12.931                    | 125.888 | .....   | 99.084      | 5.0026                     | 5.0029                              | .....                                 | .....                               |
| 14  | 5 M. H. B   | 13.016             | 13.006             | 13.022                    | 126.067 | 100.023 | 0.40106 mf  | 5.0097                     | 5.0037                              | .....                                 | .....                               |
| 15  | A+B         | 75.847             | 75.821             | 75.847                    | 251.717 | .....   | .....       | 10.0029                    | 10.0045                             | 10.0066                               | +2.1                                |
| 16  | 1 M. H. A   | 0.405              | 0.395              | 0.410                     | 100.833 | 100.023 | 99.084      | 1.0033                     | 1.0011                              | .....                                 | .....                               |
| 17  | 1 M. H. B   | 0.520              | 0.515              | 0.528                     | 101.079 | .....   | 0.10042 mf  | 1.0057                     | 1.0036                              | .....                                 | .....                               |
| 18  | A+B         | 50.596             | 50.570             | 50.595                    | 201.213 | .....   | .....       | 2.0022                     | 2.0016                              | 2.0047                                | +3.1                                |
| 19  | 0.5 M. H. A | 0.575              | 0.565              | 0.570                     | 51.17   | .....   | 49.04       | 0.5041                     | 0.5048                              | .....                                 | .....                               |
| 20  | 0.5 M. H. B | 0.480              | 0.475              | 0.478                     | 50.98   | 50.01   | 0.20089 mf  | 0.5023                     | 0.5025                              | .....                                 | .....                               |
| 21  | A+B         | 26.03              | 26.02              | 26.025                    | 102.08  | .....   | .....       | 1.0057                     | 1.0050                              | 1.0073                                | +2.3                                |



The values of  $2r+P$  are given in column 6 of the tables. The temperature of the condenser is given in column 7 of Table VIII, and its capacity at the given temperature in column 8. In Tables IX, X, and XI the temperature of the condenser was kept constant. The values of  $L$ , calculated from equation (24), are given in column 10, and in column 11 are given the corrected values of  $L$ , obtained by applying the value of  $\alpha$ , which is sometimes positive and sometimes negative. In column 12 the sums ( $L_s$ ) of the separate values of  $L$  are given, for comparison with the values opposite to them in column 11, obtained by measuring the coils in groups of two or three. The differences between the values of  $L_s$  and  $L$  are given in column 13. These differences are discrepancies which indicate errors in the measured values of the inductances, but which amount on the average to only five parts in 200,000 for the 100-millihenry coils, and are sometimes positive and sometimes negative. They represent the errors of observation, the errors of the resistances, and the imperfections in the corrections for inductance and capacity of the arms of the bridge. The errors due to differences in  $P$  and  $R$  are eliminated by the commutator; the error due to inductance in  $r$  we have seen to be inappreciable; the condenser is so good that no correction needs to be applied for its very slight absorption and leakage. In the second half of the table the results are given for a remeasurement of the coils under conditions similar to those under which the first measurements were made. It will be noticed that the values agree very closely.

In Table IX the results are given of measurements on coils of 100, 50, 10, 5, 1.0, and 0.5 millihenrys, using 4 of 100 millihenrys and 2 each of the other denominations, measuring them separately and in series in each. The residual differences between the corresponding values in columns 11 and 12, and which amount to a very few microhenrys, are as small as could be expected.

In Table X the results are given of measurements on a coil  $C$  of nominally 100 millihenrys, and of two coils,  $A$  and  $B$ , of 50 millihenrys each, measured together as one coil and in series with the coil  $C$ . The residual differences between the sums of the separate values (column 12) and the measured sum (column 11) are given in column 13. They average about 4 in 200,000, being sometimes positive and sometimes negative. In Table XI similar results are given for the two 50-millihenry coils measured separately and in series, with the residuals in column 13 averaging 6 parts in 100,000.

In both these tables the measurements are made with three different values of the resistances of the bridge ( $P$ ,  $R$ , and  $S$  being equal in every case) and three different currents for each separate value of the

Table X. — RESULTS OF MEASUREMENTS ON A COIL OF 100 MILLIHENRYS AND ON TWO 50 MILLIHENRY COILS IN SERIES.

| 1   | 2       | 3                  | 4                  | 5                         | 6         | 8              | 9                                           | 10                         | 11                                  | 12                                    | 13                                    |
|-----|---------|--------------------|--------------------|---------------------------|-----------|----------------|---------------------------------------------|----------------------------|-------------------------------------|---------------------------------------|---------------------------------------|
| No. | Coils.  | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r+P$    | $P, S$ and $C$ | Total<br>bridge<br>current<br>am-<br>peres. | $L_0$<br>milli-<br>henrys. | $L=L_0+\alpha$<br>milli-<br>henrys. | $L_0$<br>sums of<br>single<br>values. | $\Delta=L_0-L_1$<br>micro-<br>henrys. |
| 1   | C+(A+B) | 278.080            | 278.050            | 278.068                   | 806.214   | .....          | .....                                       | 202.607                    | 202.605                             | 202.597                               | -8                                    |
| 2   | (A+B)   | 74.193             | 74.154             | 74.191                    | 398.400   | P=R=250.018    | 0.12                                        | 100.124                    | 100.120                             | .....                                 | .....                                 |
| 3   | C       | 78.892             | 78.850             | 78.888                    | 407.794   | S=250.233      | .....                                       | 102.485                    | 102.477                             | .....                                 | .....                                 |
| 4   | C+(A+B) | 278.109            | 278.027            | 278.065                   | 806.208   | .....          | .....                                       | 202.614                    | 202.612                             | 202.606                               | -6                                    |
| 5   | (A+B)   | 74.204             | 74.164             | 74.201                    | 398.420   | C=1.00433      | 0.20                                        | 100.130                    | 100.126                             | .....                                 | .....                                 |
| 6   | C       | 78.891             | 78.862             | 78.893                    | 407.804   | .....          | .....                                       | 102.488                    | 102.480                             | .....                                 | .....                                 |
| 7   | C       | 78.883             | 78.856             | 78.887                    | 407.792   | .....          | .....                                       | 102.485                    | 102.476                             | .....                                 | .....                                 |
| 8   | (A+B)   | 74.177             | 74.162             | 74.187                    | 398.392   | .....          | 0.30                                        | 100.122                    | 100.118                             | .....                                 | .....                                 |
| 9   | C+(A+B) | 278.064            | 278.035            | 278.075                   | 806.168   | .....          | .....                                       | 202.604                    | 202.602                             | 202.594                               | -6                                    |
| 10  | C       | 154.872            | 154.831            | 154.868                   | 509.762   | .....          | .....                                       | 102.493                    | 102.469                             | .....                                 | .....                                 |
| 11  | (A+B)   | 149.004            | 148.966            | 149.002                   | 496.030   | P=R=200.026    | 0.12                                        | 100.135                    | 100.114                             | .....                                 | .....                                 |
| 12  | C+(A+B) | 403.837            | 403.750            | 403.806                   | 1,007.638 | S=200.194      | .....                                       | 202.597                    | 202.578                             | 202.583                               | +6                                    |

|    |         |         |         |         |           |             |       |         |         |         |       |
|----|---------|---------|---------|---------|-----------|-------------|-------|---------|---------|---------|-------|
| 13 | C+(A+B) | 403.870 | 403.800 | 403.848 | 1,007.722 | C=1.00433   | ..... | 202.614 | 202.595 | 202.597 | ..... |
| 14 | (A+B)   | 149.020 | 148.986 | 149.020 | 498.066   | .....       | 0.20  | 100.142 | 100.121 | .....   | +2    |
| 15 | C       | 154.881 | 154.856 | 154.885 | 509.796   | .....       | ..... | 102.500 | 102.476 | .....   | ..... |
| 16 | C       | 154.869 | 154.856 | 154.875 | 509.776   | .....       | ..... | 102.496 | 102.472 | .....   | ..... |
| 17 | (A+B)   | 148.987 | 148.988 | 149.005 | 498.036   | .....       | 0.30  | 100.136 | 100.115 | .....   | ..... |
| 18 | C+(A+B) | 403.823 | 403.823 | 403.836 | 1,007.698 | .....       | ..... | 202.609 | 202.590 | 202.587 | -3    |
| 19 | C+(A+B) | 598.680 | 598.750 | 598.743 | 1,343.494 | P=R=150.008 | ..... | 202.588 | 202.584 | 202.582 | -2    |
| 20 | (A+B)   | 256.943 | 256.966 | 256.982 | 683.972   | .....       | 0.30  | 100.119 | 100.114 | .....   | ..... |
| 21 | C       | 264.755 | 264.785 | 264.797 | 679.602   | S=150.138   | ..... | 102.476 | 102.468 | .....   | ..... |
| 22 | C       | 264.740 | 264.760 | 264.777 | 679.562   | .....       | ..... | 102.470 | 102.462 | .....   | ..... |
| 23 | C+(A+B) | 598.680 | 598.677 | 598.706 | 1,343.420 | .....       | 0.20  | 202.572 | 202.569 | 202.573 | +4    |
| 24 | (A+B)   | 256.950 | 256.934 | 256.971 | 683.950   | .....       | ..... | 100.116 | 100.111 | .....   | ..... |
| 25 | (A+B)   | 256.893 | 256.875 | 256.913 | 683.834   | .....       | ..... | 100.098 | 100.094 | .....   | ..... |
| 26 | C+(A+B) | 598.600 | 598.545 | 598.600 | 1,343.208 | .....       | 0.12  | 202.540 | 202.537 | 202.543 | +6    |
| 27 | C       | 264.725 | 264.697 | 264.738 | 679.484   | .....       | ..... | 102.458 | 102.449 | .....   | ..... |

**Table XI.**—RESULTS OF MEASUREMENTS ON TWO INDUCTANCE COILS OF 50 MILLIHENRYS EACH, COILS MEASURED SINGLY AND IN SERIES.

(Temperature of condenser = 24.4°.)

| 1   | 2       | 3                  | 4                  | 5                         | 6         | 8                   | 9                                           | 10                         | 11                                      | 12                                                     | 13                                     |
|-----|---------|--------------------|--------------------|---------------------------|-----------|---------------------|---------------------------------------------|----------------------------|-----------------------------------------|--------------------------------------------------------|----------------------------------------|
| No. | Coils.  | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r + P$  | $P$ and $S$ and $C$ | Total<br>bridge<br>current<br>am-<br>peres. | $L_0$<br>milli-<br>henrys. | $L = L_0 + \alpha$<br>milli-<br>henrys. | $L_0$<br>sums of<br>single<br>values mil-<br>lihenrys. | $\Delta = L_0 - L$<br>micro-<br>henry. |
| 1   | 50A     | 201.705            | 201.766            | 201.785                   | 503.546   | .....               | .....                                       | 50.113                     | 50.111                                  | .....                                                  | .....                                  |
| 2   | 50B     | 201.208            | 201.273            | 201.267                   | 502.550   | P=100.016           | 0.30                                        | 50.014                     | 50.010                                  | .....                                                  | .....                                  |
| 3   | 50A+50B | 452.915            | 453.037            | 452.988                   | 1,005.992 | S= 99.091           | .....                                       | 100.111                    | 100.111                                 | 100.121                                                | +10                                    |
| 4   | 50A+50B | 452.885            | 452.941            | 452.923                   | 1,005.862 | R=100.016           | .....                                       | 100.108                    | 100.104                                 | .....                                                  | .....                                  |
| 5   | 50B     | 201.195            | 201.235            | 201.245                   | 502.506   | .....               | 0.20                                        | 50.010                     | 50.006                                  | .....                                                  | .....                                  |
| 6   | 50A     | 201.675            | 201.725            | 201.732                   | 503.480   | C=1.00433           | .....                                       | 50.106                     | 50.104                                  | 50.110                                                 | + 6                                    |
| 7   | 50A     | 201.645            | 201.645            | 201.675                   | 503.366   | .....               | .....                                       | 50.085                     | 50.082                                  | .....                                                  | .....                                  |
| 8   | 50B     | 201.145            | 201.155            | 201.180                   | 502.376   | .....               | 0.12                                        | 49.986                     | 49.983                                  | .....                                                  | .....                                  |
| 9   | 50A+50B | 452.740            | 452.790            | 452.775                   | 1,005.566 | .....               | .....                                       | 100.074                    | 100.074                                 | 100.085                                                | +11                                    |
| 10  | 50A     | 91.133             | 91.110             | 91.143                    | 332.294   | P=R=150.008         | .....                                       | 50.106                     | 50.100                                  | .....                                                  | .....                                  |
| 11  | 50B     | 90.808             | 90.792             | 90.821                    | 331.650   | .....               | 0.12                                        | 50.009                     | 50.001                                  | .....                                                  | .....                                  |
| 12  | 50A+50B | 256.910            | 256.870            | 256.917                   | 663.842   | S=150.138           | .....                                       | 100.100                    | 100.096                                 | 100.102                                                | + 6                                    |

|    |         |         |         |         |         |             |       |         |       |
|----|---------|---------|---------|---------|---------|-------------|-------|---------|-------|
| 13 | 50A+50B | 256.933 | 256.933 | 256.975 | 663.958 | .....       | ..... | 100.112 | 0     |
| 14 | 50B     | 90.820  | 90.817  | 90.839  | 331.686 | C=1.00433   | 0.20  | 100.112 | ..... |
| 15 | 50A     | 91.148  | 91.140  | 91.165  | 332.338 | .....       | ..... | 100.112 | ..... |
| 16 | 50A     | 91.144  | 91.148  | 91.167  | 332.342 | .....       | ..... | 100.112 | ..... |
| 17 | 50B     | 90.824  | 90.822  | 90.844  | 331.696 | .....       | 0.30  | 100.112 | ..... |
| 18 | 50A+50B | 256.939 | 256.962 | 256.977 | 663.962 | .....       | ..... | 100.115 | + 2   |
| 19 | 50A+50B | 149.003 | 148.989 | 149.013 | 498.052 | P=R=200.026 | ..... | 100.112 | - 8   |
| 20 | 50B     | 24.391  | 24.389  | 24.403  | 248.832 | .....       | 0.30  | 100.112 | ..... |
| 21 | 50A     | 24.636  | 24.628  | 24.645  | 249.316 | S=200.194   | ..... | 100.112 | ..... |
| 22 | 50A     | 24.637  | 24.629  | 24.646  | 249.318 | .....       | ..... | 100.112 | ..... |
| 23 | 50B     | 24.396  | 24.385  | 24.403  | 248.832 | C=1.00433   | 0.20  | 100.112 | ..... |
| 24 | 50A+50B | 149.005 | 148.986 | 149.012 | 498.050 | .....       | ..... | 100.112 | - 7   |
| 25 | 50A+50B | 148.985 | 148.961 | 148.980 | 498.006 | .....       | ..... | 100.114 | + 3   |
| 26 | 50B     | 24.402  | 24.385  | 24.406  | 248.832 | .....       | 0.12  | 100.114 | ..... |
| 27 | 50A     | 24.647  | 24.630  | 24.651  | 249.328 | .....       | ..... | 100.114 | ..... |

**Table XII.**—SUMMARY OF VALUES OF INDUCTANCES SHOWN IN TABLE X, WITH THE DEVIATIONS FROM THE MEAN.

| 1   | 2             | 3                           | 4                              | 5                               | 6                        | 7                               | 8                                  | 9                               |
|-----|---------------|-----------------------------|--------------------------------|---------------------------------|--------------------------|---------------------------------|------------------------------------|---------------------------------|
| No. | $P = R = S$ . | Total<br>current<br>amperes | $(A + B)$<br>milli-<br>henrys. | Devia-<br>tion<br>from<br>mean. | $C$<br>milli-<br>henrys. | Devia-<br>tion<br>from<br>mean. | $C + (A + B)$<br>milli-<br>henrys. | Devia-<br>tion<br>from<br>mean. |
| 1   | 250           | 0.12                        | 100.120                        | 0.005                           | 102.477                  | 0.007                           | 202.605                            | 0.019                           |
| 2   | "             | 0.20                        | 100.128                        | .011                            | 102.480                  | .010                            | 202.612                            | .026                            |
| 3   | "             | 0.30                        | 100.118                        | .003                            | 102.476                  | .006                            | 202.602                            | .016                            |
| 4   | 200           | 0.12                        | 100.114                        | .001                            | 102.469                  | .001                            | 202.578                            | .008                            |
| 5   | "             | 0.20                        | 100.121                        | .006                            | 102.476                  | .006                            | 202.595                            | .009                            |
| 6   | "             | 0.30                        | 100.115                        | 0                               | 102.472                  | .002                            | 202.590                            | .014                            |
| 7   | 150           | 0.12                        | 100.094                        | .021                            | 102.449                  | .021                            | 202.537                            | .049                            |
| 8   | "             | 0.20                        | 100.111                        | .004                            | 102.462                  | .008                            | 202.569                            | .017                            |
| 9   | "             | 0.30                        | 100.114                        | .001                            | 102.468                  | .002                            | 202.584                            | .002                            |
|     |               | Means.                      | 100.115                        | .006                            | 102.470                  | .007                            | 202.586                            | .018                            |

**Table XIII.**—SUMMARY OF VALUES OF INDUCTANCES SHOWN IN TABLE XI, WITH THE DEVIATIONS FROM THE MEAN.

| 1   | 2             | 3                                 | 4                        | 5                               | 6                        | 7                               | 8         | 9                               |
|-----|---------------|-----------------------------------|--------------------------|---------------------------------|--------------------------|---------------------------------|-----------|---------------------------------|
| No. | $P = R = S$ . | Total<br>current<br>am-<br>peres. | $A$<br>milli-<br>henrys. | Devia-<br>tion<br>from<br>mean. | $B$<br>milli-<br>henrys. | Devia-<br>tion<br>from<br>mean. | $A + B$ . | Devia-<br>tion<br>from<br>mean. |
| 1   | 100           | 0.12                              | 50.092                   | 0.012                           | 49.993                   | 0.012                           | 100.074   | 0.033                           |
| 2   | "             | 0.20                              | 50.104                   | .000                            | 50.006                   | .001                            | 100.104   | .003                            |
| 3   | "             | 0.30                              | 50.111                   | .007                            | 50.010                   | .005                            | 100.111   | .004                            |
| 4   | 150           | 0.12                              | 50.100                   | .004                            | 50.001                   | .004                            | 100.096   | .011                            |
| 5   | "             | 0.20                              | 50.106                   | .002                            | 50.006                   | .001                            | 100.112   | .005                            |
| 6   | "             | 0.30                              | 50.107                   | .003                            | 50.008                   | .003                            | 100.113   | .006                            |
| 7   | 200           | 0.12                              | 50.107                   | .003                            | 50.007                   | .002                            | 100.111   | .004                            |
| 8   | "             | 0.20                              | 50.105                   | .001                            | 50.007                   | .002                            | 100.119   | .012                            |
| 9   | "             | 0.30                              | 50.105                   | .001                            | 50.007                   | .002                            | 100.120   | .013                            |
|     |               | Means.                            | 50.104                   | .004                            | 50.005                   | .004                            | 100.107   | .010                            |

resistances. This gives nine measurements of each coil and of the sum of the coils. These nine values of each coil and of their sums are given in Tables XII and XIII, with their deviations from the mean. In most cases the values are a little smaller with the smaller currents and smaller resistances. We have not yet ascertained why this is so; perhaps the resistances were not as accurately known as we supposed.

In Table XIV the measurements of April 21 are given, three coils of 1 henry each being taken singly and in series in groups of two and three.

The separate values found during the day for the three coils are given in Table XV. The small increase in the value of  $L$  may be due in part to uncertainty in the change of capacity of the condenser. The latter changed in temperature, according to the thermometer, by  $0^{\circ}.75$ , and that corresponds to 11 parts in 100,000 in the capacity. The slight progressive changes in the values of the inductances of the coils may be accounted for by a quarter of a degree greater change in the temperature of the condenser than indicated by the thermometer, or a slightly greater temperature coefficient. We shall investigate this further, keeping the temperature of the condenser constant, to decide whether the coils really change in the manner indicated.

**Table XV.**—RESULTS OF THE DETERMINATIONS OF THE INDUCTANCE OF THREE COILS OF 1 HENRY EACH, APRIL 21, 1905.

| Coil F.        | Coil S.        | Coil C.        |
|----------------|----------------|----------------|
| <i>Henrys.</i> | <i>Henrys.</i> | <i>Henrys.</i> |
| 0.99890        | 0.99969        | 1.01420        |
| .99892         | .999715        | 1.01421        |
| .99894         | .99970         | 1.01422        |
| .99895         | .999715        | 1.01422        |
| .99895         | .99972         |                |

The regularity of this progressive change shows that some common cause affects all the measurements, but the changes are very small indeed, amounting to only a few parts in a hundred thousand. The sensitiveness of the bridge is well shown by these results, and if we can eliminate the small residual errors of the bridge completely, it will make it possible to measure inductances with far greater accuracy than has been done heretofore.

In order to make these measurements under the most favorable circumstances, we have designed and are now constructing a bridge espe-

Table XIV.—RESULTS OF MEASUREMENTS ON THREE INDUCTANCE COILS OF 1 HENRY EACH. COILS MEASURED SINGLY AND IN SERIES IN GROUPS OF TWO AND THREE, APRIL 21, 1905.

[ $P = R = 799.819$  ohms.  $S = 800.425$  ohms.]

| 1   | 2      | 3                  | 4                  | 5                         | 6        | 8                                 | 9            | 11                        | 12                                | 13                 |
|-----|--------|--------------------|--------------------|---------------------------|----------|-----------------------------------|--------------|---------------------------|-----------------------------------|--------------------|
| No. | Coils. | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r+P$   | Tempera-<br>ture of<br>condenser. | $C$          | $L$<br>measured<br>value. | $L_s$<br>sum of<br>single values. | $\Delta = L_s - L$ |
|     |        | Ohms.              | Ohms.              | Ohms.                     | Ohms.    | °                                 | Microfarads. | Henry.                    | Henry.                            | Henry.             |
| 1   | F      | 220.953            | 220.963            | 220.960                   | 1241.740 | 19.85                             | 1.00501      | 0.99890                   | .....                             | .....              |
| 2   | C      | 230.472            | 230.470            | 230.474                   | 1280.766 | 19.9                              | 1.005005     | 1.01420                   | .....                             | .....              |
| 3   | F+C    | 851.365            | 851.375            | 851.343                   | 2502.505 | 19.95                             | 1.00500      | 2.01308                   | 2.01310                           | +0.00002           |
| 4   | S      | 221.457            | 221.458            | 221.460                   | 1242.739 | 19.95                             | 1.00500      | 0.99899                   | .....                             | .....              |
| 5   | F      | 220.973            | 220.979            | 220.978                   | 1241.776 | 19.95                             | 1.00500      | 0.99892                   | .....                             | .....              |
| 6   | S+F    | 842.310            | 842.295            | 842.275                   | 2484.369 | .....                             | 1.00500      | 1.99849                   | 1.99861                           | +0.00012           |
| 7   | S      | 221.503            | 221.498            | 221.503                   | 1242.825 | 20.25                             | 1.00495      | 0.999715                  | .....                             | .....              |
| 8   | C      | 230.517            | 230.513            | 230.518                   | 1280.854 | 20.3                              | 1.004945     | 1.01421                   | .....                             | .....              |
| 9   | S+C    | 851.930            | 851.930            | 851.903                   | 2503.625 | 20.3                              | 1.004945     | 2.01387                   | 2.01393                           | +0.00006           |
| 10  | F      | 221.030            | 221.028            | 221.032                   | 1241.882 | 20.35                             | 1.00494      | 0.99894                   | .....                             | .....              |
| 11  | F+S+C  | 1472.833           | 1472.823           | 1472.760                  | 3745.339 | 20.4                              | 1.00493      | 3.01264                   | 3.01287                           | +0.00023           |
| 12  | F+C    | 851.502            | 851.488            | 851.468                   | 2502.755 | 20.45                             | 1.00492      | 2.01313                   | 2.01315                           | +0.00002           |



|    |       |          |          |          |          |      |          |          |         |          |
|----|-------|----------|----------|----------|----------|------|----------|----------|---------|----------|
| 13 | S     | 221.512  | 221.517  | 221.517  | 1242.853 | 20.5 | 1.004915 | 0.99970  | .....   | .....    |
| 14 | F     | 221.051  | 221.048  | 221.052  | 1241.923 | 20.5 | 1.004915 | 0.99895  | .....   | .....    |
| 15 | S+F   | 842.438  | 842.428  | 842.406  | 2484.631 | 20.5 | 1.004915 | 1.99852  | 1.99865 | +0.00013 |
| 16 | C+S   | 851.977  | 851.967  | 851.945  | 2508.709 | 20.6 | 1.00490  | 2.01387  | 2.01394 | +0.00007 |
| 17 | C     | 230.541  | 230.544  | 230.545  | 1260.909 | 20.6 | 1.00490  | 1.01422  | .....   | .....    |
| 18 | S     | 221.528  | 221.531  | 221.531  | 1242.881 | 20.6 | 1.00490  | 0.999715 | .....   | .....    |
| 19 | F+C   | 851.537  | 851.530  | 851.506  | 2502.831 | 20.6 | 1.00490  | 2.01314  | 2.01317 | +0.00003 |
| 20 | F     | 221.056  | 221.058  | 221.060  | 1241.938 | 20.6 | 1.00490  | 0.99895  | .....   | .....    |
| 21 | C     | 230.545  | 230.549  | 230.550  | 1260.918 | 20.6 | 1.00490  | 1.01422  | .....   | .....    |
| 22 | S     | 221.533  | 221.536  | 221.537  | 1242.893 | 20.6 | 1.00490  | 0.99972  | .....   | .....    |
| 23 | F+C+S | 1472.930 | 1472.910 | 1472.852 | 3745.523 | 20.6 | 1.00490  | 3.01270  | 3.01289 | +0.00019 |

cially adapted to this work. The bridge we have been using has been made up of several different resistance boxes. The resistances were all immersed in oil, the temperatures were accurately taken, they were frequently measured against our standards, and every effort made to get precise results. But with all combined in a single set, with shorter and more permanent connections between the arms of the bridge and the residual inductances and capacities eliminated more completely, we hope to improve upon the results given above, and make sure of the value of the inductance of a standard coil to at least one part in ten thousand, barring the uncertainty in the ohm itself. When this has been accomplished we hope to determine some inductance coils, the dimensions of which shall have been accurately determined, and the inductances computed therefrom, in order to obtain a check upon the value of the ohm.

The Anderson bridge, used in the manner we have described above, is admirably adapted to practical use in measuring inductances of a great range of values. Its accuracy far exceeds most of the methods that have been proposed, and it is more convenient than any method we have employed that can compare with it in accuracy.

NOTE.—Since the above was written we have discovered that the serpentine spools on which the inductances  $F$  and  $S$  of 1 henry each and several of 0.1 henry are wound are slightly magnetic, and that their permeability is larger with higher magnetizing force. Consequently the inductances of the coils are not quite constant, but vary when the voltage on the bridge or the resistances of the arms of the bridge are varied. This would cause the sum of two inductances measured separately to be a little greater than when measured together, and this accounts for the discrepancies  $\Delta$  in column 13 of Table XIV, all being of the same sign, and larger when coils  $F$  and  $S$  (both being wound on serpentine spools) are taken together than when  $F$  and  $C$  or  $S$  and  $C$  are taken together,  $C$  being wound on a mahogany spool and therefore not magnetic.

In the case of coils of 0.1 henry or less the current is reduced very little when two are put in series in the bridge, and hence the resultant change in the inductance is much less.

We give in another article in this bulletin the results of tests made to determine the magnitude of the effect of this small amount of magnetic matter in the spools.

## THE USE OF SERPENTINE IN STANDARDS OF INDUCTANCE.

By EDWARD B. ROSA and FREDERICK W. GROVER.

### 1. CONSTRUCTION OF THE STANDARDS.

In 1896 Wien described<sup>a</sup> the construction of a very convenient and permanent form of inductance standard and gave measurements of several such standards by one of his own methods. Spools were turned out of serpentine and wound with insulated copper wire. They were then boiled in paraffin, which completely filled the spaces between the wires, and when cold made a solid mass which preserves its shape permanently. This form of inductance standard has since been manufactured by several of the German instrument makers, and the Bureau of Standards has purchased from Siemens & Halske, R. Franke & Co., and Hartmann & Braun a considerable number of them having values of 1.0, 0.1, 0.01, and 0.001 henry. Coils obtained from different makers agree with one another very closely. Coils of different denominations are reasonably consistent with one another and their values are nearly correct, showing that the makers have accurate standards and adjust the inductances carefully.

In attempting measurements of high precision on these coils, however, we found discrepancies that were difficult to understand until we discovered recently that the serpentine was slightly magnetic and its permeability variable, thus making the inductance of a coil a function of the current passing through it. It happened that the current employed in making the measurements did not ordinarily vary greatly, and hence the discrepancies due to the variation in the permeability of the serpentine were quite small. The small differences noted we first attributed to errors of measurement, but when we undertook a systematic search for the source of these discrepancies we traced it to the coils themselves, and then quickly located it in the varying permeability due to magnetic impurities in the serpentine. Coils wound on mahogany spools show no such effects and, of course, pure marble

<sup>a</sup> Wied. Ann. 58, p. 553.

would be free from it. The question is of so much importance that we give in this paper a considerable number of our tests and measurements in order to show the character of the effects produced and the magnitude of the variations of the inductances.

## 2. MEASUREMENTS OF INDUCTANCES USING DIFFERENT CURRENTS.

Having noticed that the inductance found varied when the resistances of the bridge were changed, we made a series of measurements on the three inductance coils F, C, and S, of one henry each, having a nearly constant voltage on the Anderson bridge, and giving the resist-

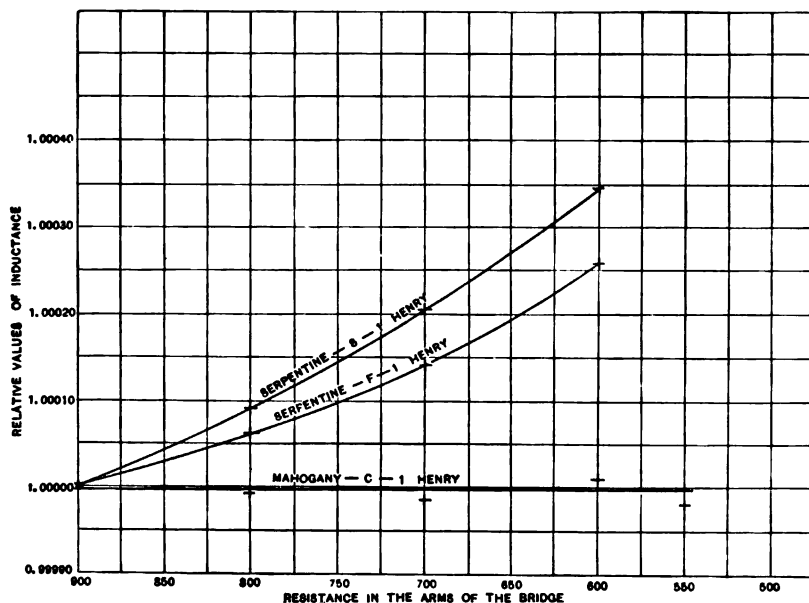


Fig. 1.—Showing the variation of the inductance of the two standards wound on serpentine as the resistances of the (equal) arms of the bridge are varied. Third standard, wound on mahogany spool, remained constant.

ances of the arms of the (square) bridge the successive values 550, 600, 700, 800, and 900 ohms. It was found that the measured inductance of the coil C (wound on mahogany) remained constant within the errors of measurement, the observations varying from the horizontal straight line (fig. 1) by not over 2 parts in 100,000 on the average. On the other hand the two coils wound on serpentine (F, Franke & Co., S, Siemens & Halske) increased steadily as the resistance was decreased, and the current therefore increased. Fig. 1 shows the changes in the relative values of the inductances of these coils.

We next measured the same coils, keeping the resistances of the arms of the bridge constant, and changing the voltage impressed on the bridge. In order to show that these changes were not due, even in part, to the heating of the bridge wires, we have frequently repeated measurements on lower voltages after making measurements at higher voltages. All the resistances were submerged in oil, and the resistances making up the four arms of the bridge were wound with relatively heavy wire having ample carrying capacity. The results of these measurements are given in fig. 2, showing, as before, a constant value for the coil C wound on a mahogany spool, but a steadily increasing value for F and S (on serpentine) as the voltage (and conse-

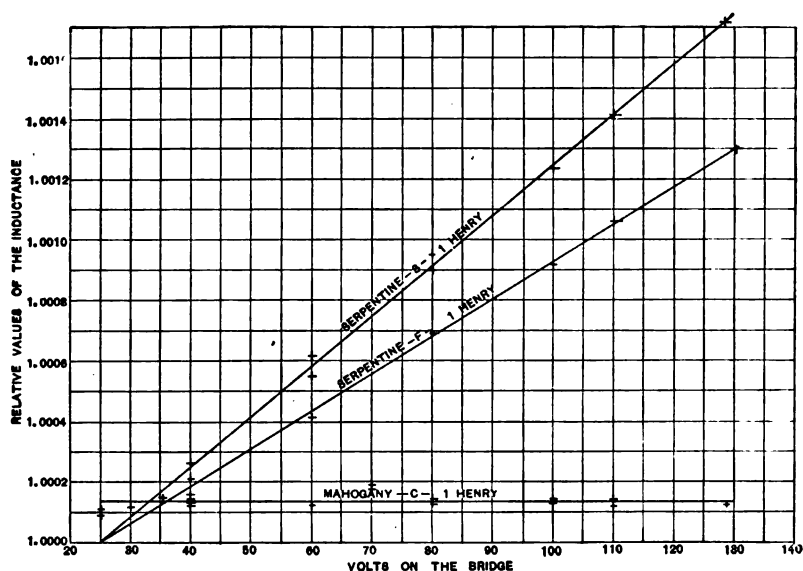


FIG. 2.—Showing variation of serpentine standards as voltage on bridge is varied, resistance remaining constant. Mahogany standard remains unchanged.

quently the current) was increased. In this case the current is carried through a much wider range than before, the maximum being about five times the minimum. The total variation of the inductance of the F coil (while the current is increased in the ratio of 1 to 5) is about 12 parts in 10,000, while the S coil varies 16 parts in 10,000 under the same circumstances. The C coil (on mahogany) varied from the constant value corresponding to the horizontal line in fig. 2 by about 2 parts in 100,000 on the average. The values of the inductances given in figs. 1 and 2 are referred to their lowest value as unity, except the C coil in fig. 2, which is referred to a value 1.0001 as its lowest value for convenience in plotting. Inasmuch as variations of two parts in

100,000 can be detected in the measurements, it would be necessary to keep the current constant within 2 or 3 per cent of its value in using the serpentine inductance, in order that the measured value of the inductance should always be the same. This is, of course, a very awkward condition to impose.

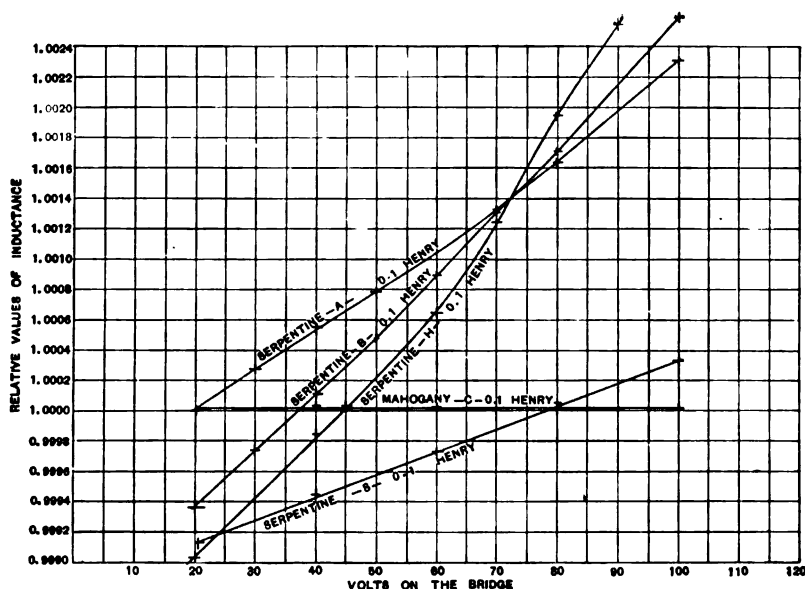


FIG. 3.—Showing variation of inductance of four serpentine 0.1 henry standards. Mahogany standard remains unchanged.

Fig. 3 gives the results of measurements made on five coils of 0.1 henry, each, as follows:

*Schedule of inductance coils, each of 0.1 henry.*

- A = Inductance coil from Franke & Co., on serpentine.
- B = Inductance coil from Franke & Co., on serpentine.
- C = Inductance coil from Carpentier, on mahogany.
- H = Inductance coil from Hartmann & Braun, on serpentine.
- S = Inductance coil from Siemens & Halske, on serpentine.

The results agree with the measurements made on the three coils of 1.0 henry each, in that the mahogany spool has a constant value as the current was increased in the ratio one to five, whereas the other four on serpentine all increased. The four coils, A, B, H, and S, in



FIG. 6.—FIVE SERPENTINE AND TWO MAHOAGANY INDUCTANCE STANDARDS.

The two larger serpentine spools are those of Figs. 1 and 2. The three smaller serpentine spools and the mahogany spool at the right are those of Fig. 3.





fig. 3, are all referred to the lowest value of  $A$  as unity; hence the differences shown in the figure are the actual differences of the coils. The fifth coil  $C$  (on mahogany) has so large a value (nearly 2.5 per cent greater than the others) that it would fall more than a meter above the others if plotted on the same scale. Hence it has been reduced to the lowest value of  $A$  as its initial value for comparison. The five determinations of the inductance of this coil at 20, 40, 60, 80, and 100 volts on the bridge lie very near the horizontal line, the mean variations being not over 2 parts in 100,000.

On the other hand, the four coils wound on serpentine increase largely as the current is increased, the current being of course proportional to the voltage on the bridge. The amount of increase varies from 12 parts in 10,000 for the  $S$  coil to 23 for  $A$ , 31 for  $B$ , and 40 for  $H$ . Since the inductance of a coil is proportional to the average permeability of its magnetic circuit, and the principal magnetic reluctance is in the region occupied by the spool, it is evident that the curves of fig. 3 represent approximately the permeabilities of the four serpentine spools. One is straight throughout, two are slightly curved, and the fourth,  $H$  is decidedly curved, showing a rapidly increasing permeability as the current is increased. They are all made from serpentine having the same appearance, but  $S$  is a relatively large spool in which the strength of field within the spool is relatively smaller, whereas  $H$  is the smallest of the four spools and therefore has the largest number of turns of wire and the strongest field, the inductances of the four spools being equal. The range of inductance in the serpentine spool  $S$  therefore corresponds to the lower end only of  $H$ ,  $A$ , and  $B$ , which are straight lines. We may therefore conclude that if the spools were all as large as  $S$ , the lines would all be sensibly straight, and conversely if all were as small as  $H$  they would be curved in much the same way.

It is a coincidence that three curves intersect at one point. Thus there is one value of the current for which these three coils have the same inductance. At lower current,  $H$  has the smallest value, and at larger currents it has the largest value. The coil  $B$  is intermediate in value both at large and small values of the current.

This explains discrepancies in the relative values of these coils which we have often noticed. That is, comparisons by bridge methods, which may be made with great precision, have not given the same ratios at different times, and we were at a loss to account for the fluctuating ratios. It may be that other observers have been troubled in the same way, for serpentine inductance standards are considerably used.

### 3. INDUCTANCE OF A COIL INCREASED BY PROXIMITY OF SERPENTINE.

In order to demonstrate further the fact that the serpentine spools are appreciably magnetic, we made a series of measurements on the same inductance coils, with one coil placed directly on another, or between two others, so that the reluctance of its magnetic circuit might be reduced by the magnetic serpentine. The following are some of the results obtained:

(a) Serpentine henry placed on (or under) mahogany henry increases the value of the latter by 2.3 millihenrys.

(b) On the other hand, mahogany henry placed on (or under) a serpentine henry makes no appreciable change in the inductance of the latter.

(c) Serpentine henry placed on a serpentine henry increases the inductance of the latter. For example, S increases F 2.4 millihenrys, and F increases S 2.2 millihenrys. F, between two serpentines, is increased 4.1 mh, and S, between two, is increased 3.5 mh.

(d) Serpentine 0.1 henry on serpentine 0.1 henry is increased about 0.25 mh, and on serpentine 1.0 henry about 1.7 mh, on an average. On 0.01 henry it is increased about 0.2 mh, and on 0.001 henry about the same.

The above results show that the smaller inductance coils also are magnetic and can affect the magnetic circuit of other coils if near them.

### 4. EFFECT OF MAGNETIC SERPENTINE IMITATED BY IRON FILINGS OR WIRE.

In order to illustrate further the effect of magnetic material in the spools of inductance coils, we have made a number of measurements on the coils with iron wire or iron filings in or near them.

(a) With a single piece of soft iron wire, 4 cm long, 1.0 mm in diameter, inserted in the axis of the mahogany spool, the inductance was increased over 2 per cent, as shown in (a) fig. 4, the form of the curve showing that the permeability of the circuit increased to a maximum and then decreased as the current in the coil increased.

(b) With six pieces of iron wire, each 5 cm long, arranged radially on the bottom of the mahogany spool (inverted), the inductance increased over 1 per cent, as shown in (b) fig. 4. In this case the permeability did not reach its maximum, the wires not being in a strong magnetic field, as in case (a). As the voltage was increased from 60 to 100, the inductance changed nearly 3 parts in 1,000, which is much greater than the change in the case of the serpentine coils.

(c) A piece of this wire 2 mm long (weighing 21 mg) inside the coil produced a sensible change in the inductance of the coil; that is, 5 parts in 100,000. A few particles of iron (weighing 9 mg) increased the inductance by 2 parts in 100,000. This is only one part iron to over a hundred thousand of the spool, if the latter were serpentine.

(d) Iron filings, sprinkled over the wooden spool, also increased the inductance, fig. 5; 2 grams of the filings increasing the inductance initially by 1 part in 800, the amount increasing further with the current, as is the case with the serpentine.

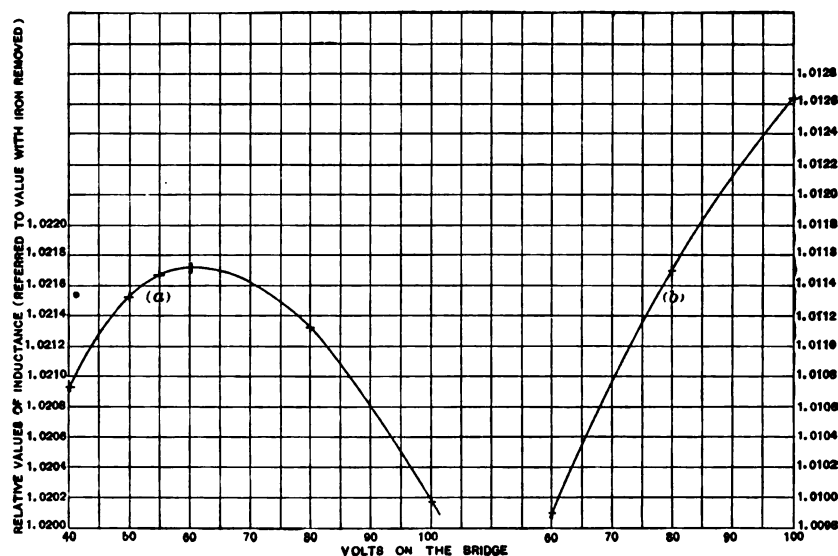


FIG. 4.—Showing change of inductance of standards; (a) when a piece of iron wire is inserted in axis of coil; (b) when six pieces of iron wire are placed radially outside the spool.

(e) Iron filings placed in the center of the F serpentine spool (8.2 grams filled a hole along the axis of the spool) increased the inductance initially by 70 parts in 10,000, and this increased further by 17 parts in 10,000 as the voltage was carried up from 40 to 120 volts. This is nearly double the rate of increase of the inductance that occurs without the filings. These experiments show that the behavior of the serpentine spools is fully explained by supposing them to contain a very small amount of iron.

##### 5. MAGNETOMETER AND CHEMICAL TESTS.

The serpentine inductances were also tested with a magnetometer, and found to be magnetic. By placing one of the serpentine spools

Table I.—RESULTS OF MEASUREMENTS ON THREE INDUCTANCE COILS OF 1 HENRY EACH; COILS MEASURED SINGLY AND IN SERIES,  
JUNE 9, 1905.

[Current through coils=0.06 ampere for first set, 0.045 ampere for second set.]

| No. | Coils. | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r+P$   | $P, R, S,$ and $C$ | Voltage<br>on<br>bridge. | $L_0$<br>henrys. | $L=L_0+\alpha$<br>henrys. | $L_0$<br>sum of<br>single<br>values. | $\Delta=L_0-L$<br>henrys. |
|-----|--------|--------------------|--------------------|---------------------------|----------|--------------------|--------------------------|------------------|---------------------------|--------------------------------------|---------------------------|
| 1   | S      | 528.996            | 529.146            | 529.117                   | 1658.122 | $P=R=599.888$      | 69.75                    | 1.00043          | 1.00041                   | .....                                | .....                     |
| 2   | F      | 528.202            | 528.332            | 528.313                   | 1656.514 | $S=600.505$        | 69.7                     | 0.99946          | 0.99945                   | .....                                | .....                     |
| 3   | S+F    | 1357.268           | 1357.550           | 1357.450                  | 3314.788 | $C=1.00474$        | 93.0                     | 1.99998          | 1.99996                   | 1.99986                              | -0.00010                  |
| 4   | F      | 528.195            | 528.341            | 528.314                   | 1656.516 | .....              | 69.7                     | 0.99946          | 0.99945                   | .....                                | .....                     |
| 5   | C      | 540.358            | 540.490            | 540.470                   | 1680.828 | .....              | 70.0                     | 1.01413          | 1.01412                   | .....                                | .....                     |
| 6   | F+C    | 1368.606           | 1368.910           | 1368.799                  | 3337.486 | .....              | 93.4                     | 2.01367          | 2.01365                   | 2.01357                              | -0.00008                  |
| 7   | S      | 529.035            | 529.148            | 529.137                   | 1658.162 | .....              | 69.75                    | 1.00046          | 1.00043                   | .....                                | .....                     |
| 8   | C      | 540.370            | 540.493            | 540.478                   | 1680.844 | .....              | 70.0                     | 1.01414          | 1.01413                   | .....                                | .....                     |
| 9   | S+C    | 1369.435           | 1369.708           | 1369.413                  | 3339.114 | .....              | 93.4                     | 2.01466          | 2.01463                   | 2.01456                              | -0.00007                  |
| 10  | S      | 528.997            | 529.146            | 529.117                   | 1658.122 | .....              | 69.75                    | 1.00044          | 1.00042                   | .....                                | .....                     |
| 11  | F      | 528.198            | 528.328            | 528.309                   | 1656.506 | .....              | 69.7                     | 0.99946          | 0.99944                   | .....                                | .....                     |
| 12  | S+F    | 1357.252           | 1357.543           | 1357.439                  | 3314.766 | .....              | 93.4                     | 1.99997          | 1.99993                   | 1.99986                              | -0.00007                  |

|    |       |          |          |          |          |       |         |         |                  |
|----|-------|----------|----------|----------|----------|-------|---------|---------|------------------|
| 13 | C     | 540.357  | 540.501  | 540.475  | 1080.838 | 70.0  | 1.01414 | 1.01412 | .....            |
| 14 | S     | 528.989  | 529.130  | 529.111  | 1058.110 | 69.75 | 1.00042 | 1.00040 | .....            |
| 15 | F     | 528.187  | 528.328  | 528.304  | 1056.496 | 69.7  | 0.99945 | 0.99944 | .....            |
| 16 | F+C   | 1368.590 | 1368.910 | 1368.791 | 3337.470 | 93.4  | 2.01368 | 2.01366 | 2.01356 -0.00010 |
| 17 | S+C   | 1368.422 | 1369.715 | 1369.609 | 3339.106 | 93.4  | 2.01465 | 2.01461 | 2.01452 -0.00009 |
| 18 | F     | 221.140  | 221.146  | 221.168  | 1242.246 | 78.8  | 0.99934 | 0.99932 | .....            |
| 19 | C     | 230.327  | 230.341  | 230.359  | 1280.628 | 79.0  | 1.01412 | 1.01410 | .....            |
| 20 | F+C   | 851.442  | 851.490  | 851.523  | 2502.956 | 96.6  | 2.01352 | 2.01350 | 2.01342 -0.00008 |
| 21 | S     | 221.712  | 221.729  | 221.745  | 1243.400 | 78.8  | 1.00026 | 1.00024 | .....            |
| 22 | F     | 221.125  | 221.144  | 221.159  | 1242.228 | 78.8  | 0.99932 | 0.99930 | .....            |
| 23 | S+F   | 842.838  | 842.894  | 842.928  | 2485.766 | 96.3  | 1.99969 | 1.99966 | 1.99954 -0.00012 |
| 24 | C     | 230.315  | 230.336  | 230.351  | 1280.612 | 79.0  | 1.01411 | 1.01410 | .....            |
| 25 | S+C   | 852.042  | 852.062  | 852.114  | 2504.138 | 96.6  | 2.01447 | 2.01444 | 2.01432 -0.00012 |
| 26 | S     | 221.696  | 221.726  | 221.736  | 1243.382 | 78.8  | 1.00025 | 1.00022 | .....            |
| 27 | F     | 221.111  | 221.131  | 221.146  | 1242.202 | 78.8  | 0.99930 | 0.99928 | .....            |
| 28 | S+F+C | 1473.257 | 1473.308 | 1473.336 | 3746.582 | 119.4 | 3.01398 | 3.01394 | 3.01360 -0.00034 |

in a magnetizing solenoid its permanent magnetism could be reversed so that, when it was removed to the magnetometer, the deflection of the latter was reversed. The normal permanent magnetism of the spools was probably due to the earth's magnetic field.

Chemical analysis shows that the serpentine of one of the spools contained 0.6 per cent of iron, which is chiefly in the form of magnetite.

The serpentine inductance coils of 1 henry each were measured alone and in series, increasing the voltage on the bridge when they were in series, so that the current through the coils was nearly the same then as when they were measured alone. This prevented the

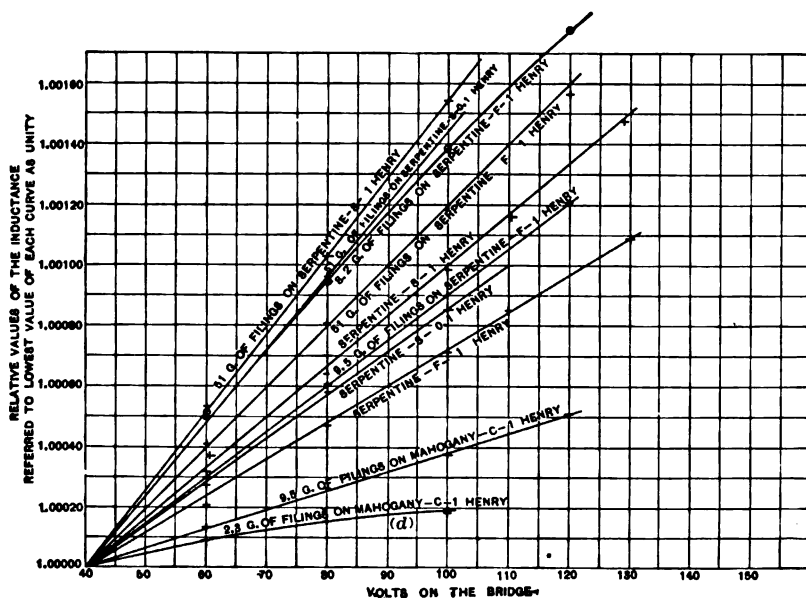


FIG. 5.—Showing effect of iron filings in imitating or increasing the effect of magnetic impurities in the serpentine.

[NOTE.—The curve "8.2 g of filings on serpentine F 1 henry" was obtained with the filings placed *inside* the coil.]

small decrease in value which had formerly resulted when the coils were measured in series, and the residual differences were this time found to be negative and nearly constant, whereas they had before been positive and variable, being smaller with the sum of the mahogany and serpentine than for two serpentines, Table I. This constant negative difference of about 8 parts in 200,000 might perhaps have been due to errors in the resistances; at any rate it is not the same as the residuals due to the variability in the inductance arising from the magnetic material in the serpentine spools.

In Table II we give a summary of the separate measurements of the three coils, taken from Table I. It will be seen that the different determinations of the coils agree very closely, showing that the accidental errors of measurement are very small. The variations would

**Table II.**—SUMMARY OF MEASUREMENTS OF THE INDUCTANCE GIVEN IN TABLE I.

| 1   | 2                             | 3                     | 4               | 5               |
|-----|-------------------------------|-----------------------|-----------------|-----------------|
| No. | Current in coils,<br>amperes. | Measured inductances. |                 |                 |
|     |                               | Coil F, henrys.       | Coil S, henrys. | Coil C, henrys. |
| 1   | 0.05                          | 0.99945               | 1.00041         | 1.01412         |
| 2   | 0.05                          | 0.99945               | 1.00043         | 1.01413         |
| 3   | 0.05                          | 0.99944               | 1.00042         | 1.01412         |
| 4   | 0.05                          | 0.99944               | 1.00040         |                 |
| 5   | 0.045                         | 0.99932               | 1.00024         | 1.01410         |
| 6   | 0.045                         | 0.99930               | 1.00022         | 1.01410         |
| 7   | 0.045                         | 0.99928               |                 |                 |

probably have been greater if the resistances had been varied, on account of variations in the resistances. The last three measurements with a little smaller current are, of course, smaller for coils F and S (serpentine spools) than the first four, but the coil C (mahogany spool) remains substantially the same throughout. In an article in Bulletin No. 2 (issued February 1, 1905), on the Absolute Measurement of Inductance, we called attention to some slight discrepancies which we were at a loss to understand. We now know that they were due to the magnetic impurities in the serpentine spools on which one of the coils was wound.

Hartmann and Braun exhibited at St. Louis some inductance coils wound on white marble. These would undoubtedly be entirely free from magnetism. Wooden spools are cheaper and when the wood is thoroughly seasoned seem to be quite permanent. This question we are investigating further. We have ordered some of these inductance standards wound on marble from Hartmann & Braun, and have designed some to be built for us in this country. When these standards have been thoroughly tested for stability and constancy with

different testing currents we shall determine their values by several different methods in order to ascertain what sensibility and how close agreement may be found among entirely different methods.



## THE SILVER COULOMETER.

By K. E. GUTHE.

1. According to Faraday's law of electrolysis, a strict proportionality exists between the quantity of electricity passing through an electrolyte and the electrochemical reaction produced by it. The latter may therefore serve for the measurement of quantity of electricity. We call the instruments employed for this purpose "voltameters" or "coulometers."

A good many different types of coulometers have been used; for example, the gas coulometer, in which the volume of the liberated gases is measured, or the iodine and the iron coulometers, in which the electrochemical change is measured by titration. The usual method, however, is the determination of the mass of a substance deposited at one of the electrodes by an electric current.<sup>a</sup> Of the last the copper coulometer and the silver coulometer are the best-known types.

The investigations of F. and W. Kohlrausch,<sup>b</sup> Rayleigh and Sidgwick,<sup>c</sup> Gray,<sup>d</sup> Schuster and Crossley,<sup>e</sup> and Glazebrook and Skinner,<sup>f</sup> have proved that the silver coulometer is by far the most reliable instrument of this sort and that it will give results accurate to 1 in 5,000 if certain specifications as to its construction and treatment are closely followed.

<sup>a</sup> A description of some unusual types is given by Danneel: *ZS. f. Electroch.*, **4**, p. 154; 1897.

<sup>b</sup> Fr. and W. Kohlrausch: *Wied. Ann.*, **27**, p. 1; 1886.

<sup>c</sup> Rayleigh and Sidgwick: *Phil. Trans.*, **175**, p. 111; 1884.

<sup>d</sup> Gray: *Phil. Mag.*, **22**, p. 389; 1886.

<sup>e</sup> Schuster and Crossley: *Proc. Roy. Soc.*, **50**, p. 344; 1892.

<sup>f</sup> Glazebrook and Skinner: *Phil. Trans.*, **183**, p. 567; 1892.

2. For this reason the International Electrical Congress, held at Chicago in 1893, adopted the silver coulometer as a standard for the measurement of electric current. Though the ampere was defined as one-tenth of the unit of current of the c. g. s. system of electromagnetic units, it was added that it "is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of 0.001118 gram per second." This value is called the electrochemical equivalent of silver.

Specifications, as referred to above, were prepared by the National Academy of Sciences and legalized in the United States in 1894. They are practically in conformity with the earlier specifications of the British Board of Trade and read as follows:

In employing the silver voltameter to measure currents of about one ampère the following arrangements shall be adopted:

The cathode on which the silver is to be deposited shall take the form of a platinum bowl not less than 10 cm. in diameter and from 4 to 5 cm. in depth.

The anode shall be a disk or plate of pure silver some 30 sq. cm. in area and 2 or 3 mm. in thickness.

This shall be supported horizontally in the liquid near the top of the solution by a silver rod riveted through its center. To prevent the disintegrated silver which is formed on the anode from falling upon the cathode, the anode shall be wrapped around with pure filter paper, secured at the back by suitable folding.

The liquid shall consist of a neutral solution of pure silver nitrate containing about 15 parts, by weight, of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

*Method of making a measurement.*—The platinum bowl is to be washed consecutively with nitric acid, distilled water, and absolute alcohol. It is then to be dried at 160° C. and left to cool in a desiccator. When thoroughly cool it is to be weighed carefully.

It is to be nearly filled with the solution and connected to the rest of the circuit by being placed on a clean insulated copper support to which a binding screw is attached.

The anode is then to be immersed in the solution so as to be well covered by it and supported in that position. The connections to the rest of the circuit are then to be made.

Contact is to be made at the key, noting the time. The current is to be allowed to pass for not less than half an hour and the time of breaking contact observed.

The solution is now to be removed from the bowl and the deposit washed with distilled water and left to soak for at least six hours. It is then to be rinsed successively with distilled water and absolute alcohol and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is to be weighed again. The gain in mass gives the silver deposited.

To find the time average of the current in amperes this mass, expressed in grams, must be divided by the number of seconds during which the current has passed and by 0.001118.

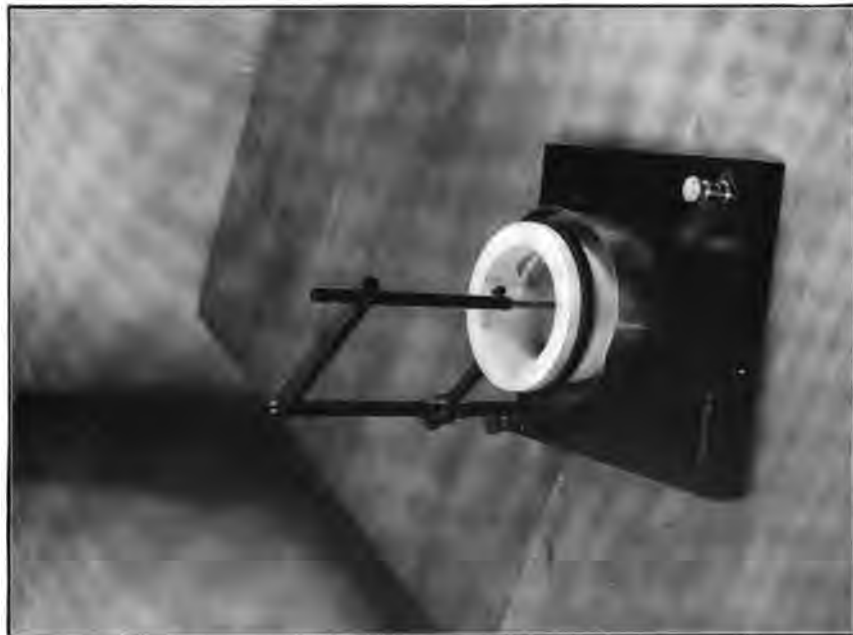


FIG. 2.—POROUS CUP SILVER COULOMETER.



FIG. 2.—POROUS CUP SILVER COULOMETER, SHOWING PARTS.



In determining the constant of an instrument by this method the current should be kept as nearly uniform as possible and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time average of the current) can be found. The current, as calculated from the voltameter results, corresponds to this reading.

The current used in this experiment must be obtained from a battery and not from a dynamo, especially when the instrument to be calibrated is an electro-dynamometer.

Other countries in which the silver coulometer has been legalized have adopted similar rules.<sup>a</sup>

3. The minute description of the form and treatment of the silver coulometer implies that a slight departure from the specifications may result in a deposit of silver not in accordance with the definition of the ampere. It was apparent that there are in the instrument disturbing factors which required a further study.

Novák,<sup>b</sup> who worked with the Kohlrausch form—i. e., a coulometer in which a small glass bowl is suspended below the anode in order to prevent any particles from falling upon the cathode bowl—describes some of the difficulties encountered and strongly emphasizes the necessity for a thorough study of the side reactions, such as the formation of the anode slime and the possible existence of a complex silver salt in the electrolyte.

Kahle<sup>c</sup> has shown that the electrolyte becomes acid by electrolysis and that on repeated use the deposits are too large. The Reichsanstalt recommends, therefore, that not more than 3 grams of silver should be deposited from 100 cm<sup>3</sup> of the solution.

Leduc<sup>d</sup> hopes to overcome all trouble by employing a large silver anode, consisting of granulated silver. He keeps the anodic current density below 0.02 ampere per cm<sup>2</sup> and recommends that the amount of silver collected at the cathode should be large, say about 30 grams.

Patterson and Guthe<sup>e</sup> obtained concordant results by keeping the solution in contact with silver oxide. Leduc speaks in favor of this method, but Richards's results with the same type show great variations in the amount of silver deposited by the same quantity of electricity.

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<sup>a</sup> The Electrician, 27, p. 325; 1891. ZS. f. Instrk., 21, p. 180; 1901.

<sup>b</sup> Novák: Studie o voltmetru na stříbro, Proc. Roy. Bohemian Ac. Sci., Prague, 1, pp. 387-432; 1892. This paper seems to have been published in the Bohemian language only, and I failed to find any reference to it in the literature on the subject. I am indebted to Mr. C. M. Jansky for a translation of it.

<sup>c</sup> Kahle: ZS. f. Instrk., 18, pp. 229 and 267; 1898.

<sup>d</sup> Leduc: J. de Phys., 1, p. 561; 1902.

<sup>e</sup> Patterson and Guthe: Phys. Rev., 7, p. 257; 1898.

Table I.—RESULTS OF MEASUREMENTS ON THREE INDUCTANCE COILS OF 1 HENRY EACH; COILS MEASURED SINGLY AND IN SERIES,  
JUNE 9, 1905.

[Current through coils=0.05 ampere for first set, 0.045 ampere for second set.]

| No. | Coils. | $r$<br>position 1. | $r$<br>position 2. | $r$<br>mean<br>corrected. | $2r+P$   | $P, R, S,$ and $C$ | Voltage<br>on<br>bridge. | $L_0$<br>henrys. | $L=L_0+\alpha$<br>henrys. | $L_0$<br>sum of<br>single<br>values. | $\Delta=L_0-L$<br>henrys. |
|-----|--------|--------------------|--------------------|---------------------------|----------|--------------------|--------------------------|------------------|---------------------------|--------------------------------------|---------------------------|
| 1   | S      | 528.996            | 529.146            | 529.117                   | 1658.122 | $P=R=599.888$      | 69.75                    | 1.00043          | 1.00041                   | .....                                | .....                     |
| 2   | F      | 528.202            | 528.332            | 528.313                   | 1656.514 | $S=600.505$        | 69.7                     | 0.99946          | 0.99945                   | .....                                | .....                     |
| 3   | S+F    | 1357.268           | 1357.550           | 1357.450                  | 3314.788 | $C=1.00474$        | 93.0                     | 1.99998          | 1.99996                   | 1.99986                              | -0.00010                  |
| 4   | F      | 528.195            | 528.341            | 528.314                   | 1656.516 | .....              | 69.7                     | 0.99946          | 0.99945                   | .....                                | .....                     |
| 5   | C      | 540.358            | 540.490            | 540.470                   | 1680.828 | .....              | 70.0                     | 1.01413          | 1.01412                   | .....                                | .....                     |
| 6   | F+C    | 1368.606           | 1368.910           | 1368.799                  | 3337.486 | .....              | 93.4                     | 2.01367          | 2.01365                   | 2.01357                              | -0.00008                  |
| 7   | S      | 529.035            | 529.148            | 529.137                   | 1658.162 | .....              | 69.75                    | 1.00046          | 1.00043                   | .....                                | .....                     |
| 8   | C      | 540.370            | 540.493            | 540.478                   | 1680.844 | .....              | 70.0                     | 1.01414          | 1.01413                   | .....                                | .....                     |
| 9   | S+C    | 1369.435           | 1369.708           | 1369.413                  | 3339.114 | .....              | 93.4                     | 2.01468          | 2.01463                   | 2.01456                              | -0.00007                  |
| 10  | S      | 528.997            | 529.146            | 529.117                   | 1658.122 | .....              | 69.75                    | 1.00044          | 1.00042                   | .....                                | .....                     |
| 11  | F      | 528.198            | 528.328            | 528.309                   | 1656.506 | .....              | 69.7                     | 0.99946          | 0.99944                   | .....                                | .....                     |
| 12  | S+F    | 1357.252           | 1357.543           | 1357.439                  | 3314.766 | .....              | 93.4                     | 1.99997          | 1.99993                   | 1.99986                              | -0.00007                  |

|    |       |          |          |          |          |                              |       |         |         |                  |
|----|-------|----------|----------|----------|----------|------------------------------|-------|---------|---------|------------------|
| 13 | C     | 540.357  | 540.501  | 540.475  | 1680.838 | .....                        | 70.0  | 1.01414 | 1.01412 | .....            |
| 14 | S     | 528.999  | 529.130  | 529.111  | 1658.110 | .....                        | 69.75 | 1.00042 | 1.00040 | .....            |
| 15 | F     | 528.187  | 528.328  | 528.304  | 1656.496 | .....                        | 69.7  | 0.99945 | 0.99944 | .....            |
| 16 | F+C   | 1368.590 | 1368.910 | 1368.791 | 3337.470 | .....                        | 93.4  | 2.01368 | 2.01366 | 2.01356 —0.00010 |
| 17 | S+C   | 1369.422 | 1369.715 | 1369.609 | 3339.106 | .....                        | 93.4  | 2.01465 | 2.01461 | 2.01452 —0.00009 |
| 18 | F     | 221.140  | 221.146  | 221.168  | 1242.246 | .....                        | 78.8  | 0.99934 | 0.99932 | .....            |
| 19 | C     | 230.327  | 230.341  | 230.359  | 1260.628 | .....                        | 79.0  | 1.01412 | 1.01410 | .....            |
| 20 | F+C   | 851.442  | 851.490  | 851.523  | 2502.956 | $P=R=799.910$<br>$S=800.662$ | 96.6  | 2.01352 | 2.01350 | 2.01342 —0.00008 |
| 21 | S     | 221.712  | 221.729  | 221.745  | 1243.400 | $C=1.00474$                  | 78.8  | 1.00026 | 1.00024 | .....            |
| 22 | F     | 221.125  | 221.144  | 221.159  | 1242.228 | .....                        | 78.8  | 0.99932 | 0.99930 | .....            |
| 23 | S+F   | 842.838  | 842.894  | 842.928  | 2485.766 | .....                        | 96.3  | 1.99969 | 1.99966 | 1.99954 —0.00012 |
| 24 | C     | 230.315  | 230.336  | 230.351  | 1260.612 | .....                        | 79.0  | 1.01411 | 1.01410 | .....            |
| 25 | S+C   | 852.042  | 852.062  | 852.114  | 2504.138 | .....                        | 96.6  | 2.01447 | 2.01444 | 2.01432 —0.00012 |
| 26 | S     | 221.696  | 221.726  | 221.736  | 1243.382 | .....                        | 78.8  | 1.00025 | 1.00022 | .....            |
| 27 | F     | 221.111  | 221.131  | 221.146  | 1242.202 | .....                        | 78.8  | 0.99930 | 0.99928 | .....            |
| 28 | S+F+C | 1473.257 | 1473.308 | 1473.336 | 3746.582 | .....                        | 119.4 | 3.01398 | 3.01394 | 3.01360 —0.00034 |

TABLE I.  
EXPERIMENT A.

| Time.        | Temperature. | Current,<br>in amperes. | Pot. diff.,<br>in volts. | Resistance,<br>in ohms. |
|--------------|--------------|-------------------------|--------------------------|-------------------------|
| <i>h. m.</i> |              |                         |                          |                         |
| 1 15         | 21.2         | 0.378                   | 1.860                    | 4.921                   |
| 16           | 21.2         | .380                    | 1.854                    | 4.879                   |
| 17           | 21.2         | .381                    | 1.845                    | 4.843                   |
| 18           | 21.4         | .3775                   | 1.820                    | 4.821                   |
| 20           | 21.6         | .393                    | 1.885                    | 4.797                   |
| 22           | 21.8         | .395                    | 1.882                    | 4.765                   |
| 25           | 22.0         | .3975                   | 1.882                    | 4.735                   |
| 28           | 22.2         | .3995                   | 1.880                    | 4.708                   |
| 33           | 22.7         | .401                    | 1.880                    | 4.688                   |
| 42           | 23.2         | .404                    | 1.880                    | 4.654                   |
| 53           | 23.6         | .406                    | 1.878                    | 4.625                   |
| 2 02         | 24.0         | .4075                   | 1.876                    | 4.604                   |
| 14           | 24.3         | .409                    | 1.874                    | 4.582                   |

EXPERIMENT B.

|      |       |       |       |       |
|------|-------|-------|-------|-------|
| 2 47 | 22.8  | 0.350 | 1.36  | 3.886 |
| 48   | 23.0  | .353  | 1.346 | 3.818 |
| 49   | 23.0  | .353  | 1.341 | 3.803 |
| 50   | 23.0  | .3525 | 1.341 | 3.804 |
| 51   | 23.0  | .352  | 1.341 | 3.809 |
| 52   | 23.05 | .351  | 1.342 | 3.823 |
| 53   | 23.1  | .3505 | 1.343 | 3.831 |
| 54   | 23.1  | .350  | 1.344 | 3.840 |
| 55   | 23.15 | .3495 | 1.346 | 3.851 |
| 57   | 23.2  | .348  | 1.348 | 3.873 |
| 3 00 | 23.25 | .347  | 1.350 | 3.890 |
| 03   | 23.4  | .3455 | 1.352 | 3.913 |
| 07   | 23.5  | .344  | 1.356 | 3.942 |
| 14   | 23.6  | .341  | 1.359 | 3.985 |
| 18   | 23.7  | .340  | 1.361 | 4.003 |
| 25   | 24.0  | .3365 | 1.37  | 4.071 |
| 35   | 24.0  | .333  | 1.378 | 4.138 |
| 41   | 24.2  | .331  | 1.38  | 4.169 |
| 50   | 24.2  | .3275 | 1.383 | 4.223 |
| 56   | 24.2  | .326  | 1.385 | 4.248 |



solutions was again determined as before, care being taken to mix the electrolyzed solution from each vessel thoroughly before taking out the sample.

Calculation showed that in both cases the concentration had slightly increased, probably owing to the evaporation of the liquid during electrolysis. The remaining electrolyte in each instrument was then treated with hydrochloric acid, filtered, and the filtrate evaporated to dryness in a porcelain dish. There was a slight residue of a light-greenish color, which, when heated, disintegrated and became black. It was then dissolved in nitric acid, but in the solution no silver chloride was formed on the addition of hydrochloric acid. A surplus of ammonia colored the solution slightly blue, showing that some copper was present, which was probably contained in the original electrolyte.

The same test was applied to the heavy anode solution which had collected in the beaker during the experiment (B), but in this case also no silver that had escaped precipitation by the hydrochloric acid was found in the filtrate.

5. The main problem is either to prevent the formation of complex ions or to hinder the anode solution from reaching the cathode. Attempts in the first direction by adding an oxidizing agent, for example, hydrogen peroxide, to the solution, have been unsuccessful so far, while the second problem has to a certain extent been solved. Richards places the silver rod, which forms the anode, in a fine-grained porous cup and removes from time to time the solution collecting at the bottom.

The author fully corroborated Richards's results, but proposed a different form of the anode. The bottom of a wide porous cup is filled with granulated silver, and upon this a large silver plate is pressed. In this type the rather inconvenient frequent removal of the solution was found to be unnecessary. The heavy solution is prevented by the porous cup from rapid diffusion and breaks up to a large extent when it remains in contact with silver (see figs. 1 and 2). This secondary reaction, he believes, gives rise to the formation of the well-known dark anode slime, which is pure silver when the anode is pure.

Formerly this was thought to be peroxide of silver. Novák believes he has found two distinct modifications, according to the current density employed. Investigations at the Reichsanstalt have shown, however, that if pure silver is used at the anode no peroxide is formed. The black powder at the anode is, in the main, silver in a form similar to platinum black. It is, however, not impossible that some oxygen may be present.

The phenomenon reminds us of the appearance of finely divided

copper on a copper anode, in a copper sulphate solution. As Fischer<sup>a</sup> has shown, the copper goes into solution partly as cuprous ions, the amount sometimes reaching 50 per cent of the total anodic copper loss. On reaching the cooler body of the electrolyte, the cuprous sulphate breaks up into cupric sulphate and pure copper; similarly the silver may go into solution as an ion, in a lower state of oxidation, and be broken up again in the immediate neighborhood of the anode. While the usual anodes were very soon covered with anode slime after the current of one ampere was started through the coulometer, silver, freshly deposited on the rods and left in the solution, did not show the tendency to blacken, and this suggests a decrease in the secondary chemical reaction. Possibly this is due simply to the increase in the size of the electrode. Experiments with anodes of electrolytic silver will be made.

Other arrangements may be adopted to prevent the anode solution from reaching the cathode. The latter may, for instance, be suspended above the anode, or the cathode may be in form of a plate suspended in a glass vessel as Gray has done, or the two electrodes be placed in different vessels connected by a siphon. From a practical point of view the latter types are, however, less convenient than the two described above. Moreover Van Dijk<sup>a</sup> has lately shown that when a siphon is employed the deposit is still a little larger than in the porous cup coulometer.

6. Preliminary experiments have shown that Bancroft's proposition,<sup>b</sup> that rotating electrodes be used, promises good results. A constant stirring of the solution in the neighborhood of the anode reduces the formation of the anode slime, or, as I believe, decreases the number of ions of lower oxidation than the normal. Thus, three methods, either separately or combined, will be tried in our future work on the subject: (a) addition of oxidizing agent to the solution; (b) electrolyzed silver as anode; (c) rotating electrodes. It is hoped that some means may be found to avoid the porous cup altogether, or at least allow the use of cups of loose texture. Another source of trouble may arise from the contact of the solution with filter paper. The organic substances contained in the latter may act chemically upon neutral silver nitrate solution. Filter paper should therefore not be used in the preparation of the electrolyte nor in the coulometer. ||

7. The following table shows a comparison between different types. In the first column the description of the instrument is given, and in

<sup>a</sup> Fischer: *ZS. anorg. Chem.*, 48, p. 177; 1904.

<sup>b</sup> Van Dijk: *Arch. neerl. Sci. exactes et nat.*, 9, p. 442; 1904.

<sup>c</sup> *Trans. Intern. Electr. Congr.*, St. Louis, 2, p. 106; 1905.

the following column the differences between the mass of the deposit and that in the porous cup coulometer, if in the latter one gram of silver were deposited. The filter paper type includes the one used by Van Dijk and Kunst,<sup>a</sup> who surrounded the anode by a thimble of filter paper, as used in Soxhlet's extraction apparatus. On the average the filter paper coulometer yields 1.00045 grams to one gram in the standard or porous cup type. Kahle compared the siphon type and the silver oxide type with the ordinary form. In order to include his results in this table, the average difference between the filter paper coulometer and the standard have been added to the numerical values found by him.

TABLE II.

| No. | Type.                                                                             | Kahle.    | Richards. |           | Guthe.    | Van Dijk. | Average.  |
|-----|-----------------------------------------------------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
|     |                                                                                   | 1898.     | 1899.     | 1902.     | 1904.     | 1904.     |           |
|     |                                                                                   | <i>mg</i> | <i>mg</i> | <i>mg</i> | <i>mg</i> | <i>mg</i> | <i>mg</i> |
| 1   | Anode surrounded by filter paper.....                                             |           | 0.82      | 0.34      | 0.48      | 0.28      | 0.45      |
| 2   | Anode surrounded by granulated silver and filter paper .....                      |           |           |           | 0.57      |           | 0.57      |
| 3   | Anode surrounded by filter paper; solution saturated with Ag <sub>2</sub> O ..... | 0.95      | 2.03      | 1.12      |           |           | 1.2       |
| 4   | Anode surrounded by porous cup; solution saturated with Ag <sub>2</sub> O .....   |           |           | 0.55      | 0.55      |           | 0.55      |
| 5   | Glass dish under anode.....                                                       | 0.45      |           |           |           | 0.12      | 0.3       |
| 6   | Anode and kathode in different vessels, connected by a siphon.                    | -0.05     |           |           |           | 0.11      | 0.03      |

We see that the usual filter-paper type (No. 1 in the above table) with a silver nitrate solution gives a deposit 1 part in 2,000 larger than the standard porous cup coulometer. It should, however, be remarked that the porous cup does not entirely prevent the diffusion of the complex ion into the kathode space, and this will become the more apparent the longer the time of electrolysis. It would therefore be a decided advantage if a method could be devised in which the use of

<sup>a</sup> Van Dijk and Kunst: Proc. Roy. Ac. Amsterdam, Jan. 21, 1904, and Ann. Phys., 14, p. 569; 1904.

a porous cup is unnecessary. The low values obtained by Van Dijk may possibly be due to a looser texture of the cups, but it is difficult to decide this question on account of the unreliability shown by two ordinary coulometers put in series and treated in exactly the same way. Kohlrausch's form gives intermediate results, as was to be expected.

As the investigations referred to show, we can rely upon the porous cup coulometer to at least within 1 part in 10,000, even for independent series of experiments. It would therefore be a decided step in advance if one of these types was substituted for the one now in common use. This would necessitate a change in the accepted value of the electrochemical equivalent of silver, as will be shown later—i. e., if in the future the unit of electric current should still be defined on the basis of the c. g. s. system.

8. Whether or not there are other disturbing factors of minor importance only an extended research with the porous cup coulometer will show. Some of the earlier experiments are not conclusive in this respect; they were made with the usual type, and often the differences observed amount to less than what we may expect. Besides, different investigators frequently contradict each other.

In the silver coulometer a neutral solution of silver nitrate is recommended. To test the neutrality it seems best to precipitate the silver by means of neutral sodium chloride solution and test the filtrate with methylorange. If the crystals contain acid it may be well to melt them in a silver or platinum crucible.

As Kahle has shown, a neutral solution becomes acid on being used. Richards explains this by a deionization of hydroxyl ions with the formation of silver oxide, and a consequent excess of ionized hydrogen in solution. That the mere presence of acid can not be held responsible for the trouble follows from Rodgers and Watson's and Leduc's investigations with an originally acid solution. The latter found the deposits 1 part in 5,000 lighter than without the addition of the acid to the electrolyte. In fact, if the original amount of free acid surpasses a definite percentage, then on electrolysis the amount decreases. I believe that the addition of acid or some other oxidizing agent to the anode side of a porous cup coulometer will be of help in either preventing the formation of the complex ion or in breaking it up. Experiments in this direction are in progress.

The Reichsanstalt recommends that acid be not used, lest impurities of the anode pass into solution. The influence of impurities in the solution seems, however, to be of little importance, as shown by Rayleigh and Mrs. Sidgwick, who added a large proportion of copper

sulphate to the solution, and by Leduc, who added copper sulphate and also potassium nitrate. The deposits did not contain any of these impurities. If the current is not too large, it may be stated as a general rule that metals which require a higher cathodic difference of potential than silver will not be found in the deposits. The presence of a small amount of nitrite does not influence the result, but in a silver-nitrate solution saturated with nitrite the deposit becomes too heavy.

The presence of rarer metals would seem to be more serious, but they are hardly ever found in appreciable quantities in commercial silver, and, moreover, their electrochemical equivalents are not very different from that of silver. Extreme care in the selection of material for the electrolyte as well as the anode seems, therefore, to be unnecessary. We may even use a soluble electrode of a metal like zinc and still obtain satisfactory results, as Richards and Heimrod proved.

9. One great objection is found in the tendency to looseness which the silver obtained from nitrate solution frequently shows. The addition of a small proportion of silver acetate greatly improves the texture of the deposit, but, as was shown by Rayleigh and Mrs. Sidgwick, the deposit is always too heavy, possibly owing to an inclusion of liquid.

Richards suggests that in this case the concentration of the oxide-complex is larger than in the nitrate solution, and therefore the deposit heavier. Lord Rayleigh employed silver chlorate as electrolyte and obtained results closely agreeing with those given by a silver nitrate coulometer. Deposits from cyanide solution are pure white and show no tendency to looseness. Leduc<sup>a</sup> has made some experiments with potassium silver cyanide, but the amount of silver collected was entirely too small, which he attributed to a simultaneous development of hydrogen and silver at the kathode and to an occlusion of the former in the silver. Farup,<sup>b</sup> however, has shown that hydrogen is not produced, but that the silver is dissolved by potassium cyanide if air is present in the solution. He employs, therefore, a silver coulometer with potassium silver cyanide as electrolyte, but saturates the solution with hydrogen. The results obtained by him are quite satisfactory, though I believe that with larger deposits trouble may arise due to occlusion of the liquid in the deposits. For small currents, however, this form seems to be very useful.

It is claimed that deposits from pure silver nitrate solution keep some of the liquid included. This will show itself in a decrease of

<sup>a</sup> Leduc: *Rapports Congr. Internat. de Phys.*, 1900, 2, p. 440.

<sup>b</sup> Farup: *ZS. f. Electroch.*, 8, p. 569; 1902.

weight when the deposit is heated to nearly red heat, the nitrate being decomposed. According to Rayleigh and Mrs. Sidgwick, there was sometimes no loss on heating, but perhaps more often a slight decrease. Kahle advises a treatment for ten to twenty minutes with water at a temperature of  $70^{\circ}$  to  $90^{\circ}$  C., to insure the complete removal of the mother liquid. Richards also found a slight amount included; but Gray claims that with proper washing the plates may be heated without any sensible loss of weight. This statement has been recently corroborated by Van Dijk,<sup>a</sup> who, after washing and drying at  $150^{\circ}$ , heated the deposits in an electric furnace to  $500^{\circ}$  and  $600^{\circ}$  C. without noticing a loss in weight. The silver may either be freshly formed on a platinum surface or on another former silver deposit. Only in the case that a new deposit is formed on an old one which had been heated to  $500^{\circ}$  C. a decrease could be noticed, but this is explained by the cracking of the old crystals, due to the heating and consequent inclusion of liquid in the fissures. It seems, therefore, unnecessary to correct for included mother liquid in a deposit properly washed and dried.

10. The question of the solubility of silver in different liquids is one of great importance.

According to Richards and Heimrod, pure silver when boiled with a silver-nitrate solution will produce nitrite of silver in small quantities, but the Reichsanstalt makes the statement that this is not the case, and that therefore, under normal conditions, a reduction to nitrite can not be observed in a silver coulometer. Kahle, Myers, and Merrill<sup>b</sup> observed a decrease of the weight of silver when it was treated with warm water, but Richards, Collins and Heimrod, Leduc, and Guthe could not detect any change when the deposit was left standing *under* water for hours, while Van Dijk asserts that in water of  $80^{\circ}$  C. not more than 0.1 mg is dissolved in several hours for each gram of silver deposited.

The concentration of the silver nitrate solution seems to affect only the texture of the deposit, which has a tendency to looseness if the solution is too weak in relation to the current. Though Gray considers it a mistake to use solutions containing more or even as much as 10 per cent of silver nitrate, all other observers deny any influence of concentration, and generally recommend high concentration, i. e., from 15 per cent to 30 per cent.

The Reichsanstalt has found that silver oxide is almost insoluble in concentrated nitrate solution, but the experiments referred to above show that solutions treated with the oxide are favorable to the formation of the complex ion and will yield too much silver.

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<sup>a</sup> Van Dijk, Arch. nêerl, 10, p. 287, 1905.

<sup>b</sup> Merrill: Phys. Rev., 10, p. 67, 1900.

11. Schuster and Crossley stated that the deposits of silver in vacuo were about 1 part in 1,000 larger than those obtained from solutions surrounded by air, and these again larger than those formed in an atmosphere of oxygen. The former result was verified by Kahle, Richards, and Myers.<sup>a</sup> The latter also found an increase when the liquid was saturated with nitrogen, but a decrease when the dissolved gas was carbon dioxide. It is reasonable to suppose that the increased weight of the deposit is due to the removal of oxygen from the solution and not to a change in pressure. In addition, Merrill showed that an increase of pressure to 103 atmospheres has no appreciable effect.

Rayleigh and Mrs. Sidgwick observed an increase of deposit with increase of temperature; Leduc a decrease; Richards, Collins, and Heimrod obtained as well at 60° C. as at 0° C. a larger deposit than at 20° C.

12. Almost all observers agree that the size of the cathode makes no difference as long as the current density is kept within about 1 ampere per 50 to 500 cm<sup>2</sup> of cathode surface. An objection has been made to the use of the ampere as one of the fundamental electrical standards, because the ordinary size of silver coulometer allows only currents of the magnitude of one ampere to be measured. An increase in the current limit may possibly be obtained by the use of rotating cathodes, as now employed so successfully in chemical analysis. With a rotating cathode a considerably larger current can be allowed than with a stationary cathode. According to Schuster and Crossley too great a current density at the anode is accompanied by a smaller deposit; according to Leduc just the opposite is the case, while Merrill could find no measurable effect due to a variation in the size of the electrodes. The author's experiments lead to the conclusion that in the porous-cup coulometers the size of the anode does not come into account. It is, however, well to keep in mind that the drop of potential should not be large enough to allow a decomposition of water. In all ordinary forms of silver coulometers the potential difference is sufficiently small.

With the usual type of coulometer Kahle as well as Van Dijk obtained a somewhat larger deposit on a silver cathode than on platinum, and considers, therefore, as normal deposits those obtained on silver. Richards and Heimrod confirm this observation, but find that with a porous-cup coulometer no such difference appears. The author also obtained identical results when the cathode was platinum as when silver had been previously deposited on it. The explanation of Kahle's results is to be sought in the action of silver upon the heavy anode liquid mentioned above.

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<sup>a</sup>Myers: Wied. Ann., 55, p. 288, 1895.

## THE ELECTROCHEMICAL EQUIVALENT OF SILVER.

13. The electrochemical equivalent of silver has been determined repeatedly by absolute measurements, i. e., by means of instruments which allow a calculation of the current in terms of the fundamental units of mass, length, and time. Among the earlier investigations only those of Rayleigh and Mrs. Sidgwick and of Fr. and W. Kohlrausch can be considered accurate. In order to express all measurements in terms of the same standard, the different values found have been reduced to those given by the porous-cup coulometer, and these are given in the last columns of the following table. Since in most cases the exact conditions of the experiments are unknown, these corrected values will simply give a general idea of the agreement between different observers.

As will be seen, Richards's and Van Dijk's corrections differ considerably. In deciding upon my own corrections I have taken the average of these two and those proposed by myself elsewhere.<sup>a</sup>

It was thought unnecessary to take into account the possible effect of included mother liquid.

From this list we see that the results obtained so far are not very satisfactory.

The most reliable experiments made in the United States, England, Germany, and the Netherlands lead to an average of 1.1178 mg for the electrochemical equivalent of silver, while the last two French determinations give a value of about 1 in 1,000 higher than this. Redeterminations in absolute measure with the use of a reliable form of coulometer are highly desirable.

14. The electrochemical equivalent of silver may also be expressed in terms of the electromotive force of a standard cell, i. e., by comparing the electromotive force of the cell with the potential difference produced by the current at the terminals of a known resistance. The electrochemical equivalent will depend upon the value chosen for the electromotive force of the standard cell. The legalized value for the Clark cell is 1.434 volts. But this is probably too high. In Germany the electromotive force of the Clark cell is derived from silver-coulometric measurements, and the Reichsanstalt has chosen as the working value 1.4328 volts at 15° C. In the following table the electrochemical equivalent of silver is calculated as well for an electromotive force = 1.434 as for 1.433 volts.

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<sup>a</sup>Transactions of the International Electrical Congress, St. Louis, II, p. 104, 1905.



TABLE III.

| Observer.                      | Year. | Coulometer.            | Method.                  | Electrochemical equivalent. |                             |           |        |
|--------------------------------|-------|------------------------|--------------------------|-----------------------------|-----------------------------|-----------|--------|
|                                |       |                        |                          | Found.                      | Corrected by—               |           |        |
|                                |       |                        |                          |                             | Richards<br>and<br>Heimrod. | Van Dijk. | Guthe. |
|                                |       |                        |                          | mg                          | mg                          | mg        | mg     |
| Mascart <sup>a</sup>           | 1884  | Usual type.....        | Current balance.....     | 1.1156                      | .....                       | 1.1155    | 1.1153 |
| Fr. and W. Kohlrausch....      | 1884  | Usual type.....        | Tangent galvanometer.... | 1.1183                      | 1.1173                      | 1.1182    | 1.1177 |
| Rayleigh and Sidgwick....      | 1884  | Usual type.....        | Current balance.....     | 1.1179                      | 1.1175                      | 1.1178    | 1.1176 |
| Gray.....                      | 1886  | Plate coulometer.....  | Sine galvanometer.....   | 1.1183                      | .....                       | .....     | 1.1178 |
| Koepseel <sup>b</sup>          | 1887  | Usual type.....        | Current balance.....     | 1.1174                      | .....                       | .....     | 1.1169 |
| Pellat and Potier <sup>c</sup> | 1890  | Usual type.....        | Current balance.....     | 1.1192                      | .....                       | 1.1191    | 1.1189 |
| Patterson and Guthe.....       | 1898  | Silver-oxide type..... | Electro-dynamometer....  | 1.1192                      | 1.1175                      | 1.1180    | 1.1177 |
| Pellat and Leduc <sup>d</sup>  | 1903  | Leduc's type.....      | Current balance.....     | 1.1195                      | .....                       | 1.1192    | 1.1190 |
| Van Dijk and Kunst.....        | 1904  | Usual type.....        | Tangent galvanometer.... | 1.1182                      | .....                       | 1.1180    | 1.1178 |

<sup>a</sup> Mascart: J. de Phys., 2, p. 100, 1882, and 3, p. 283, 1884.<sup>b</sup> Koepseel: Wied. Ann., 31, p. 250, 1887.<sup>c</sup> Pellat and Potier: J. de Phys., 9, p. 381; 1890.<sup>d</sup> Pellat and Leduc: C. R., 136, p. 1649; 1903.

TABLE IV.

| Observer.                           | Year. | Electrochemical equivalent. |               |                |               |
|-------------------------------------|-------|-----------------------------|---------------|----------------|---------------|
|                                     |       | E=1.434 volts.              |               | E=1.433 volts. |               |
|                                     |       | Usual type.                 | Porous cup C. | Usual type.    | Porous cup C. |
|                                     |       | <i>mg</i>                   | <i>mg</i>     | <i>mg</i>      | <i>mg</i>     |
| Carhart <sup>a</sup> .....          | 1882  | 1.1172                      | 1.1167        | 1.1180         | 1.1175        |
| Rayleigh and Sidgwick .....         | 1884  | 1.1183                      | 1.1178        | 1.1192         | 1.1187        |
| Von Ettinghausen <sup>b</sup> ..... | 1884  | 1.1180                      | 1.1175        | 1.1188         | 1.1183        |
| Glazebrook and Skinner .....        | 1892  | 1.1183                      | 1.1178        | 1.1191         | 1.1186        |
| Perot and Fabry <sup>c</sup> .....  | 1898  | 1.1193                      | 1.1188        | 1.1120         | 1.1196        |
| Kahle .....                         | 1898  | 1.1173                      | 1.1167        | 1.1180         | 1.1175        |
| Guthe .....                         | 1904  | 1.1174                      | 1.1168        | 1.1181         | 1.1176        |

<sup>a</sup> Carhart: *Am. Jour. Sci.*, 28, p. 374; 1884.

<sup>b</sup> Von Ettinghausen: *ZS. f. Electrotechnik*, 2, p. 484; 1884.

<sup>c</sup> Perot and Fabry: *Ann. Fac. des Sci. Marseille*, 8, p. 201; 1898.

In the case of Perot and Fabry, who used a Clark cell at 0° C. and found its electromotive force to be 1.4522 volts, using 1.118 mg as the electrochemical equivalent, the difference of 0.0164 volt given by the Reichsanstalt has been used to reduce to 15° C., instead of the ratio given by them. The latter would give 1.1180 mg in the first column and corresponding values in the others and make the agreement with the earlier experiments a very close one.

The large differences between the earlier and the more recent comparisons can hardly be due to the silver coulometer alone. Doubtless the Clark cell comes in for its share.

Wolff and Carhart and Hulett have lately discovered an electrolytic method of preparing mercurous sulphate. Cadmium standard cells, in which this substance is used, show, according to preliminary reports, an excellent agreement among themselves, and no variation in their electromotive force in course of time as far as can be ascertained during a relatively short period.

With the improvement of our standard of electromotive force and the construction of a reliable silver coulometer, a wide and interesting field for research has been opened, and it is to be expected that the new determination of the volt and the ampere, which are in progress in the different countries, will show an agreement considerably better than the older ones.

## HISTORY OF THE STANDARD WEIGHTS AND MEASURES OF THE UNITED STATES.<sup>a</sup>

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By LOUIS A. FISCHER.

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The subject of weights and measures is one of such great interest and importance and is attracting so much attention in this country and in England at the present time that a short account of the steps taken to secure uniformity in the United States is deemed an appropriate subject to bring to the attention of this convention.

The attention of the United States Government has long been directed toward securing uniformity in this country, and in the effort to secure international agreement in weights and measures it has always shown the greatest interest. The history of the original Confederation of States and of the constitutional government of the United States is full of evidences of the perplexities arising from the diversity of weights and measures throughout their jurisdiction and of the desirability of a uniform system.

The weights and measures in common use in this country at the time of the American Revolution were all of English origin and were in use in England at that period. The principal units were the yard, the avoirdupois pound, the gallon, and the bushel. More or less authentic copies of the English standards of the denominations mentioned had been brought over from time to time and adopted by the different colonies. Divergencies in these weights and measures were, however, quite common, due no doubt to the fact that the system of weights and measures of England was not itself well established, and hence the copies brought to this country were often adjusted to different standards.

That this condition was recognized very early is made evident by the Articles of Confederation which contained the following clause: "The

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<sup>a</sup> An address delivered before the First Annual Meeting of the Sealers of Weights and Measures of the United States at the Bureau of Standards, Washington, D. C., 1905.

United States in Congress assembled shall also have the sole and exclusive right and power of regulating the alloy and value of coin struck by their own authority, or by that of the respective States—fixing the standard of weights and measures throughout the United States.” This power was transferred to Congress by the Constitution of the United States in article 1, section 8, the language being as follows: “The Congress shall have Power . . . To coin Money, regulate the Value thereof, and of foreign Coin, and fix the Standard of Weights and Measures;”.

While Congress was not slow to take action in regard to coinage, it seems not to have been inclined to come to a decision in regard to weights and measures, though apparently willing enough to consider the subject. Washington, in his first annual message to Congress, January, 1790,<sup>a</sup> stated that “uniformity in the currency, weights, and measures of the United States is an object of great importance, and will, I am persuaded, be duly attended to.” In accordance with Washington’s suggestion, the matter was referred to a select committee of the House of Representatives with instructions to prepare a bill, and it was also ordered that the matter be referred to the Secretary of State to prepare and report to the House a proper plan for establishing uniformity in the weights and measures.<sup>b</sup> Jefferson was then Secretary of State, and in response to the above request made a report, in which he proposed two distinct plans. The first was substantially to “define and render uniform and stable the existing system \* \* \* to reduce the dry and liquid measures to corresponding capacities by establishing a single gallon of 270 cubic inches and a bushel of eight gallons, or 2,160 cubic inches \* \* \*.” The second plan was “to reduce every branch to the same decimal ratio already established for coin, and thus bring the calculations of the principal affairs of life within the arithmetic of every man who can multiply and divide plain numbers.”<sup>c</sup>

No action was taken, however, by the House and in his second message to Congress, on December 8, 1790, Washington again called the attention of that body to the importance of the subject.<sup>d</sup> A few days later the House ordered that the report of Jefferson, referred to above, be communicated to the Senate. On March 1, 1791, the Senate committee to which the matter had been referred reported that it would not be eligible to make a change in the weights and measures, as a

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<sup>a</sup> Messages and Papers of the Presidents, 1, p. 66.

<sup>b</sup> Congressional Register, 3, p. 106.

<sup>c</sup> Journal of the H. R., Childs & Swaine, p. 106.

<sup>d</sup> Messages and Papers of the Presidents, 1, pp. 83.

proposition had been made to the French and British Governments to obtain an international standard.<sup>a</sup> This report was accepted and the matter rested there, although Washington, on October 25, 1791, repeated his former recommendations in his third annual message to Congress, in the following language:<sup>b</sup>

A uniformity in the weights and measures of the country is among the important objects submitted to you by the Constitution and if it can be derived from a standard at once invariable and universal, must be no less honorable to the public councils than conducive to the public convenience.

A week later the Senate appointed a committee to take into consideration the subject of weights and measures. The committee reported on the 4th of April, 1792, recommending the adoption of the second plan proposed by Jefferson, which was an entirely decimal system. Again no definite action was taken. The matter was considered in a desultory way by Congress from time to time, but no agreement was reached notwithstanding that the repeated recommendations of Washington were followed by those of Adams. A sufficient explanation for the disinclination of Congress to act in a matter of such admitted importance was the difficulty of agreeing upon a plan.

The Fifth Congress, second session, in 1799, passed an act ordering that the surveyor (of each port of the United States) shall from time to time, and particularly on the first Monday of January and July in each year, examine and try the weights, measures, and other instruments used in ascertaining the duties on imports with standards to be provided by each collector at the public expense for that purpose; and when disagreements and errors are discovered he shall report the same to the collector and obey and execute such directions as he may receive for the correction thereof agreeably to the standards aforesaid.<sup>c</sup>

This was the first act passed by Congress in regard to weights and measures, but in view of the fact that no standards had ever been adopted the legislation was not put into operation until about thirty-five years after its passage, when certain standards, which will be referred to later, were adopted by the Treasury Department.

After the war of 1812 the question of uniformity in weights and measures was again brought to the attention of Congress, and in 1819 a committee of the House of Representatives proposed to adopt absolute standards conforming to the weights and measures in common use; to obtain through a commission copies of the yard, the bushel, the wine gallon, and the pound supposed to conform to those in common use in the United States; to preserve these standards and to dis-

<sup>a</sup>Journal of the Senate, p. 143; John L. Fenno.

<sup>b</sup>Messages and Papers of the Presidents, 1, p. 108.

<sup>c</sup>Statutes at Large, 1, p. 643.

tribute copies of them; to compare the length measure with the length of the second's pendulum and also with that of an arc of the terrestrial meridian; to connect them by determining the weight of a certain bulk of distilled water, and to define the bushel and the gallon by the weight of water which they contain.<sup>a</sup> No further record of the report is found, and it may be assumed that no action upon it was taken. The Senate had, by a resolution adopted March 3, 1817—two years prior to the above report—requested the Secretary of State to prepare and report a "statement" relative to the regulations and standards for weights and measures in the several States and relative to the proceedings in foreign countries for establishing uniformity in weights and measures, together with such propositions relative thereto as might be proper to adopt in the United States.

John Quincy Adams was at that time Secretary of State, and four years later—namely, on February 22, 1821—he submitted an elaborate report to the House of Representatives, in which, among other recommendations, the following are found:

- (1) To fix the standard with the partial uniformity of which it is susceptible for the present, excluding all innovations.
- (2) To consult with foreign nations for the future and ultimate establishment of universal and permanent uniformity.

As before, Congress took no action, probably because the situation at that time was extremely complicated. Neither the metric system in France nor the system in common use in England was well established. In France the law making the metric system compulsory had been repealed, and the metric system was in use side by side with the ancient weights and measures, thus producing endless confusion. In England the situation was not much better; the ale gallon of 282 cubic inches and the wine gallon of 231 cubic inches were both in use until 1824, when the new imperial gallon, containing 10 pounds of water, and of a capacity of about  $277\frac{1}{4}$  cubic inches, was adopted, together with the bushel of 8 gallons. Neither of these measures was in use in this country, and hence the United States could not at that time adopt either the system in use in England or the one in France without introducing radical changes in the weights and measures already in use, nor was there at that time any positive assurance that either the English or the metric system would be permanent.

While Congress had been considering the matter, most of the States had, independently of one another, secured and adopted standards. Most of the standards thus adopted were brought from England; nevertheless, standards of the same denomination differed widely

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<sup>a</sup> Executive Doc. No. 73, 30th Cong., 1st sess., Senate.

among themselves, thus perpetuating confusion in the commerce between adjacent States.

Though confusion in commercial transactions might be overlooked, uncertainty in regard to the coinage could not be tolerated, and on May 19, 1828, a certain troy pound was adopted as the standard for coinage by Congress in an "Act to continue the Mint at the City of Philadelphia, and for other purposes." The section 2 of the act referred to reads as follows:

*And be it further enacted, That, for the purpose of securing a due conformity in weight of the coins of the United States \* \* \* the brass troy pound weight procured by the minister of the United States at London, in the year one thousand eight hundred and twenty-seven, for the use of the mint, and now in the custody of the Mint at Philadelphia, shall be the standard troy pound of the Mint of the United States, conformably to which the coinage thereof shall be regulated.*

The troy pound thus adopted had been procured in the year 1827 by Albert Gallatin, minister of the United States at London, and brought to this country by special messenger, who delivered it to the director of the Mint at Philadelphia. The weight was of brass and an exact copy of the imperial troy pound of Great Britain, according to the statement of Captain Kater, who made the comparison between the two standards. The casket and accompanying packages were retained under seal until Mr. Adams, President of the United States, visited Philadelphia and verified the seal of Mr. Gallatin and the other facts in regard to its authenticity.

This ceremony took place on October 12, 1827, and the full certificate of President Adams in regard to the seal, which he readily recognized, and to the whole transaction and consequent accuracy of the weight was added to the vouchers in the case. He declared, in conclusion, his belief that the brass weight then exhibited was the identical pound copy of the imperial standard troy pound of Great Britain referred to in the aforesaid certificates.<sup>a</sup> The foregoing facts were communicated to Congress through the Committee on the Mint and resulted in the passage of the act cited above.

While the act of Congress of 1828 only made this pound the standard for coinage, it virtually became the fundamental standard of the United States from which the avoirdupois pound in common use was derived.

On May 29, 1830, two years after the mint pound had been legalized for coinage, the Senate passed a resolution directing the Secretary of the Treasury to cause a comparison of the weights and measures in use at the principal custom-houses to be made, and to report to the Senate at its next session.

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<sup>a</sup> Report on Weights and Measures, Franklin Institute, 1834, Appendix No. VII.

Steps were promptly taken by the Treasury Department to comply with the resolution of the Senate, and the preliminary report of F. R. Hassler, Superintendent of the Coast Survey, to whom the investigation had been intrusted, was transmitted to the Senate on March 3, 1831,<sup>a</sup> this being followed by a more complete report in June, 1832.

As was anticipated, large discrepancies were found to exist among the weights and measures in use at the different ports, some being too small and others too large, but the average value of the various denominations agreed fairly well with the weights and measures in use in Great Britain at the time of the American Revolution.

Without waiting for authority from Congress the Treasury Department took immediate steps to correct the evil by having constructed, under the direction of Mr. Hassler, the necessary weights and measures for the customs service. The divergencies among the weights and measures in use in the customs service were directly opposed to the spirit of the Constitution, which requires that all duties, imposts, and excises shall be uniform throughout the United States,<sup>b</sup> and the Secretary of the Treasury felt fully authorized in taking steps to secure uniformity when discrepancies were once known to exist.

Before weights and measures could be constructed, however, it was necessary for the Treasury Department to determine upon certain units and to adopt material representatives of these units. Those finally adopted were the yard of 36 inches, the avoirdupois pound of 7,000 grains, the gallon of 231 cubic inches, and the bushel of 2,150.42 cubic inches. The standard yard adopted was the 36 inches comprised between the twenty-seventh and the sixty-third inches of a certain 82-inch brass bar, prepared for the Coast Survey by Troughton, of London. This bar had been brought to the United States by Hassler in 1813, and the 36-inch space referred to was supposed to be identical with the English standard at 62° F., though it had never been directly compared with that standard.

It is evident from the reports of Mr. Hassler that he regarded the English yard as the real standard of length of the United States and the Troughton scale merely as a copy whose length should be corrected if it was subsequently found to differ from the English yard; and this view was taken by those who subsequently had charge of our standards, as will be shown later on.

The avoirdupois pound adopted by Mr. Hassler as the standard for the Treasury Department was derived from the troy pound of the mint according to the equivalent, 1 avoirdupois pound equals  $\frac{128}{155}$

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<sup>a</sup> See H. R. Doc. No. 299, 22d Cong., 1st sess.

<sup>b</sup> Sec. VIII, clause 1.



pounds troy. This was the accepted relation in this country as well as in England; hence both the troy and avoirdupois pounds adopted were in practical accord with the similar standards of Great Britain.

#### CAPACITY STANDARDS.

The units of capacity, namely, the wine gallon of 231 cubic inches and the Winchester bushel of 2,150.42, were adopted, because, as intimated, they represented more closely than any other English standards the average of the capacity measures in use in the United States at the date of Mr. Hassler's investigation. The wine gallon was introduced as a wine measure into England in 1707, during the reign of Queen Anne, but it was abolished in 1824, when the new imperial gallon, containing 10 pounds of water, was made the standard. This last statement applies also to the bushel of 2,150.42 cubic inches. This bushel is the earliest English capacity measure of which we have any record, a copy of it made by order of Henry VII being still in existence. But this bushel had also been abolished in England, it having been superseded by the bushel of 8 imperial gallons. Therefore neither the gallon nor the bushel adopted by the United States Treasury Department was in accord with the legal capacity standards of England, but they were smaller by about 17 per cent and 3 per cent, respectively, and these differences exist at the present time. Not only did they differ from the new standards in Great Britain, but they also differed from the discarded English standards from which they were derived for the reason that Mr. Hassler selected the temperature of the maximum density of water, namely, 39.2° F.<sup>a</sup> as the temperature at which the United States measures were standard, whereas their English prototypes were standard at 62° F.

Such, then, were the fundamental standards adopted upon the recommendation of Mr. Hassler by the United States Treasury Department, and to which the weights and measures for the customs service were made to conform. The construction of the weights and measures for this purpose was pushed with almost feverish haste, and so well satisfied was Congress with the progress made that the following resolution was passed and approved June 14, 1836:

*Resolved by the Senate and House of Representatives of the United States of America in Congress assembled, That the Secretary of the Treasury be, and he hereby is, directed to cause a complete set of all weights and measures adopted as standards and now either made or in progress of manufacture for the use of the several custom-houses,*

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<sup>a</sup> According to the determination made by Mr. Hassler on the expansion of water, 39.83° F. was the temperature of maximum density. See report of Alexander D. Bache, Superintendent of Weights and Measures, 46-47. Ex. Doc. No. 73, 30th Cong., 1st sess.

and for other purposes, to be delivered to the governor of each State in the Union, or such person as he may appoint for the use of the States, respectively, to the end that a uniform standard of weights and measures may be established throughout the United States.

While the act does not specifically adopt the standards above described, the practical effect of it was to make them the standards for the United States, inasmuch as the weights distributed to the States in accordance with the act were in almost every case adopted by the State legislatures soon after their receipt.

The act of 1836 was supplemented in 1838 by a joint resolution of Congress, which directed the Secretary of the Treasury to furnish balances to the States. By 1838 the weights for the States were reported finished, and during the following year the weights for the custom-houses were completed and delivered.<sup>a</sup>

By 1850 practically all the States admitted to the Union had been supplied with complete sets of weights and measures, and in addition sets were presented to England, France, Japan, and Siam. As new States were admitted they were also supplied with sets of standards, the last set being supplied to North Dakota in 1893.

In order to carry out the provisions of the acts of 1836 and 1838 the Office of Weights and Measures, under the direction of the Superintendent of the Coast Survey, had been established, and all the standards adopted at the beginning of the work, and subsequently, were in charge of this Office, with the exception of the troy pound of the mint, which has always remained at Philadelphia.

In October, 1834, the British imperial yard and troy pound made in 1758, of which the Troughton scale and the mint pound were supposed to be exact copies, were destroyed by the burning of the Houses of Parliament. When the new imperial standards to replace them were completed in 1855 two copies of the yard and one copy of the avoirdupois pound were presented to the United States, arriving in this country in 1856. One of these bars, namely, bronze yard No. 11, was very soon after compared with the Troughton scale, the result showing that the accepted 36 inches of the Troughton scale was 0.00087 inch longer than the British imperial yard.<sup>b</sup> The second bar received from England was subsequently compared with the Troughton scale and fully corroborated the result obtained from the comparison with bronze No. 11. The new yards, and especially bronze No. 11, were far superior to the Troughton scale as standards of

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<sup>a</sup> H. R. Doc. 159, 28th Cong., 2d sess.

<sup>b</sup> See Report of the Secretary of the Treasury on the construction and distribution of weights and measures in 1857. S. Doc. No. 27, 34th Cong., 3d sess.

length, and consequently they were accepted by the Office of Weights and Measures as the standards of the United States, and all comparisons were afterwards referred to the imperial yard through these two standards. They were twice taken to England and recompared with the imperial yard, once in 1876 and again in 1888.

The avoirdupois pound presented with the two yards was also compared with the United States avoirdupois pound derived from the mint pound, the result showing a very satisfactory agreement. The advent of the new pound did not, therefore, disturb the position of the troy pound of the mint or of the avoirdupois pound derived from the mint pound.

#### METRIC SYSTEM.

The next and perhaps the most important legislation enacted by Congress was the act of 1866 legalizing the metric system of weights and measures in the United States. The act, which was passed July 28, 1866, reads as follows:

*Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,* That from and after the passage of this act it shall be lawful throughout the United States of America to employ the weights and measures of the metric system, and no contract or dealing or pleading in any court shall be deemed invalid or liable to objection because the weights or measures expressed or referred to therein are weights or measures of the metric system.

SEC. 2. *And be it further enacted,* That the tables in the schedule hereto annexed shall be recognized in the construction of contracts and in all legal proceedings as establishing in terms of the weights and measures now in use in the United States the equivalents of the weights and measures expressed therein in terms of the metric system; and said tables may be lawfully used for computing, determining, and expressing in customary weights and measures the weights and measures of the metric system.

#### MEASURES OF LENGTH.

| Metric denominations and values. |                              | Equivalents in denominations in use.        |
|----------------------------------|------------------------------|---------------------------------------------|
| Myriameter.....                  | 10,000 meters.               | 6.2137 miles.                               |
| Kilometer.....                   | 1,000 meters.                | 0.62137 miles, or 3,280 feet and 10 inches. |
| Hectometer.....                  | 100 meters.                  | 328 feet and 1 inch.                        |
| Dekameter.....                   | 10 meters.                   | 39.37 inches.                               |
| Meter.....                       | 1 meter.                     | 39.37 inches.                               |
| Decimeter.....                   | $\frac{1}{10}$ of a meter.   | 3.937 inches.                               |
| Centimeter.....                  | $\frac{1}{100}$ of a meter.  | 0.3937 inch.                                |
| Millimeter.....                  | $\frac{1}{1000}$ of a meter. | 0.0394 inch.                                |

## MEASURES OF CAPACITY.

| Metric denominations and values. |                   |                                      | Equivalents in denominations in use. |                         |
|----------------------------------|-------------------|--------------------------------------|--------------------------------------|-------------------------|
| Names.                           | Number of liters. | Cubic measure.                       | Dry measure.                         | Liquid or wine measure. |
| Kiloliter or stere.              | 1,000             | 1 cubic meter .....                  | 1.308 cubic yards.                   | 264.17 gallons.         |
| Hectoliter ....                  | 100               | $\frac{1}{10}$ of a cubic meter .    | 2 bushels and 3.35 pecks.            | 26.417 gallons.         |
| Dekaliter .....                  | 10                | 10 cubic decimeters.                 | 9.08 quarts .....                    | 2.6417 gallons.         |
| Liter .....                      | 1                 | 1 cubic decimeter ..                 | 0.908 quart .....                    | 1.0567 quarts.          |
| Deciliter .....                  | $\frac{1}{10}$    | $\frac{1}{10}$ of a cubic decimeter. | 6.1022 cubic inch.                   | 0.845 gill.             |
| Centiliter .....                 | $\frac{1}{100}$   | 10 cubic centimeters                 | 0.6102 cubic inch.                   | 0.338 fluidounce.       |
| Milliliter .....                 | $\frac{1}{1000}$  | 1 cubic centimeter..                 | 0.061 cubic inch..                   | 0.27 fluid dram.        |

## MEASURES OF SURFACE.

| Metric denominations and values. |                       | Equivalents in denominations in use. |
|----------------------------------|-----------------------|--------------------------------------|
| Hectare .....                    | 10,000 square meters. | 2.471 acres.                         |
| Are .....                        | 100 square meters.    | 119.6 square yards.                  |
| Centare .....                    | 1 square meter.       | 1,550 square inches.                 |

## WEIGHTS.

| Metric denominations and values. |                  |                                                      | Equivalents in denominations in use. |
|----------------------------------|------------------|------------------------------------------------------|--------------------------------------|
| Names.                           | Number of grams. | Weight of what quantity of water at maximum density. | Avoirdupois weight.                  |
| Millier or tonneau...            | 1,000,000        | 1 cubic meter .....                                  | 2204.6 pounds.                       |
| Quintal .....                    | 100,000          | 1 hectoliter .....                                   | 220.46 pounds.                       |
| Myriagram .....                  | 10,000           | 10 liters .....                                      | 22.046 pounds.                       |
| Kilogram or kilo ....            | 1,000            | 1 liter .....                                        | 2.2046 pounds.                       |
| Hectogram .....                  | 100              | 1 deciliter .....                                    | 3.5274 ounces.                       |
| Dekagram .....                   | 10               | 10 cubic centimeters.....                            | 0.3527 ounces.                       |
| Gram .....                       | 1                | 1 cubic centimeter.....                              | 15.432 grains.                       |
| Decigram .....                   | $\frac{1}{10}$   | $\frac{1}{10}$ of a cubic centimeter ...             | 1.5432 grains.                       |
| Centigram .....                  | $\frac{1}{100}$  | 10 cubic millimeters.....                            | 0.1543 grain.                        |
| Milligram .....                  | $\frac{1}{1000}$ | 1 cubic millimeter.....                              | 0.0154 grain.                        |

While the above act was being considered, Congress also considered a resolution authorizing the Secretary of the Treasury to furnish the States with metric weights and measures. Strange to say, this resolution, which logically should follow, was approved one day before the act legalizing the use of the metric system. It was a joint resolution and read as follows:

*Be it resolved by the Senate and House of Representatives of the United States of America in Congress assembled, That the Secretary of the Treasury be, and he is hereby, authorized and directed to furnish to each State, to be delivered to the governor thereof, one set of standard weights and measures of the metric system for the use of the States, respectively.*

The work of making and adjusting these standards fell, naturally, upon the Office of Weights and Measures, and the first question that had to be considered was that of standards. The practice followed by other countries which had adopted the metric system of accepting the meter and the kilogram of the archives of France as fundamental standards was followed by the United States. The question was mainly one of securing authentic copies of these standards. Fortunately the Office of Weights and Measures had several copies of both standards of more or less authenticity on hand, but without hesitation an iron bar, known as the "committee meter," and a platinum kilogram, known as the "Arago kilogram," were selected.

#### COMMITTEE METER.

The committee meter is one of fifteen similar bars, whose lengths were ascertained in the process of constructing the original meter by the French committee of weights and measures in 1799; hence its name, "committee meter."

The committee referred to was composed of members of the National Institute of France and of deputies from foreign countries. Mr. J. G. Tralle, the deputy from the Helvetic Republic, had been placed in charge of the construction of the meters, and when the bars were distributed among the members of the committee he secured two of them, one of which he presented to Mr. Hassler. This bar was therefore of the highest authenticity. As before stated, it is made of iron, with a cross section of 9 by 29 mm, and its length is defined by the end surfaces, which are remarkably plane when one considers the age in which the bars were made. The bar bears the stamp of the committee, namely, a small ellipse, whereof three quadrants are shaded and the fourth one clear, except for the number 10,000,000, which indicates the number of meters in a meridian quadrant of the earth. It also bears the mark :· at one end, by which it was distinguished during the comparison with the other meters. In Mr. Hassler's report on the

construction of the meters<sup>a</sup> it is stated, on the authority of Mr. Tralle, that all the meters agreed with the true meter within one-millionth part of the toise.<sup>b</sup>

When Mr. Hassler came to the United States in 1805 he brought with him the committee meter, which he soon after presented to the Philosophical Society of Philadelphia, Pa. Shortly after, when he was put in charge of the survey of the coast, the meter was placed at his disposal by the Philosophical Society, and he made it the standard of length for that work, and until 1890 all base measurements of the Survey were referred to this meter.<sup>c</sup>

In view of the foregoing, it was but natural that this bar should be selected as the standard to which the State meters should conform.

#### ARAGO PLATINUM KILOGRAM.

The Arago kilogram was procured in 1821 by Mr. Gallatin while minister of the United States to France and was sent to this country, together with a platinum meter. The certificate of Arago, the celebrated physicist, which accompanied these standards, states that the kilogram differs from the original kilogram des Archives by less than 1 mg. The weight is a platinum cylinder with flat bases, the edges being slightly rounded. The height and diameter are nearly equal, being approximately 39.5 mm each. There is no stamp or distinguishing mark of any kind, except near the center of one base there is a faint lathe or tool mark of circular form, thus:  $\odot$ . The weight is contained in a square mahogany box, on the cover of which is a circular silver plate bearing the inscription "Kilogramme comparé pour son Poids à l'Étalon Prototype des Archives de France, et vérifié par M. Arago. Fortin fecit." No particulars of Arago's comparison with the kilogram des Archives were furnished, and consequently it is not known what means were used by him in making his comparison nor whether he reduced his weighings to vacuo. It was not until 1879 that the Arago kilogram was compared with any other standards of recognized authority. It is true that it was compared between 1852 and 1873 with a couple of kilograms in the possession of the Office of Weights and Measures, but as both of these weights were of brass and of unknown density, no great reliance could be attached to the results. In 1879, however, it was taken to England and there compared with the British platinum kilogram in the custody of the Standards Office.

<sup>a</sup>H. R. Doc. No. 299, 22d Cong., 1st sess., pp. 75, 76.

<sup>b</sup>The toise was the French standard of length prior to the adoption of the meter, and all the geodetic measurements upon which the meter was based were made with the toise. Its length is 1.949+ meters.

<sup>c</sup>Special Publication No. 4, U. S. Coast and Geodetic Survey.



FIG. 1.—THE SILBERMANN KILOGRAM.





This comparison indicated that the Arago kilogram was 4.25 mg light, but this result could not be considered conclusive, on account of certain assumptions made in the reduction to vacuo and also in regard to the correction to the British kilogram.

In 1884 the weight was taken from the Standards Office in London, where it had been since 1879, to the International Bureau of Weights and Measures at Paris and there compared with two auxiliary kilograms whose values in terms of the kilogram of the Archives were known with the greatest accuracy. The results obtained from the comparison confirmed that previously obtained from the comparison with the British kilogram, the result giving

$$K_a = 1,000 \text{ g} - 4.63 \text{ mg}$$

As the weights supplied to the States were to be made of brass, it was more convenient to compare them with a brass standard, and in order to do this two secondary brass standards were carefully compared between the years 1873-1876 with the Arago kilogram and afterwards used in all the work of adjustment and verification. One of the kilograms, known as the Silbermann kilogram, was presented to the United States by France in 1852, together with a number of other weights and measures. The other kilogram used was one made in the Office of Weights and Measures and was identical in form and material with the kilograms subsequently furnished to the States.

The unit of capacity in the metric system being defined as the volume of the mass of 1 kilogram of pure water at the temperature of maximum density, the most convenient way to adjust such measures, and in fact all capacity measures, is by weighing the water they contain. The only two material standards that need to be considered, therefore, in connection with the metric weights and measures furnished to the States in accordance with the act of 1866 are the committee meter and the Arago kilogram described above.

By the end of 1880 practically all the States had been supplied with sets of metric weights and measures consisting of the following denominations:

|                         |                                                                                                                                                                        |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Length measures .....   | { One brass-line meter.<br>One steel-end meter.                                                                                                                        |
| Capacity measures ..... | { One liter made of brass.<br>One decaliter made of brass.<br>One myriagram made of brass.<br>One kilogram made of brass.<br>One $\frac{1}{2}$ kilogram made of brass. |
| Weights .....           | { One gram made of brass.<br>One set of small silver weights<br>from 4 decigrams to 1 milli-<br>gram.                                                                  |

It is necessary at this point to go back a few years and give an account of the establishment of the International Bureau of Weights and Measures, since the present fundamental standards of length and mass for practically the whole civilized world result from the establishment of that institution.

In reponse to an invitation of the French Government, the following countries sent representatives to a conference held in Paris on August 8, 1870, to consider the advisability of constructing new metric standards:

|                |           |                |
|----------------|-----------|----------------|
| Austria,       | Italy,    | Spain,         |
| Ecuador,       | Norway,   | Switzerland,   |
| France,        | Peru,     | Turkey,        |
| Great Britain, | Portugal, | United States, |
| Greece,        | Russia,   | Colombia,      |

in all 15 countries. This conference was of short duration, on account of the war then raging between France and Germany.

A second conference was held two years later, at which 30 countries were represented, the United States again being among this number. At this conference it was decided that new meters and new kilograms should be constructed to conform with the original standards of the Archives, and a permanent committee was appointed to carry out this decision. The preparation of the new standards had advanced so far by 1875 that the permanent committee appointed by the conference of 1872 requested the French Government to call a diplomatic conference at Paris to consider whether the means and appliances for the final verification of the new meters and kilograms should be provided, with a view to permanence, or whether the work should be regarded as a temporary operation.

In compliance with this request a conference was held in March, 1875, at which 19 countries were represented, the United States as usual being of this number.

On May 20, 1875, 17 of the 19 countries represented signed a convention which provided for the establishment and maintenance of a permanent International Bureau of Weights and Measures to be situated near Paris and to be under the control of an international committee elected by the conference, the committee to consist of 14 members, all belonging to different countries.

In addition to the primary work of verifying the new metric standards the bureau was charged with certain duties, the following being the most important:

(1) The custody and preservation, when, completed of the international prototypes and auxiliary instruments.



FIG. 2.—THE UNITED STATES PROTOTYPE METER.





FIG. 3.—THE UNITED STATES PROTOTYPE KILOGRAM.



(2) The future periodic comparison of the several national standards with the international prototypes.

(3) The comparison of metric standards with standards of other countries.

The expenses of the bureau were to be defrayed by contributions of the contracting Governments, the amount for each country depending upon the population and upon the extent that the metric system was in use in the particular country.

In accordance with the terms of the convention the French Government set aside a plat of ground in the park of St. Cloud just outside of Paris, and upon this ground, which was declared neutral territory, the International Bureau of Weights and Measures was established.

The construction of the meters and kilograms had been intrusted to a special committee, and early in 1887 the committee completed its work and the new meters and kilograms were turned over to the International Bureau for comparison with the standards of the Archives and with one another.

It had been decided as early as 1873 that the new standards should be made of an alloy of 90 per cent platinum and 10 per cent iridium, and that the meters should be line standards of the cross section shown in the illustration. Altogether 31 meters and 40 kilograms were constructed. By 1889 the entire work was completed and in September of that year a general conference was held at Paris, and by it the work of the international committee was approved.

The meter and kilogram which agreed most closely with the meter and kilogram of the Archives were declared to be the international meter and the international kilogram. These two standards, with certain other meters and kilograms, were deposited in a subterranean vault under one of the buildings of the International Bureau, where they are only accessible when three independent officials with different keys are present. The other standards were distributed by lot to the various Governments contributing to the support of the International Bureau. Those falling to the United States were meters Nos. 21 and 27 and kilograms Nos. 4 and 20.

Meter No. 27 and kilogram No. 20 were brought under seal to this country by Mr. George Davidson, of the Coast and Geodetic Survey, and on January 2, 1890, they were opened at the White House and accepted by President Harrison, who certified that they were received in good condition, and that he confidently believed that they were the standards referred to in the report. The other two standards were received the following July and were deposited in the Office of Weights and Measures, where those accepted as national standards

by the President had already been taken.<sup>a</sup> On April 5, 1893, the Superintendent of Weights and Measures, with the approval of the Secretary of the Treasury, decided that the international meter and kilogram would in the future be regarded as the fundamental standards of length and mass in the United States, both for metric and customary weights and measures.

This action did not in any way affect the metric weights and measures of the United States inasmuch as the meter and kilogram of the Archives had always been regarded as our fundamental metric standards, and the international meter and kilogram had identical values so far as could be determined by the most refined measurements.

The effect of this decision on the customary weights and measures also left them practically undisturbed. All comparisons made immediately prior to 1893 indicated that the relation of the yard to the meter fixed by the act of 1866<sup>b</sup> was by chance the exact relation between the international meter and the British imperial yard within the error of observation. A subsequent comparison made between the standards just mentioned indicates that the legal relation adopted by Congress is in error by one ten-thousandth of an inch; but in view of the fact that certain comparisons made by the English Standards Office between the imperial yard and its authentic copies show variations as great, if not greater than this, it can not be said with certainty that there is a difference between the imperial yard of Great Britain and the United States yard derived from the meter.

The case of the pound was slightly different, inasmuch as the relation of the kilogram to the pound, fixed by the act of 1866, was only approximate. In the act mentioned the kilogram was stated to be equal to 2.2046 pounds avoirdupois; but as 2.204622 was known to be a more precise value, and since it did not conflict with the legal value the avoirdupois pound was declared to be equal to  $\frac{1}{2.204622}$  kilograms.

Neither the troy pound of the mint nor the copies of the imperial yard in the possession of the Office of Weights and Measures were satisfactory standards. The mint pound is made in two pieces, the knob being screwed into the body; hence its density can not be determined by weighing in water on account of danger of leakage. Moreover, it is made of brass not plated, and therefore liable to alteration by oxidation.

<sup>a</sup> Upon the establishment of the Bureau of Standards on July 1, 1901, all standards and other property in possession of the Office of Weights and Measures passed under the control of the Bureau.

<sup>b</sup> The value of the yard, in accordance with the above declaration, is 1 yard =  $\frac{1}{3600}$  meter.



The bronze yard No. 11, which was an exact copy of the British imperial yard both in form and material, had shown changes when compared with the imperial yard in 1876 and 1888 which could not reasonably be said to be entirely due to changes in No. 11. Suspicion as to the constancy of the length of the British standard was therefore aroused.

On the other hand, the new meters and kilograms represented the most advanced ideas of standards, and it therefore seemed that greater stability in our weights and measures as well as higher accuracy would be secured by accepting the international meter and kilogram as fundamental standards.

Time has amply proved the wisdom of this action, and therefore when the Bureau of Standards was established in July, 1901, the decision made by the Office of Weights and Measures in 1893 to adopt the meter and kilogram as fundamental standards was fully accepted by this Bureau.

In conclusion I wish to state that in preparing so brief an account of so great a subject many matters of importance and interest have necessarily been omitted, but if I have succeeded in giving you an outline of the growth of our weights and measures, I shall have accomplished all that I had in mind when this paper was prepared.



## WATTMETER METHODS OF MEASURING POWER EXPENDED UPON CONDENSERS AND CIRCUITS OF LOW POWER FACTOR.

By EDWARD B. ROSA.

The power factor of an alternating current flowing into and out of a good condenser, or a cable on open circuit, is so small as to make its measurement by a wattmeter somewhat difficult. Measurements made by the resonance method<sup>a</sup> and by the calorimetric method<sup>b</sup> gave power factors for some paraffined paper condensers of less than half of 1 per cent. These methods are, however, not adapted to general laboratory work. A simple wattmeter method of measuring power involves two corrections. First, for the power expended upon the fixed coil of the wattmeter, which is measured along with that of the condenser, and, second, for the change of phase of the potential current due to the combined inductance and capacity of the potential circuit. The correction (which is chiefly due to the inductance of the coil of the instrument which carries the potential current) is made small by using a large resistance in the potential circuit. Incandescent lamp filaments have extremely small inductance and capacity, and may be employed in series with one another, as the high resistance of the potential circuit; or wire coils may be employed if wound so as to avoid both inductance and capacity. The potential current, being very small, requires a relatively delicate suspension in order to give a satisfactory deflection. I have found difficulty in securing sufficient stability and sensitiveness at once, and have hence been led to the use of a series of null methods for measuring the power factor of a condenser or cable current, thereby avoiding the measurement of a deflection, and dispensing with the requirement of so great stability. These methods were devised and employed in a long series of measurements in 1898, but the work was interrupted before its completion, and circumstances have since prevented me from resuming the work.

<sup>a</sup> Rosa and Smith: *Phys. Rev.*, Jan., 1899.

<sup>b</sup> Rosa and Smith: *Phys. Rev.*, Feb., 1899.

The most obvious null method consists in using a variable inductance in the potential circuit. The difference of phase between the two currents being nearly  $90^\circ$ , if the potential current is slightly retarded by an added inductance the phase difference can be made  $90^\circ$  and the deflection reduced to zero. Knowing the value of the added inductance, the frequency of the current, and the resistance, the change of phase, and hence the power factor, can be readily computed.

#### USING AN AUXILIARY COIL ON THE WATTMETER.

There are, however, several other methods of getting a difference of phase of  $90^\circ$  and securing a zero deflection, any one of which can be employed for this purpose, according to the instruments one has available. These all depend upon the use of an auxiliary coil of fine wire wound over the fixed coil of coarse wire, having about the same number of turns as the fixed coil and made exactly equivalent to the fixed coil magnetically. This equivalence is shown by causing the same current to flow through the main coil and the auxiliary coil in opposite directions. The resulting magnetic field is then zero at the position of the suspended coil if the latter is not deflected when the current flowing through the main and auxiliary coils in opposite direction passes through the suspended coil also. Suppose, then, the condenser current  $i_1$  passes through the fixed coil, the potential current  $i_2$  passes through the suspended coil, and a small current  $i_3$  (in phase with  $i_2$ ) passes through the auxiliary coil. Then if  $K$  is the constant of the instrument, the deflection  $d_1$  of the suspended coil due to current  $i_1$  in the main coil and  $i_2$  in the suspended coil will be

$$d_1 = Ki_1i_2 \cos \phi_1$$

where  $\phi_1$  is the angular difference of phase between  $i_1$  and  $i_2$ .

The current  $i_3$  through the auxiliary coil would by itself (supposing there is no current in the main coil) produce a deflection  $d_2$  such that

$$d_2 = Ki_2i_3.$$

If the current  $i_3$  flows in such a direction as to make the deflection  $d_2$  opposite to  $d_1$ , then when the currents  $i_1$  and  $i_3$  flow simultaneously, the deflection is the difference between  $d_1$  and  $d_2$ ; and when this is made zero by adjusting  $i_3$  we have

$$i_2i_3 = i_1i_2 \cos \phi_1$$

$$\therefore \cos \phi_1 = \frac{i_3}{i_1}$$

Another way of expressing this result is to say that the current  $i_3$  (which is in phase with  $i_2$ , but *reversed by the connections*, and therefore differs in phase by  $180^\circ$  from  $i_2$ ) is added to  $i_1$ , and the vector sum is thereby made to differ by  $90^\circ$  in phase from  $i_2$ . The ratio of  $i_3$  to  $i_1$  then gives the angle through which  $i_1$  has been turned, to make it differ by  $90^\circ$  from  $i_2$ . Or, again, the magnetic field of  $i_3$ , added to the magnetic field of  $i_1$ , gives a resultant magnetic field which differs by  $90^\circ$  in phase from the potential current  $i_2$ . Hence, knowing  $i_3$ , we can find the power factor  $\cos \phi_1$ . The corrections for the resistance of the fixed coil and the inductance of the suspended coil being applied, we have  $\cos \phi$ , the true power factor.

The auxiliary or compensation current  $i_3$  may be secured by several different devices, as follows:

(1) From the terminals of a noninductive resistance  $r_3$  in the main circuit a shunt circuit is carried to the auxiliary coil, having a second condenser in series with it. Thus the current  $i_3$  is proportional to the main current  $i_1$ , to the resistance  $r_3$ , to the capacity of the secondary condenser  $C_3$ , and to the frequency of the current. That is

$$i_3 = p C_3 i_1 r_3 \text{ and } \frac{i_3}{i_1} = \cos \phi_1 = p C_3 r_3$$

Thus, knowing the capacity of the secondary condenser  $C_3$ , the frequency of the current ( $p = 2\pi n$ ), and the variable resistance  $r_3$ , we readily compute the power factor  $\cos \phi_1$ , which, corrected as before, gives  $\cos \phi$ .

(2) A portion of the potential current  $i_2$  is shunted off through the auxiliary coil, and the value of  $i_3$  becomes known if  $i_2$  and the resistances of the divided circuit are known.

(3) By transforming down from the high e. m. f.  $E$  impressed upon the condenser and shunt circuit, a small e. m. f.  $e_3$  is obtained, differing in phase by  $180^\circ$  almost exactly. A current  $i_3 = \frac{e_3}{R_3}$  is thus obtained, which differs in phase from the shunt current  $i_2$  by  $180^\circ$ , and this is used as a compensation current. The resistance  $R_3$  is varied until the deflection is zero, and  $e_3$  is measured by an alternating current voltmeter;  $i_3$  thus becomes known, and from it  $\cos \phi_1$  as before.

When the capacity of the condenser is small, and therefore the main current  $i_1$  is small, it may be better to pass  $i_1$  through the suspended coil and the potential current through the fixed coil. This gives rise to two other arrangements of the circuits. But the principle is the same in every case, the compensation current  $i_3$  being determined when the deflection of the wattmeter is zero, and the phase angle then found by a very simple calculation.

The measurements which I made by these various methods on the same condensers showed closely agreeing results, although the apparatus employed did not permit of measurements of the highest precision. I proceed to give a description of these methods, and a few examples of their use in measuring the power absorbed in some paraffined-paper condensers.

#### METHOD 1.—SIMPLE DEFLECTION METHOD.

The connections of the wattmeter to the source of alternating current and the condenser are shown in fig. 1, and the phase diagram is shown in fig. 2.  $F$  is the fixed coil and  $M$  the movable coil of the wattmeter,  $i_1$  the main condenser current, and  $i_2$  the potential current

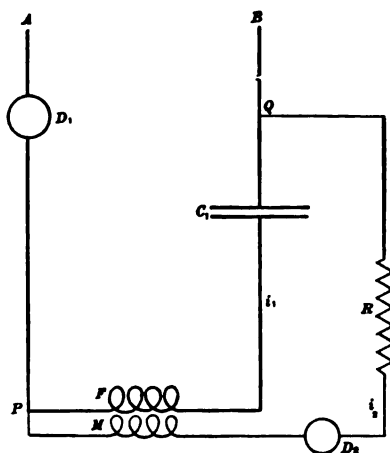


FIG. 1.—Connections for Method 1.

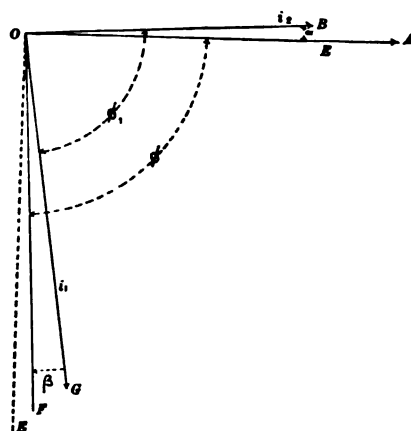


FIG. 2.—Phase diagram for Method 1.

passing through the large noninductive resistance  $R$ . An alternating potential of several hundred volts (up to 1,500 in my experiments) was applied to  $A B$ .  $D_1$  is the electrodynamic or alternating current ammeter for measuring the current  $i_1 + i_2$ , and  $D_2$  measures  $i_2$  alone. Inasmuch as these two currents differ in phase by nearly  $90^\circ$ , and  $i_2$  is relatively small, the reading of  $D_1$  is practically unaffected by the current  $i_2$ . The noninductive resistance is several thousand ohms, and if it is known accurately, and also the voltage  $E$ , the instrument  $D_2$  may be omitted and  $i_2$  calculated from  $E$  and  $R$ . The two coils  $F$  and  $M$  are joined together at the instrument in order that they may have the same potential and so avoid electrostatic attraction between them, a serious source of error when relatively high potentials are used, if the coils are not joined together.

$OA$  (fig. 2) represents the phase of the electromotive force  $E$ . The potential current lags behind it by an angle  $\alpha$ , due to the inductance of the moving coil and of the resistance  $R$ . The high resistance itself often possesses a greater capacity than inductance, and hence the combined inductance of the circuit ( $L_s$ ) may be less than that of the moving coil of the wattmeter alone. It may even be negative, and therefore the potential current may be ahead in phase of  $E$ . In my experiments the inductance and capacity of the resistance  $R$  were both very small, and  $L_s$  was taken as that due to the known inductance of the moving coil of the wattmeter. The line  $OG$  represents the condenser current  $i_1$ , differing in phase from  $i_2$  by the angle  $\phi_1$ .  $OF$  would have been the phase of this current if the fixed coil of the wattmeter had no resistance. The angle  $\beta$  is the difference of phase  $GOF$ , due to this resistance, and  $\tan \beta = r \div \frac{1}{pC} = pCr$ , where  $p = 2\pi n$ ,  $n$  being the frequency of the alternating current,  $C$  the capacity of the condenser, and  $r$  the resistance of the wattmeter coil  $F$  and the connections; that is, the resistance between  $P$  and  $Q$  when the condenser is short circuited. We then have the following expressions,  $d$  being the deflection of the wattmeter,  $K$  the constant of the latter,  $\phi_1$  the measured difference of phase  $BOG$  and  $\phi$  the corrected difference  $AOF$ , due to the condenser.

$$d = Ki_1i_2 \cos \phi_1, \text{ or } \cos \phi_1 = \frac{d}{Ki_1i_2}$$

$$\tan \alpha = \frac{pL_s}{R}, \tan \beta = pCr.$$

$$\phi = \phi_1 - \alpha + \beta$$

As stated above,  $\phi_1$  is the measured phase difference, and  $\alpha$  and  $\beta$  are respectively the small angular corrections due to the inductance of the potential circuit and the resistance of the fixed coils of the wattmeter

Table I.—RESULTS BY METHOD 1.

[ $n=120$  cycles;  $E=1,400$  volts.]

| Condenser employed. | $i_1$           | $i_2$           | $K$   | $d$  | $\cos \phi_1$ | Power factor, uncorrected. |
|---------------------|-----------------|-----------------|-------|------|---------------|----------------------------|
|                     | <i>Amperes.</i> | <i>Amperes.</i> |       |      |               | <i>Per cent.</i>           |
| Condenser No. 4..   | 1.653           | 0.1470          | 23300 | 26.0 | 0.00459       | 0.459                      |
| Condenser No. 5..   | 3.494           | 0.1666          | 23100 | 76.0 | 0.00565       | 0.565                      |
| Condenser No. 6..   | 3.494           | 0.1707          | 23100 | 75.0 | 0.00545       | 0.545                      |

and its connections.  $\cos \phi$  is then the power factor of the condenser. Table I gives some measurements made on three condensers, the second and third having a capacity about double the first.

The corrections  $\alpha$  and  $\beta$  are here very slight, so that  $\phi$  is sensibly equal to  $\phi_1$ . The condensers are Stanley paraffined paper condensers, and No. 4 has a sensibly smaller energy loss than the others.

#### METHOD 2.—INDUCTANCE IN SHUNT CIRCUIT.

Method 2 is similar to the first, except that a variable inductance  $L_s$ , fig. 3, is inserted in the shunt circuit and the current  $i_s$  is made to

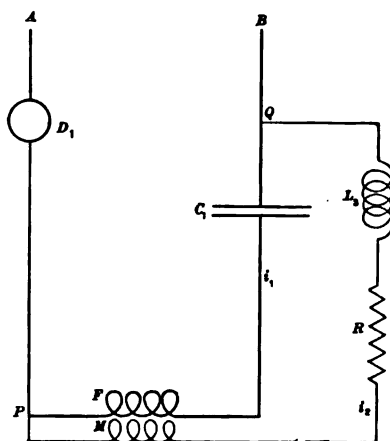


FIG. 3.—Connections for Method 2.

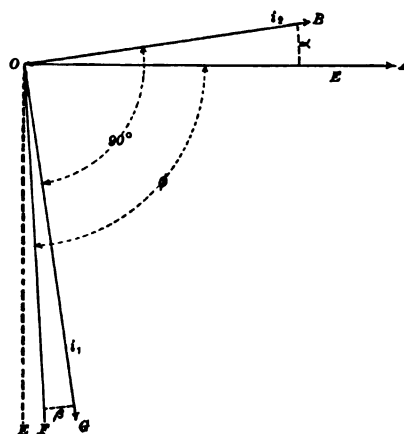


FIG. 4.—Phase diagram for Method 2.

lag by a larger angle than before, so that  $\phi_1$  is now  $90^\circ$ , and the deflection is zero. If  $L_s$  is the same as before and  $L_2$  is the added inductance,

$$\tan \alpha = \frac{p(L_2 + L_s)}{R}$$

$$\tan \beta = pCr$$

$$\phi = 90^\circ - \alpha + \beta$$

This is a very convenient method if a variable known inductance of suitable magnitude is at hand, and it gives excellent results. The frequency of the current must be known more accurately than before, for  $\alpha$  is now a much more important quantity than in method 1, since the value of  $\cos \phi_1$  depends mainly on  $\alpha$ . But  $d$ ,  $K$ ,  $i_1$ , and  $i_2$ , the quantities measured in method 1, need not be measured at all in this method.



Table II.—RESULTS BY METHOD 2.

[ $n=120$ ,  $E=1,260$  volts.]

| Condenser employed.  | $p=2\pi n$ | $L_2+L_3$ | $R$   | $\cos \phi_1$ | Power factor, uncorrected. |
|----------------------|------------|-----------|-------|---------------|----------------------------|
|                      |            | Henry.    | Ohms. |               | Per cent.                  |
| Condenser No. 4..... | 754        | 0.0530    | 8,548 | 0.00468       | 0.468                      |
| Condenser No. 5..... | 754        | 0.0627    | 8,548 | 0.00553       | 0.553                      |

These results do not agree exactly with those by the first method. The difference is partly due to errors of the experiment, and partly to a change in the temperature of the condensers and of the wave form of the electromotive force employed. For want of a suitable variable inductance, very little was done with this method.

### METHOD 3. AUXILIARY CONDENSER SHUNT ON THE MAIN CIRCUIT.

In this and the following methods the auxiliary compensation coil A is employed. As already explained, this coil is of fine wire and is

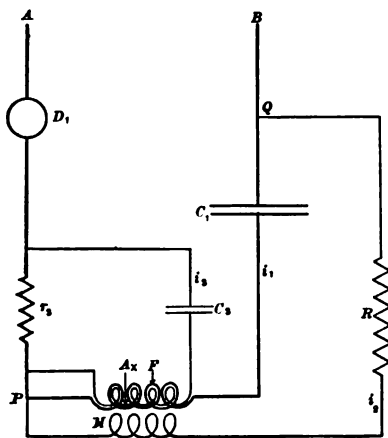


FIG. 5.—Connections for Method 3.

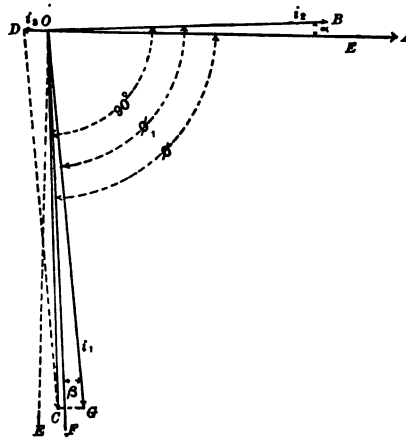


FIG. 6.—Phase diagram for Method 3.

wound on the outside of the main coil of the wattmeter, but thoroughly insulated from the latter. It is magnetically equivalent to the main coil, so that a current flowing in series through the two in opposite directions produces no magnetic field at the position of the moving coil.

A small variable resistance  $r_s$ , fig. 5, is inserted in series with the main current  $i_1$ , and an auxiliary condenser  $C_2$  is placed in series with the com-

pensation coil on the terminals of this resistance. The compensation current  $i_2$  is equal to  $pC_3 r_3 i_1$ , and is nearly  $90^\circ$  in advance in phase of  $i_1$ ; that brings it practically opposite in phase to  $i_2$ , as appears in fig. 6. We may compound  $i_1$  and  $i_2$  just as though they were in parallel in the same wire. The resultant is  $OC$ , and when  $i_2$  has such a value that there is no deflection of the movable coil,  $CO$  is exactly  $90^\circ$  ahead of  $OB$ . Then, since  $BOG = \phi_1$ ,

$$\cos \phi_1 = \frac{i_2}{i_1} = p C_3 r_3$$

Thus, it is unnecessary to measure  $E$ ,  $i_1$ ,  $i_2$ , or  $R$ , but only  $C_3$  and  $r_3$ , in addition to the frequency of the current. To apply the corrections  $\alpha$  and  $\beta$  to obtain the value of  $\phi$ , it is of course necessary to determine  $C$ ,  $r$ , and  $R$  approximately, as in method 1. The results obtained with five paraffined paper condensers are given in Table III. Nos. 2, 3, and 4 have a capacity of about 1.7 microfarads and 5 and 6 have a capacity of about 3.2 microfarads. These condensers were intended for 500 volt circuits. In these experiments, however, they were subjected to voltages between 910 and 1,386, most of the readings being taken at 1,260. This is a very convenient method.

Table III.—RESULTS BY METHOD 3.

| Condenser employed. | $E$           | $i_1$           | $p=2\pi n$ | $C_3$                 | $r_3$        | $\cos \phi_1 = p C_3 r_3$ |
|---------------------|---------------|-----------------|------------|-----------------------|--------------|---------------------------|
|                     | <i>Volts.</i> | <i>Amperes.</i> |            | <i>Farads.</i>        | <i>Ohms.</i> |                           |
| Condenser No. 2.... | 1,260         | 1.673           | 745        | $1.70 \times 10^{-6}$ | 3.46         | 0.00438                   |
| Condenser No. 2.... | 1,260         | 1.686           | 745        | $1.70 \times 10^{-6}$ | 3.48         | .00440                    |
| Condenser No. 3.... | 1,260         | 1.633           | 745        | $1.70 \times 10^{-6}$ | 3.48         | .00440                    |
| Condenser No. 3.... | 1,260         | 1.643           | 745        | $1.70 \times 10^{-6}$ | 3.48         | .00440                    |
| Condenser No. 4.... | 1,386         | 1.756           | 745        | $1.70 \times 10^{-6}$ | 3.50         | .00443                    |
| Condenser No. 4.... | 1,260         | 1.673           | 745        | $1.70 \times 10^{-6}$ | 3.52         | .00445                    |
| Condenser No. 2+3.  | 1,260         | 3.305           | 745        | $1.70 \times 10^{-6}$ | 3.62         | .00458                    |
| Condenser No. 3+4.  | 1,260         | 3.286           | 745        | $1.70 \times 10^{-6}$ | 3.66         | .00463                    |
| Condenser No. 5.... | 910           | 2.329           | 745        | $1.70 \times 10^{-6}$ | 4.35         | .00551                    |
| Condenser No. 5.... | 910           | 2.319           | 745        | $1.70 \times 10^{-6}$ | 4.36         | .00552                    |
| Condenser No. 5.... | 1,260         | 3.176           | 745        | $1.70 \times 10^{-6}$ | 4.26         | .00540                    |
| Condenser No. 6.... | 910           | 2.277           | 745        | $1.70 \times 10^{-6}$ | 4.20         | .00531                    |
| Condenser No. 6.... | 1,260         | 3.146           | 745        | $1.70 \times 10^{-6}$ | 4.12         | .00521                    |
| Condenser No. 5+6.  | 910           | 4.810           | 745        | $1.70 \times 10^{-6}$ | 4.50         | .00570                    |

### EFFECT OF THE SELF AND MUTUAL INDUCTANCE OF THE FIXED AND AUXILIARY COILS.

The self-inductance of the fixed coils of the wattmeter employed in these measurements is 0.075 millihenry, of the auxiliary coil 0.111 millihenry, and the mutual inductance of the two is 0.072 millihenry. At a frequency of 120,  $p = 754$ , and the reactance of the auxiliary coil is 0.083 ohm. This is wholly negligible in its effect upon the magnitude or phase of the compensation current  $i_3$ . The self-inductance of the fixed coil has no appreciable effect upon the magnitude or phase of the main current  $i_1$ . The mutual inductance between the coils, however, may have an appreciable effect upon the magnitude of the compensation current  $i_3$  in the auxiliary coil, provided the electromotive force  $e_3$  on the compensation circuit is small. Thus, the back e. m. f. in this circuit, due to the current  $i_1$  in the fixed coil, is  $pMi_1$ , and this is equal to 0.16 volt when  $i_1 = 3$  amperes and  $p = 754$ . In methods 4 and 5,  $e_3$  was usually 45 to 70 volts, and hence the correction  $\gamma$  due to mutual inductance is only one or two units in the last decimal place of the values given for  $\cos \phi_1$  in Tables IV and V. In method 3, however,  $e_3$  was smaller and the correction is 5 to 6 in the last decimal place of the values of  $\cos \phi_1$  in Table III.

The expression for  $\cos \phi_1$  in which the mutual inductance of the fixed and auxiliary coils is taken into account, is

$$\text{For Method 3, } \cos \phi_1 = \frac{i_3}{i_1} = pC(r_3 - pM)$$

$$\text{For Method 4, } \cos \phi_1 = \frac{i_3}{i_1} = \frac{\left(\frac{i_1}{i_2}r_3 - pM\right)}{R_3 + r_3}$$

$$\text{For Method 5, } \cos \phi_1 = \frac{i_3}{i_1} = \frac{\left(\frac{e_3}{i_1} - pM\right)}{R_3}$$

There is also a slight correction  $\delta$  to be made in method 3, for the potential current  $i_3$ , which also passes through  $r_3$ ; that is, the resultant of  $i_1$  and  $i_3$  passes through  $r_3$ . This makes a slightly larger current and shifts the phase a little. The shifting of the phase, however, does no appreciable harm. The correction  $\gamma$  can be made insignificant by increasing  $r_3$  and using an auxiliary condenser of correspondingly smaller capacity, while the correction  $\delta$  is reduced to an insignificant quantity by having the potential current  $i_3$  sufficiently small in comparison with  $i_1$ . In fact, it is easy under most circumstances to eliminate the three corrections  $\beta$ ,  $\gamma$ ,  $\delta$  by properly proportioning the coils,

leaving only a small correction  $\alpha$  to be applied, due to the resistance of the fixed coils and connections.

#### METHOD 4.—SHUNT ON POTENTIAL CIRCUIT.

In this case, fig. 7, the compensation current  $i_3$  is shunted off from  $i_2$ , the resistance  $r_3$  being in the potential circuit through the moving

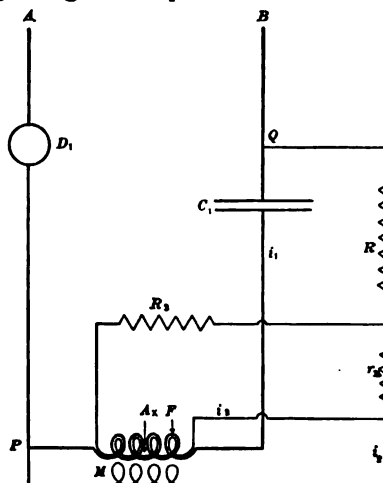


FIG. 7.—Connections for Method 4.

coil instead of the main circuit as in method 3. The compensation current is thus in phase with  $i_3$ , but by reversing the terminals of the compensation coil it has the same effect as though it were opposite in phase. Thus, fig. 6 represents this method also.

Therefore, since

$$i_3 = i_2 \frac{r_3}{R_3 + r_3}, \quad \cos \phi_1 = \frac{i_3}{i_1} = \frac{i_2}{i_1} \frac{r_3}{R_3 + r_3}$$

Table IV.—RESULTS BY METHOD 4.

[E=1,260 volts.]

| Condenser employed.   | $i_1$           | $i_2$          | $r_3$        | $R_3$ | $\cos \phi_1$ | Power factor uncorrected. |
|-----------------------|-----------------|----------------|--------------|-------|---------------|---------------------------|
|                       | <i>Amperes.</i> | <i>Ampere.</i> | <i>Ohms.</i> |       |               | <i>Per cent.</i>          |
| Condenser No. 5 ..... | 3.016           | 0.1396         | 507          | 3800  | 0.00544       | 0.544                     |
| Condenser No. 5 ..... | 3.037           | 0.1729         | 507          | 4830  | 0.00540       | 0.540                     |
| Condenser No. 6 ..... | 2.962           | 0.1735         | 507          | 5100  | 0.00529       | 0.529                     |

**METHOD 5. USING A TRANSFORMER FOR THE COMPENSATION CURRENT.**

In this case the high potential winding of a transformer is joined across the terminals *A B* of the circuit, fig. 8, and the low potential

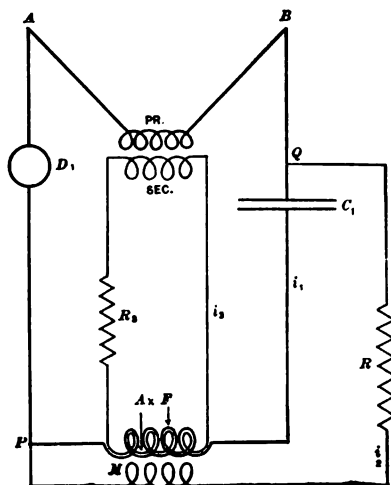


FIG. 8.—Connections for Method 5.

coil supplies the current  $i_3$  to the compensation coil through a resistance  $R_s$ . This current is opposite in phase to  $i_1$ , as in methods 3 and 4.

$$i_3 = \frac{e_s}{R_s}$$

$$\cos \phi_1 = \frac{i_3}{i_1} = \frac{e_s}{i_1 R_s}$$

The small electromotive force  $e_s$  may be determined by the ratio of transformation, or by direct measurement. It is very convenient to put a voltmeter on the secondary of the transformer and measure  $e_s$  directly, and then get  $E$  by multiplying by the ratio of transformation. Some of the results obtained by this method are given in Table V.

The transformer used here is the potential transformer which was employed to get the voltage on the main condenser circuit. In these experiments it had a ratio of about 14, and hence, for 90 volts on the secondary there was 1,260 on the primary. The secondary consisted of two equal coils, and the compensation coil was joined to the terminals of one of them. The slight current used (about a hundredth of an ampere) did not alter the ratio of transformation of the transformer.

## METHOD 6. SHUNT ON MAIN CIRCUIT.

This method corresponds to method 4, but the condenser is placed in the moving coil circuit, fig. 9, and a portion of  $i_1$  is shunted off into

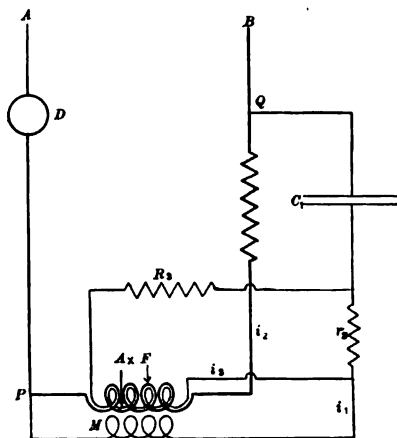


FIG. 9.—Connections for Method 6.

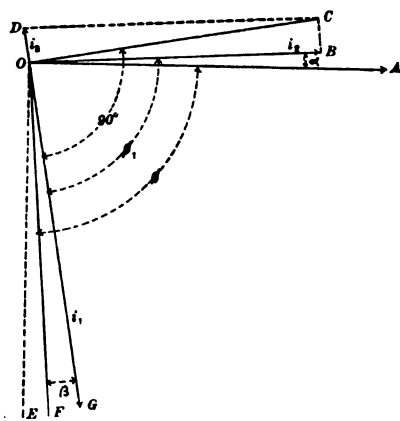


FIG. 10.—Phase diagram for Method 6.

the compensation coil. Thus,  $i_3$  is opposite in phase to  $i_1$  and is combined with  $i_2$  so that the resultant of  $i_2$  and  $i_3$ ,  $OC$ , is  $90^\circ$  different in phase from  $i_1$ , fig. 10.

Thus: 
$$i_3 = i_1 \frac{r_3}{R_3 + r_3}$$

This method is especially adapted to small condensers, where the current  $i_1$ , is very small and  $i_2$  can be increased above its usual value.

Table V.—RESULTS BY METHOD V.

[ $E=1,260$  volts=voltage on the condensers.  $e=45$  volts=voltage on the compensation circuit.]

| Condensers employed.    | $R_3$ | $i_3 = \frac{e}{R_3}$ | $i_1$          | $\cos \phi_1 = \frac{i_3}{i_1}$ |
|-------------------------|-------|-----------------------|----------------|---------------------------------|
| FEBRUARY 11.            |       |                       |                |                                 |
|                         |       | <i>Ampere.</i>        | <i>Ampere.</i> |                                 |
| Condenser No. 2 .....   | 6000  | 0.00750               | 1.633          | 0.00459                         |
| Condenser No. 3 .....   | 6250  | 0.00720               | 1.614          | 0.00446                         |
| Condenser No. 4 .....   | 6000  | 0.00750               | 1.633          | 0.00459                         |
| Condenser No. 2+3 ..... | 2950  | 0.01525               | 3.206          | 0.00476                         |
| Condenser No. 3+4 ..... | 2965  | 0.01517               | 3.181          | 0.00477                         |
| Condenser No. 5 .....   | 2560  | 0.01758               | 3.146          | 0.00559                         |
| Condenser No. 6 .....   | 2730  | 0.01648               | 3.089          | 0.00533                         |
| Condenser No. 5+6 ..... | 1310  | 0.03435               | 6.330          | 0.00542                         |

Table V.—RESULTS BY METHOD V—Continued.

| Condensers employed.    | $R_3$ | $i_3 = \frac{e}{R_3}$ | $i_1$           | $\cos \phi_1 = \frac{i_3}{i_1}$ |
|-------------------------|-------|-----------------------|-----------------|---------------------------------|
| FEBRUARY 12.            |       |                       |                 |                                 |
|                         |       | <i>Amperes.</i>       | <i>Amperes.</i> |                                 |
| Condenser No. 2 .....   | 6290  | 0.00715               | 1.604           | 0.00445                         |
| Condenser No. 3 .....   | 6530  | 0.00689               | 1.573           | 0.00438                         |
| Condenser No. 4 .....   | 6290  | 0.00715               | 1.594           | 0.00448                         |
| Condenser No. 2+3 ..... | 3080  | 0.01461               | 3.146           | 0.00464                         |
| Condenser No. 3+4 ..... | 3050  | 0.01475               | 3.105           | 0.00475                         |
| Condenser No. 5 .....   | 2650  | 0.01698               | 3.022           | 0.00561                         |
| Condenser No. 6 .....   | 2790  | 0.01613               | 2.978           | 0.00541                         |
| Condenser No. 5 .....   | 2680  | 0.01679               | 3.042           | 0.00552                         |

## METHOD 7. AUXILIARY CONDENSER ON TRANSFORMER.

This corresponds to method 5, where the transformer is used as a source of  $i_3$ , but as in method 6, the condenser under test is placed in the moving coil circuit. In order to bring the compensation current  $90^\circ$  out of phase with  $i_3$ , with which it is combined, an auxiliary con-

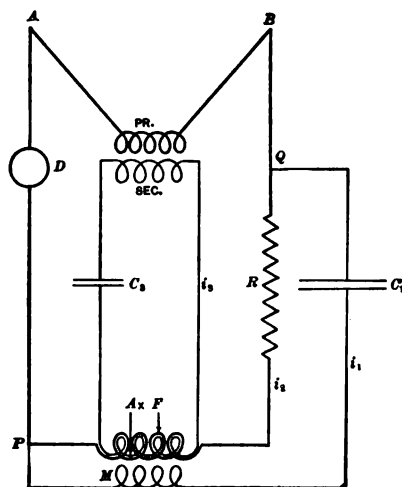


FIG. 11.—Connections for method 7.

denser  $C_3$  is placed in the  $i_3$  circuit instead of a resistance, fig. 11. The phases are shown in fig. 10. Thus:

$$i_3 = p C_3 e_3 \qquad i_2 = \frac{E}{R}$$

$$\cos \phi_1 = \frac{i_3}{i_2} = p C_3 e_3 \frac{R}{E} = \frac{p C_3 R}{n}$$

where  $n$  is the ratio of transformation of the potential transformer.

Or, the formula may be written  $\cos \phi_1 = \frac{p C_3 e_3}{i_2}$ , and  $e_3$  determined directly by a voltmeter and  $i_2$  by an ammeter or electro-dynamometer.

**Table VI.**—SUMMARY OF QUANTITIES TO BE MEASURED IN THE VARIOUS METHODS.

|               |     |     |       |       |     |     |     |       |       |       |       |
|---------------|-----|-----|-------|-------|-----|-----|-----|-------|-------|-------|-------|
| Method 1..... | $d$ | $K$ | $i_1$ | $i_2$ |     |     |     |       |       |       |       |
| Method 2..... |     |     |       |       | $p$ | $L$ | $R$ |       |       |       |       |
| Method 3..... |     |     |       |       | $p$ |     |     | $r_3$ | $C_3$ |       |       |
| Method 4..... |     |     | $i_1$ | $i_2$ |     |     |     | $r_3$ |       | $R_3$ |       |
| Method 5..... |     |     | $i_1$ |       |     |     |     |       |       | $R_3$ | $e_3$ |
| Method 6..... |     |     | $i_1$ | $i_2$ |     |     |     | $r_3$ |       | $R_3$ |       |
| Method 7..... |     |     |       | $i_2$ | $p$ |     |     |       | $C_3$ |       | $e_3$ |

The quantities in Table VI are as follows:

$d$  is the deflection of the electro-dynamometer in scale divisions.

$K$  is the constant of the electro-dynamometer.

$i_1$  is the main current, through the condenser under test.

$i_2$  is the potential current, through the large resistance  $R$ .

$p$  is  $2\pi$  times the frequency.

$L$  is the variable inductance added to the potential circuit.

$R$  is the large potential resistance, as free as possible from inductance and capacity.

$r_3$  is the small resistance shunted by the auxiliary circuit.

$C_3$  is the auxiliary condenser, of constant value.

$R_3$  is the resistance in series with the auxiliary coil.

$e_3$  is the relatively small electromotive force supplying the auxiliary current.

Thus it appears that each method requires the determination of three or four quantities, and a choice of method will be determined in part by what instrumental facilities are available for the work. All the methods are capable of giving good results, but the null methods are more satisfactory than method 1, unless one has a wattmeter which is both sensitive and stable, and which has a nearly uniform field, so that the constant  $K$  does not vary too rapidly. All the methods require the two correction terms  $\alpha$  and  $\beta$  to be applied to derive the true power factor  $\cos \phi$  from the measured power factor  $\cos \phi_1$ . These two corrections are of opposite sign, as already explained, and should be small. In the experiments here described the resistance of the fixed coils of the wattmeter and the leads through which the main



current flowed—that is, from  $P$  to  $Q$  with the condenser short circuited (figs. 1, 3, 5, etc.)—was 0.08 ohm. When the current is 1.6 amperes,  $i^2_1 r$  is 0.20 watt, and this requires a correction of about 0.0001 in the power factor. The correction for inductance in the moving coil is of the same order of magnitude and hence the results of the measurements cited above as examples are not very different from the true power factor  $\cos \phi$ . At the time these measurements were made I did not have facilities for measuring the various quantities involved in these several methods with sufficient accuracy to make a crucial test of the methods. The results, however, show that the various methods agree substantially, and there is no reason to doubt the entire reliability of any of them. The values given above for the power factor are somewhat larger than those found for the same condensers by the calorimetric method,<sup>a</sup> but in those experiments the high electro-motive force impressed upon the condensers was obtained by resonance from a lower electro-motive force and the harmonics were therefore largely suppressed. In all the work described in this article the high electro-motive force was obtained by transforming up, and hence the harmonics were retained and magnified by the condensers. Since the power factor is higher for higher frequencies, it is higher for the harmonics and therefore for the distorted wave. This is one of the subjects to be investigated when this work is resumed.

By taking careful note of the temperature of the condensers, and determining accurately the frequency of the current and the exact values of the  $\alpha$  and  $\beta$  corrections, we would obtain not only a crucial test of the various methods, but also data as to how the energy losses vary with the frequency, temperature, and wave form. I hope soon to repeat these measurements with improved apparatus, and hope to obtain results of sufficient precision to give valuable information of this character.

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<sup>a</sup> Rosa and Smith: Phys. Rev., Jan., 1899.



## THE RELATIVE INTENSITIES OF METAL AND GAS SPECTRA FROM ELECTRICALLY CONDUCTING GASES.<sup>a</sup>

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By P. G. NUTTING.

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From an arc may be obtained a nearly pure spectrum of the electrodes, the spectrum of the surrounding atmosphere under certain conditions scarcely appearing at all. On the other hand, the light from a discharge tube may give a pure spectrum of the atmosphere about the electrodes, but no spectrum of the electrodes themselves. Various kinds of arcs, sparks, and discharge tubes under various conditions may exhibit mixed spectra in all intermediate proportions.

Arc, spark, and low-pressure discharge are but particular cases of a general case in which a gas forms part of an electric circuit otherwise metallic. It is the purpose of this paper to discuss the many conditions that may govern the preponderance of the spectrum of the electrodes over the spectrum of the intervening gas, select the predominant influences and then if possible to map out the conditions necessary for the production of a given pure spectrum with lines of a given character. To obtain coordinate data over the wide range necessary to warrant general conclusions, the spectra of twenty-one different metals were photographed each under a uniform schedule of eighty-one different conditions.

The relative intensity of the spectrum of the electrodes depends primarily, of course, on the relative proportion of its vapor present in the part of the arc, spark, or discharge tube observed. We are

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<sup>a</sup> This paper is the third of a series of four on the general subject of the preponderance of one spectrum over another in a mixture of two spectra. The first of this series dealt with the conditions governing the relative intensities of the spectra of two mixed elementary gases under low-pressure discharge. (*The Spectra of Mixed Gases*, Bulletin No. 1, pp. 77-81, November, 1904.) The second dealt with the relative intensities of the two different spectra exhibited by the electro-negative elementary gases. (*On Secondary Spectra*, Bulletin No. 1, pp. 83-93, November, 1904.) The fourth paper of the series is to be on the spectra of alloys.

concerned first with the conditions governing the amount of electrode metal vaporized and then with the laws governing spectral preponderance in mixed gases, namely; (1) increasing the relative amount of a gas present in a mixture of conducting gases increases the relative intensity of its spectrum; (2) other things being equal, in a steady discharge the spectrum of that component of a mixture having greater atomic weight will be brighter;<sup>a</sup> (3) in a disruptive or forced oscillatory discharge the atomic weight effect is less.

When the internal oscillations are free (instead of being forced) as in the arrangement used by Ramsey, Lilienfeld, and others to obtain the spectrum of argon in that of ordinary air, perhaps a fourth law would be necessary. Konen (Beibl., 1904, p. 1000) has cited the case of mixtures containing oxygen as apparent exceptions to the atomic-weight law (2) above. But oxygen, it must be remembered, has only a very faint (visible) anode glow. Hence to make a fair test of the law we must either take the original faintness of the anode glow into account, or else use the cathode-glow spectrum for comparisons. I have always done the latter. In explanation of the exceptional absence of luminosity in the oxygen anode column, I would suggest that there may be the usual amount of radiation in this case, but that it is chiefly in the infra-red region. The anode column is known to be a region of frequent ionization and recombination,<sup>b</sup> hence we should expect it would be a region of great chemical changes as well. In oxygen, ozone would be rapidly formed and decomposed; hence the radiation from the anode column would consist largely of the strong infra-red ozone bands recently studied by Angstrom.<sup>c</sup>

The metallic or nonmetallic character of the atmosphere surrounding a spark appears to make no difference whatever with the behavior of the spectrum of the electrodes. A spark between aluminum electrodes in an atmosphere of mercury vapor shows identically the same changes on the introduction of capacity, inductance, spark in series, etc., that it exhibits in hydrogen. On the other hand, the spectrum of an iron spark preserves its invariant character as well in mercury vapor as in air, oxygen, or hydrogen.

The vaporization of the electrodes with which we are concerned in the arc, spark and vacuum tube is by no means a mere temperature effect. The amount of vaporization of an electrode appears to bear no simple relation either to vapor pressure or temperature. It does,

<sup>a</sup> The Spectra of Mixed Gases, *Astrophys. J.* 19, p. 107; Mar., 1904.

<sup>b</sup> Thomson: *Conduction of Electricity Through Gases*, p. 486, ¶ 269.

<sup>c</sup> *Arkiv. för "Matematik," Ast. och Fysik* 1, pp. 347-353; 1904. *Beiblätter*, 28, p. 1142; Nov., 1904.

however, vary enormously with the electrical conditions in the conducting gas, and for this reason a brief discussion of the phenomena of gas conduction is here given.

#### ELECTRICAL CONDUCTION IN THE ARC, SPARK, AND DISCHARGE TUBE.

Our first problem is to find a relation—a sort of generalized Ohm's law—between the current passing through a gas, the drop in potential and certain constants relating to the gas and the metal used as electrodes. This relation involves the solution of a differential equation containing an unknown function. This function I have constructed, but it gives a nonintegrable form to the differential equation. However, the particular case of steady current admits of a graphical solution (due to W. Kaufmann<sup>a</sup>) and covers a wide variety of the phenomena with which we are concerned.

Consider a circuit containing an e. m. f.  $E_0$ , an ohmic resistance  $R$ , inductance  $L$ , capacity  $C$ , and a column of gas through which the drop of potential is expressible by the (experimentally determined) function  $e(i)$  of the current. Equating energy supplied to energy taken up by the circuit,

$$E_0 i = i^2 R + Li \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt} + i e(i)$$

The final term,  $ie(i)$ , represents the energy used up in the discharge in whatever manner the drop of potential  $e$  varies with the current.

As an illustration, consider the energy used by a column of nitrogen 10 cm long and at 1 mm pressure, with platinum electrodes carrying 1 milliampere of current. The anode fall of potential is about 20 volts, the fall through the gas about 600 volts (60 volts per cm), and the cathode fall 260 volts. Hence the total energy used will be about 0.9 watt, of which about a third is used up at the cathode, two-thirds in the gas, and a negligible amount at the anode. If the column were shortened to 5 cm the energy used at the electrodes would be the same, while in the gas but half the previous amount of energy would be used up, and the total fall of potential would be 580 instead of 880 volts. In an ordinary illuminating arc carrying a current of 10 amperes, of the total drop in potential of approximately 50 volts, about half is at the electrodes, so that the luminescent gas receives about half the energy supplied. In the spark we may be reasonably certain that the energy losses are distributed in a manner intermediate between

<sup>a</sup> Ann. d. Phys. 2, pp. 158-176; 1900.

that of the arc and the discharge tube mentioned above. In every case it appears to require a greater expenditure of energy to force a (positive) current from a gas into a metal than from a metal into a gas.

Now, the loss of energy at the electrodes is not within the electrode itself, nor is it in the adjacent gas outside; it is localized right at the surface separating the electrode from the conducting gas, and represents energy used in forcing the current from the metal to the gas or vice versa. Apparently all the electrode loss of electrical energy reappears as heat energy, about half being given to the gas and half to the electrode. In the spark with capacity, probably a large part of this energy is used up directly in vaporizing the surface layer of the metal, the discharge being so sudden that there is no time for the heat to be communicated to the adjacent metal or gas. In the arc a considerable part of this surface heat is conducted back and radiated from the solid electrode.

The electrical energy equation above can only be solved when the function  $e(i)$  is known. This function may be constructed from the properties of its derivative in the following manner: since  $e(i)$  has in general a single maximum and a single minimum, but no real roots, write for the derivative  $de = a(e - e_1)(e - e_2)di$ , where  $e_1$  and  $e_2$  are the ordinates of the maximum and minimum. By integration

$$e = e_2 - \frac{e_1 - e_2}{ce^{ai(e_2 - e_1)} - 1},$$

a function having the general properties of the characteristic curve for a gas, but which does not give a solvable form when substituted in the energy equation.

When the current is steady the energy equation becomes

$$E_0 = iR + e(i),$$

from which the value of the current may be obtained (Kaufmann, l. c.) graphically, it being the abscissa of the point of intersection of the line  $E_0 - iR$  with  $e(i)$ . (The abscissa of the point of intersection of  $y = \varphi_1(x)$  and  $y = \varphi_2(x)$  is a root of the equation  $\varphi_1(x) - \varphi_2(x) = 0$ .) Since in a conducting gas but little energy is stored in comparison with what is dissipated, the  $e(i)$  in the last equation is the same as that in the general equation above.

Some typical characteristic curves,  $e(i)$ , are shown in fig. 1. The curve marked  $g$  shows the drop of potential as a function of the current in its general form. It is to be noted that: (1) the current starts with a brush discharge, during which the drop increases with increasing current; (2) there is a maximum drop at which the gas

breaks down; (3) after the gas breaks down for a time the drop decreases as the current increases; (4) there is a minimum drop at which the drop is independent of the current, (5) followed by a region in which drop and current increase together. This form of curve may be realized in a gas at moderate pressures with the electrodes at a moderate distance apart.<sup>a</sup>

The curve *a*, fig. 1, is characteristic of an ordinary arc. The maximum is very high (say 20,000 volts), while the minimum is low, only about 50 volts. The downward sloping part of the curve is the portion usually observed, using but little series resistance and low e. m. f. To start an arc, like any other form of gas conductor, one must either (1)

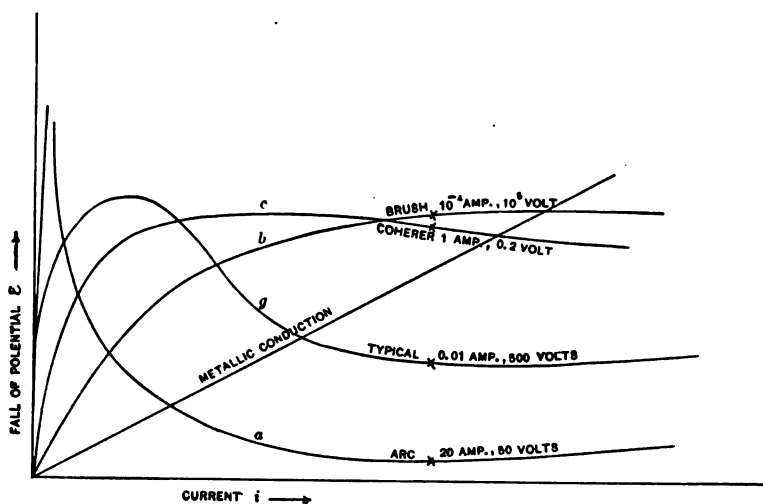


FIG. 1.—Characteristic conduction curves.

raise the e. m. f. above the maximum drop or (2) depress the maximum drop below the e. m. f.; the latter may be done by bringing the electrodes into contact, starting ionization by other means or lowering the pressure on the intervening gas.

The curve *b* is characteristic of a brush discharge at high pressures between electrodes wide apart or pointed. The electromotive forces concerned are very large, while the currents are small. *c* is typical of very small distances and high pressures. It was obtained by Guthe and Trowbridge,<sup>b</sup> using steel balls nearly in contact ( $10^{-7}$  cm apart) as a coherer. The maximum drop is only about one-fourth volt, while

<sup>a</sup> Studied more in detail, this curve shows steps and other irregularities indicating saturation, ionization by impact, discharge not covering electrodes, etc., but which need not be considered here.

<sup>b</sup> Phys. Rev., 11, p. 29; 1900.

the corresponding current is nearly half an ampere. All these curves are seen to be special forms of the type curve  $g$ , and all are capable of representation by the function  $e(i)$  above constructed.

Given the characteristic curve for a gas under given conditions, then by means of the Kaufmann diagram (fig. 2) we may find what the current will be in a circuit containing an e. m. f.  $E_0$  and an ohmic resistance  $R$ . The diagram is drawn for a circuit containing a gas at rather low pressure, with a moderately large e. m. f. controlled by considerable resistance. Keeping the e. m. f.  $E_0$  constant, if the external resistance be increased, the line  $E_0 - iR$  will take a greater and greater slope and intersect the  $e(i)$  curve farther and farther to the left. At a certain slope it will intersect the curve in three places and the current may become intermittent. Finally

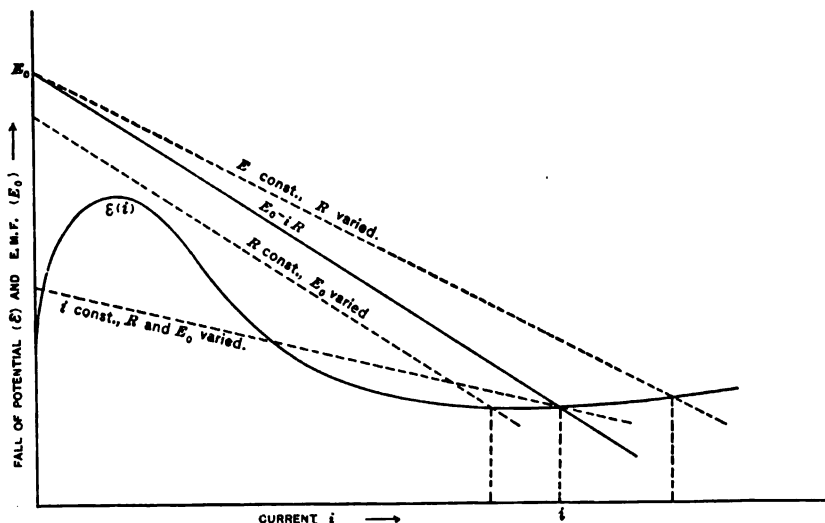


FIG. 2.—Kaufmann diagram.

it will become tangent to the under side of the gas curve and the heavy current will cease abruptly, leaving only the brush discharge corresponding to the upper intersection. Diminishing the external resistance, the current will not start again where it ended so abruptly, but only when the resistance line becomes tangent to the curve on its upper side. By taking a higher e. m. f., only single intersections will occur as the resistance is varied, hence the current will never become intermittent and the whole  $e(i)$  curve may be followed through experimentally, as it can not be when the e. m. f. used is just sufficient to break down the gas. Stark<sup>a</sup> has worked out the relation between the critical minimum current for the arc (when the

<sup>a</sup> Ann. d. Phys. 12, pp. 694-699; 1903.



arc breaks) and e. m. f. and electrode metal. The relation to the density of the surrounding atmosphere may be seen from figs. 1 and 2, since as the pressure is diminished, curve *a*, fig. 1, is deformed into curve *g*.

When a spark or Plücker tube is excited by an induction coil or transformer, it is the e. m. f.  $E_0$  that is variable, while the resistance remains constant. In this case the  $E_0-iR$  line moves parallel to itself, and the resultant current flowing through the gas has quite a different wave form from what it would have if the current were controlled by varying the resistance alone. Fig. 3, taken from the author's paper on Rectifying Effects, shows a typical curve for the current through a

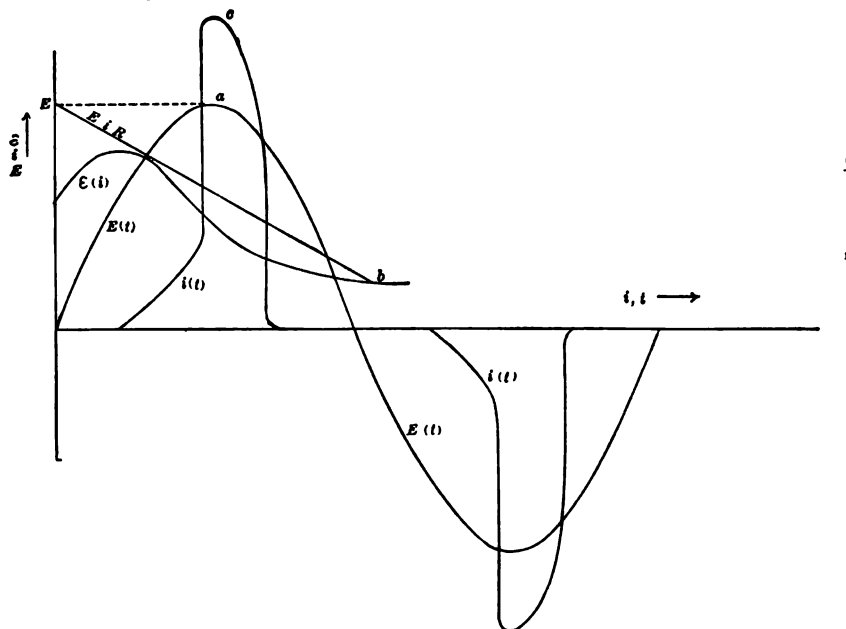


FIG. 3.—Alternating current curve through gases.

gas upon which a sine alternating e. m. f.  $E(t)$  is impressed. Starting with the point *a* on the curve  $E(t)$  giving the e. m. f. as a function of time, the point *c* on the curve  $i(t)$  giving current as a function of time is obtained by means of the intermediate points *E* and *b*, assuming that the characteristic curve  $e(i)$  for the gas is of the form shown and that sufficient ohmic resistance has been used to give the line  $E-iR$  the slope indicated. Fig. 3 of course represents the current only in the ideal case of a circuit containing neither inductance nor capacity. When inductance ( $L$ ) is added, instead of  $E-iR$  we must use the line  $E-iR-L\frac{di}{dt}$  (a line in three dimensions  $E, i, \frac{di}{dt}$ ) to find the current

graphically. Without constructing this three dimensional diagram it is easy to see that the general effect of adding inductance is to increase the slope of the  $E-iR$  line while the current is starting and to decrease the slope when the current is decreasing. Hence the vertical portions of the current curve  $i(t)$ , fig. 3, will be replaced by sloping lines which will always be convex toward the axis of abscissæ and sharp corners will all be rounded off.

When a low voltage transformer (say 1,000 volts) is used to excite a Plücker tube with but little resistance or inductance in series, the current, if it pass at all beyond a mere brush discharge, will pass with brilliant flashes of light. This sudden and very great increase in the current is what we should expect, since in this case in the Kaufmann diagram the line  $E-iR$  is nearly horizontal and after it has once passed over the crest of the  $e(i)$  curve, intersects the latter only at a very large value of the current.

But it is only after capacity has been added that sufficient quantity of electricity can be supplied to actually attain the enormous currents predicted by the Kaufmann diagram for circuits of low resistance and inductance. The spark with capacity runs through the characteristic  $e(i)$  curve for a gas up to large values of  $i$ ; without capacity, an intermittent or alternating current runs over the small current end of the curve only. When the spark passes several times during each alternation, all discharges after the first are made over a modified and much lower characteristic curve. The effect of capacity on the form of the current curve,  $i(t)$ , fig. 3, is the opposite of the inductance effect, it sharpens the corners and increases the discontinuities. But quantity not being independent of the current, like  $\frac{di}{dt}$ , we can not

extend the Kaufmann diagram to cover the case of capacity as was possible with inductance, nor do the equations appear to admit of other than a graphical solution. Aside from making a very heavy maximum current possible, adding capacity has another very important effect generally overlooked, namely, that of reducing the potential of the discharge, or more strictly speaking the maximum e. m. f. of the spark circuit. If the supply of energy is limited, as it is in an induction coil or small transformer, the voltage may be cut down to 10 or 20 per cent of its value when no capacity is in circuit, and cutting down the voltage nearly to that necessary to just cause a spark to pass, increases the discontinuities in the current curve, and hence may actually strengthen the metallic spectrum. I have succeeded in showing this in a very striking manner. A 2-mm spark in hydrogen at about

atmospheric pressure was produced by a 300-watt 5,000-volt transformer controlled by noninductive resistance in its 100-volt side. With only 50 watts supplied, capacity was introduced until the spark barely passed, and thus a brilliant metallic spectrum, sharing the spectral energy about equally with the hydrogen, was obtained. But when the energy supply was increased from 50 to 200 watts the metallic spectrum was weakened to less than one-fourth of the total energy. Introducing inductance did not appear to alter this voltage effect.

Another aspect of the effect of capacity on the spark should be mentioned here. If a spark with large capacity in parallel and fed with a steady supply of current is examined with a rotating mirror, bright sparks appear at regular intervals. On decreasing the capacity gradually the sparks appear smaller and nearer together, until at a capacity of a few centimeters they are no longer distinguishable. The spark has now become an arc, and from this point of view an arc may be regarded as a rapid succession of very minute sparks.

Hartmann,<sup>a</sup> working with a magnesium arc carrying a current of 4 amperes, found that the line *Mg* 4481 appeared much brighter with only 20 volts e. m. f. than it did on a 120-volt circuit. A Kaufmann diagram (see fig. 2) indicates a very plausible explanation. The 20-volt current is just on the point of breaking, hence the  $E-iR$  line will cut the characteristic gas curve in three points, while the 120-volt line cuts it but once. The 20-volt arc should then be of the nature of a rapid succession of very short sparks, and might well show spark lines in its spectrum.

#### EXPERIMENTAL RESULTS.

Experimental work was undertaken chiefly with the object of providing a wide range of homogeneous data relating to the conditions governing the relative intensity of the spectrum of the electrodes to the spectrum of the intervening gas in arc, spark, and discharge tube. Spectra of 21 different metals were photographed with a large spectrograph, all under a uniform schedule of 81 different conditions. Three different gases were used as atmospheres; pure hydrogen, pure oxygen, and air, each being used at three different pressures, namely, 760 mm, 380 mm, and 100 mm, a considerable number of metals being tried at 4 mm as well. On each plate nine spectra were taken with the same metal in the same gas at the same pressure, but with nine different modes of excitation, thus making 81 different spectra of each metal.

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<sup>a</sup> *Astroph. J.* 17, p. 145; 1903.

The spectra of the following metals were examined:

|            |            |            |
|------------|------------|------------|
| Silver.    | Chromium.  | Palladium. |
| Aluminium. | Copper.    | Platinum.  |
| Gold.      | Iron.      | Antimony.  |
| Bismuth.   | Mercury.   | Silicon.   |
| Carbon.    | Magnesium. | Tin.       |
| Cadmium.   | Nickel.    | Tellurium. |
| Cobalt.    | Lead.      | Zinc.      |

The excitation schedule was as follows:

1. Large capacity—about 0.04 mf.
2. Moderate capacity—about 0.02 mf.
3. Small capacity—about 0.006 mf.
4. No capacity; i. e., 5,000-volt, a. c. arc.
5. Series spark with small capacity—0.006 mf—over both sparks.
6. Same as (1), but with large inductance added—0.9 millihenry.
7. Same as (2), with same inductance as (6).
8. Same as (3), with same inductance as (6).
9. Same as (5), with same inductance as (6).

In the last four cases capacity was varied rather than inductance, because the inductance effect is sharply marked and persistent, while the capacity effects are more varied. These nine spectra, each about 15 cm long and 4 mm wide, were photographed side by side on the same plate, so that very slight variations could easily be traced.

A spark about 2 mm in length was used. This was excited usually by a 5,000-volt, 300 watt transformer, but sometimes a 5,000-volt continuous current from a generator set was used instead. The condenser consisted of large glass plates in air and had a capacity of about 0.15 microfarad, variable in steps of 0.006 mf. The inductance spool had an inductance of about 0.9 millihenry, variable in 0.009 millihenry steps. The series spark used was between 15 mm brass balls in air and was lengthened out to nearly its maximum before an exposure was made. The tube for holding the electrodes and inclosing the spark is shown in fig. 4. It was designed so that it could easily be opened for cleaning and interchanging electrodes and yet withstand a powerful and long-continued discharge. Electrodes *E*, 2 mm in diameter and 20 mm long, were slipped into a roll of platinum foil *P*, which was inclosed in a small glass tube sealed off at *S* except for a wire leading to the outside. The ground joint *J* permitted easy removal of the whole. The design prevented conduction of heat to the joint to such an extent that stopcock grease could be used (this contained 1 part rubber, 1 part vaseline,  $\frac{1}{2}$  part paraffin). A bulb *B* proved useful at low pressures as an auxiliary gas reservoir in keeping the pressure constant. A quartz window *W*, 40 mm in diameter and 40 mm away from the spark, was attached with a hard grade of fusible cement.

The whole makes a very serviceable spark tube. Gases were prepared electrolytically from a solution of potassium hydroxide and dried with phosphorus pentoxide. At each filling, the tube was flushed several times with the gas while a strong discharge was passing.

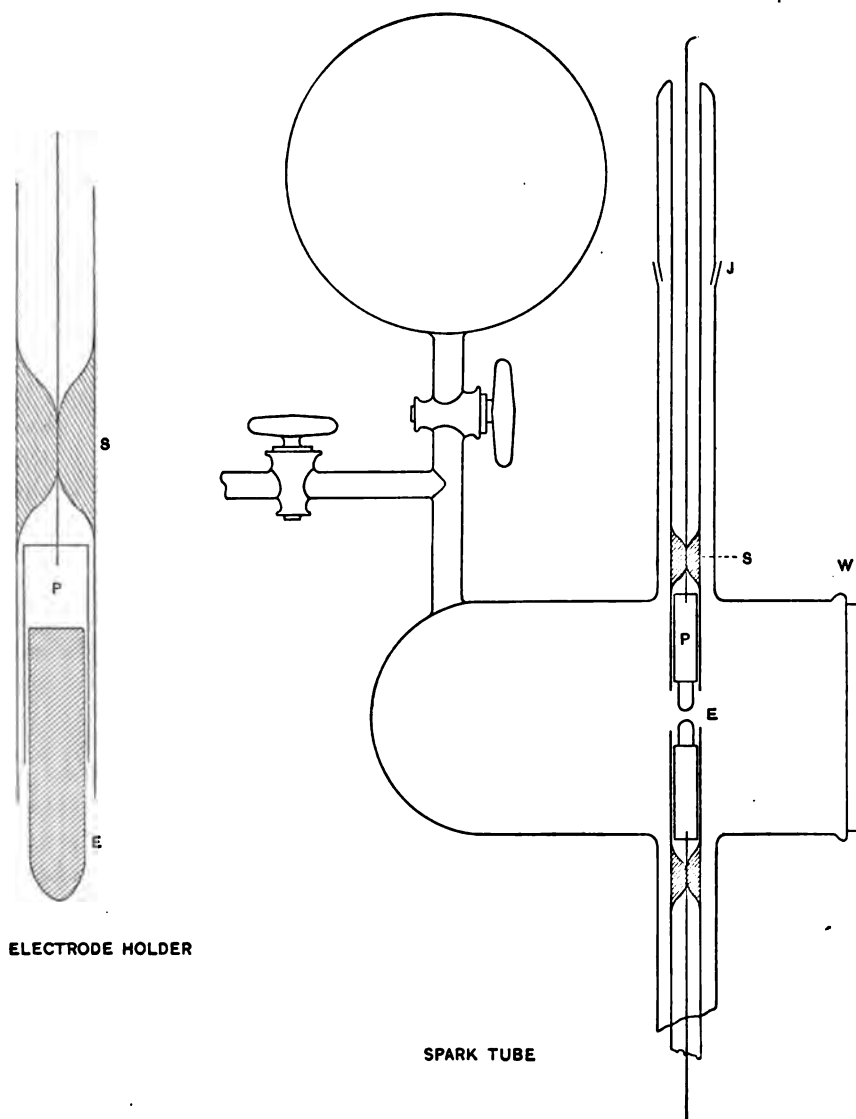


FIG. 4.—Spark tube.

The spectrograms gave of course an integrated effect so far as time was concerned, but effects in different parts of the spark were kept

separate by throwing a real image of the spark on the slit. Inductance and capacity effects are frequently very different in different parts of the spark; when not specifically stated, the effects here described relate to the central portion of the spark. Changes in the individual spectra were ignored as far as possible in estimating the relative intensity of gas and electrode spectra. When the gas spectrum changed from primary to secondary, the secondary was compared with the metallic spectrum. When inductance cut out part of a metallic spectrum, the remaining spectrum was compared with the gas spectrum.

Throughout the numerous variations in the excitation, the resistance in the primary (100-volt) side of the transformer was the quantity kept constant, with the object of keeping the amount of energy used as nearly constant (300 watts) as possible. The spark gap was usually lengthened out until the spark just passed in the gas at one atmosphere pressure with 0.05 mf capacity in parallel.

In the data given below the intensity of the metallic spectrum is given as a percentage of the whole spectral energy of the spark, electrode and atmosphere together; 30 indicates that 30 per cent of the spectral energy of the aluminium spark—in hydrogen, say—is in the aluminium spectrum, while 70 per cent is in the hydrogen spectrum. These proportions were roughly estimated from prints of the spectrograms, the whole spectrum between the limits  $350\text{ }\mu\mu$  and  $650\text{ }\mu\mu$  being taken into consideration. While the accuracy attained is not great, it is ample to determine the conditions affecting spectral preponderance, and fully as great as the constancy of other experimental conditions would warrant.

(1) *Variation with electrode metal.*—Some metals give relatively a very much stronger spectrum than others under the same conditions. The series runs from Cu 10, Pt 15, Ni 20 to Pb 75, Bi 80, Te 80, other metals being rather evenly distributed between. The numbers relate to sparks with large capacity in hydrogen at atmospheric pressure. The series appears to bear no relation to the fusion temperature series, to the atomic weight series, or to the cathode fall series, nor would the extreme infra-red<sup>a</sup> and ultra-violet<sup>b</sup> portions of the spectra, if taken into account, alter the order of the series very materially.

(2) *Effect of spark atmosphere.*—As a rule, the relative intensity of the electrode spectrum does not vary greatly when the intervening gas is changed. Hydrogen, oxygen, air, and in two cases with aluminium and iron, mercury vapor were used. The spark usually

<sup>a</sup> Coblenz: Phys. Rev. 20, p. 122; Feb., 1905.

<sup>b</sup> Pfüger: Ann. d. Phys. 18, p. 890; 1904.

appears much brighter in oxygen than in hydrogen, particularly when capacity effects are compared, but the metal spectrum is relatively little if any brighter than in hydrogen. With a majority of metals there appears to be a slight increase from hydrogen to oxygen, while air gives an intermediate intensity. Cobalt in hydrogen 30, in oxygen 55, in air 40 per cent, is typical. In an atmosphere of mercury vapor the spectra of iron and aluminium were considerably weaker than in the lighter gases, as we should expect from the atomic weight law.

(3) *Effect of gas pressure.*—Changing the pressure on the atmosphere about a spark over a wide range (2 mm to 800 mm) does not alter the preponderance of electrode spectrum over gas spectrum sufficiently to be perceptible, and this is true for all the 21 metals tested in the various gases used, and whether excited with or without capacity, inductance, or series spark. This is a most remarkable result, and by no means to be expected. In a very few cases (out of the 1,600)—palladium in oxygen with capacity, antimony in hydrogen with capacity and inductance, palladium in air with inductance, nickel and zinc in hydrogen, and magnesium in air with neither capacity nor inductance—slightly different percentages were observed at high and low pressures, but the differences were well within the possible errors due to other causes. Comparing the photographs published by Hale and Kent,<sup>a</sup> of the spectra of an iron spark at high pressures, with my own, it would appear that the independence of pressure held as well at high pressures, but the broadening, reversal, and overlapping of the lines at high pressures make the results uncertain.

(4) *Effect of varying capacity.*—The effect of varying the capacity in parallel with the spark from 0.04 mf—about all that a 2 mm 5,000-volt 300-watt spark will carry—down to 0.006 mf is scarcely perceptible. As the critical capacity—about 0.01 mf at 760 mm<sup>b</sup> and wave length 450  $\mu$ —is passed, the gas spectrum passes over from secondary to primary, but the intensity of both relative to the metallic spectrum is the same. It is only at very much smaller capacities—probably less than  $10^{-5}$  mf—that a spark goes over into an arc—i. e., that a rotating mirror fails to resolve the discharge into separate pulses.

<sup>a</sup> *Astrophys. J.*, 17, p. 154; 1903.

<sup>b</sup> See B. S. Bulletin No. 1, p. 87, for data on critical capacity. The 180 sets of spectra taken for the present article show well the critical capacity effect in its relation to wave length, pressure, and inductance and independence of electrodes, gas, spark length, and voltage; thus adding considerable weight to the conclusions drawn in the preceding paper.

(5) *Effect of a spark in series.*—Putting in circuit a spark in air with 0.006 mf capacity about both the auxiliary and observed spark, produced the same effect as simply adding a large capacity—say, 0.04 mf. The series spark has a great practical advantage over large capacity at low pressures—1 mm and less—in that it will break up a glow discharge and give a strong metallic spectrum and secondary gas spectrum even at pressures so low that adding capacity has no effect whatever.

(6) *Effect of removing all capacity and inductance.*—A spark without capacity is of course an arc, in this case it was an arc at about 4,000 volts fed at about 0.05 ampere. It was usually very faint and frequently at low pressure degenerated into a mere glow about the electrodes, particularly in hydrogen. In hydrogen all electrodes, except those of Bi, Hg, Pb, and Pd, show a relatively much weaker spectrum than in the capacity spark. But in oxygen and air all metals showed relatively the same spectral intensity without capacity as with capacity. This effect could hardly have been anticipated and has an important bearing on the general theory following.

(7) *Inductance effect.*—The effect of inductance in cutting out the air lines from a spark spectrum has been studied before by several workers, but from the results of my own work, I am inclined to question the existence of any such effect, at any rate in atmospheres of hydrogen, nitrogen, or mercury vapor, in the case of any of the twenty-one metals studied. Added inductance *does* change the secondary spectrum of a gas over into the primary,<sup>a</sup> in a 2 mm spark as well as in a 40 mm Plücker tube, but there is no possibility of anyone familiar with both spectra mistaking the effect. Oxygen lines appear to be swamped by metallic when inductance is added, but what really occurs may be quite different. Oxygen has but an extremely faint primary anode glow spectrum, at least in the visible region. Hence the effect of adding inductance is to change the oxygen spectrum over into an invisible form and *not* to swamp it with the spectrum of the electrodes. The oxygen lines in the air spectrum being affected in a similar manner, the air spectrum is apparently but not really displaced by the metallic when inductance is added.

(8) *Inductance with variable capacity and series spark.*—Four of the nine spectrograms on each plate showed the resultant variations of these opposed effects. An inductance of 0.9 millihenry—ten times the critical inductance—was connected in, while capacity was varied from very large to very small and a series spark added. While metallic and gas spectra each show wide variations—various lines appear or dis-

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<sup>a</sup> B. S. Bulletin 1, p. 88.



appear in the metallic spectra while gas spectra change from primary to secondary and vice versa—yet the relative total intensity of metal to gas spectra remains unchanged within an error of 5 to 15 per cent in their estimation. This invariability under such directly opposed conditions is strong confirmation of effects described in paragraphs 4, 5, and 7. Some very striking differences were observed in the behavior of metallic spectra in oxygen and hydrogen, but they are foreign to the subject of this paper.

An abridged table of results is given below. High, low, and mod-

**Table I.—RATIO OF METALLIC ENERGY TO THE TOTAL ENERGY OF SPARK.**

| Element. | Large capacity and auxiliary spark. |       |       | Inductance with 0.01 mf capacity. |       |       | Arc at 4,000 volts, 0.05 ampere. |       |       |
|----------|-------------------------------------|-------|-------|-----------------------------------|-------|-------|----------------------------------|-------|-------|
|          | H                                   | O     | Air   | H                                 | O     | Air   | H                                | O     | Air   |
| Ag.....  | 40                                  | 20    | 30    | 30                                | 90    | 80    | 10                               | 20    | 60    |
| Al.....  | 30                                  | 50    | 35    | 35                                | 50    | 25    | 10                               | 40    | 10    |
| Au.....  | 35                                  | 40    | 40    | 35                                | 70    | 60    | 30                               | 30    | 50    |
| Bi.....  | 80                                  | 80    | 70    | 80                                | 90    | 90    | 70                               | 90    | 70    |
| C.....   | 15                                  | 5     | 10    | 10                                | 5     | 10    | 10                               | 10    | 10    |
| Cd.....  | 45                                  | 45    | 55    | 70                                | 90    | 90    | 20                               | 70    | 90    |
| Co.....  | 30                                  | 55    | 40    | 40                                | 90    | 90    | 40                               | 30    | 30    |
| Cr.....  | 40                                  | 75    | ..... | 50                                | 95    | ..... | 20                               | 80    | ..... |
| Cu.....  | 10                                  | 30    | 15    | 10                                | 90    | 20    | 10                               | 70    | 10    |
| Fe.....  | 25                                  | 70    | 75    | 10                                | 90    | 90    | 10                               | 60    | ..... |
| Hg.....  | 75                                  | ..... | ..... | 90                                | ..... | ..... | 70                               | ..... | ..... |
| Mg.....  | 60                                  | 65    | 50    | 60                                | 90    | 90    | 60                               | 80    | 30    |
| Ni.....  | 20                                  | 50    | 40    | 15                                | 90    | 50    | 10                               | 50    | 30    |
| Pb.....  | 75                                  | 80    | 70    | 90                                | 95    | 85    | 80                               | 80    | 50    |
| Pd.....  | 40                                  | 45    | 65    | 55                                | 85    | 80    | 40                               | 70    | 25    |
| Pt.....  | 15                                  | 20    | 10    | 10                                | 80    | 40    | 10                               | 25    | 10    |
| Sb.....  | 35                                  | 30    | 35    | 65                                | 70    | 40    | 10                               | 45    | 20    |
| Si.....  | 35                                  | 30    | 30    | 30                                | 70    | 40    | 10                               | 55    | 15    |
| Sn.....  | 45                                  | 30    | 30    | 35                                | 95    | 80    | 15                               | 65    | 15    |
| Te.....  | 80                                  | 75    | 30    | 90                                | 90    | 70    | 30                               | 20    | 25    |
| Zn.....  | 30                                  | 30    | 30    | 40                                | 85    | 40    | 10                               | 30    | 20    |

erate pressures are grouped together, as are the effects of large capacity and series spark. The inductance effects quoted relate to 0.9 millihenry used with 0.01 microfarad capacity. The group headed "arc"

relates to sparks without other capacity than lead wires a meter long. Results for carbon (Acheson graphite) are very uncertain on account of the lack of intensity of the carbon spectrum in comparison with compound spectra and the spectra of the impurities (calcium and titanium) present. These foreign spectra were left out of account in the estimate of the ratio of carbon to gas spectra. Values for oxygen with inductance are given as they appeared with no account taken of the shift to infra-red. The mercury spark was about 3 mm long from mercury to mercury through a hole in the side of an auxiliary glass tube.

(9) *Effect of current density.*—Varying the current has a pronounced effect in intensifying the spectra of the electrodes at the relative expense of the spectrum of the intervening gas. The effect is well known and considerable data is available. Varying the current varies the amount of vaporization and ionization, and these factors chiefly govern spectral predominance. Vaporization is, of course, not so much due to a heating of the electrode as a whole as to a skin effect due to the great energy lost in the electrode fall of potential at the surface. Whether the electric current first causes a local heating and this heating vaporizes the metal, or whether the current tears the particles directly from the surface and this electrical vaporization causes the heating afterwards we are unable to state, but evidence for the latter process appears stronger. This energy loss at the surface of an electrode being equal to the product of current and electrode fall, the latter being roughly proportional to the current, we should have vaporization approximately proportional to the square of the current. Since the luminosity of a conducting gas is very nearly proportional to the current, the intensity of the spectrum of the spark or arc atmosphere would be proportional to the current. On the other hand the luminosity of the vapor of the electrodes depends on the variable amount of vapor present as well as on the current. Since the amount of the electrodes vaporized is roughly proportional to the square of the current, the luminosity of the electrode vapor would be proportional, approximately, to the cube of the current. Hence a heavy current would greatly assist the preponderance of metallic over gas spectrum.

(10) *Effect of electrode temperature.*—Raising the temperature of an electrode increases the vapor pressure of the metal, but diminishes the electrode fall of potential,<sup>a</sup> hence the most advantageous temperature will be an intermediate one, varying widely with the substance used. But in the case of nearly all metals of high melting point both coeffi-

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<sup>a</sup>Cunningham: Phil. Mag., 9, 193; Feb., 1905.

cients are very small, and the effect of electrode temperature is negligible.

(11) *Effect of distance between electrodes.*—By varying the length of the column of the conducting gas one may obtain metal and gas spectra in any proportion from pure metal to pure gas. Current, pressure, and distance from an electrode of the observed portion are the chief variable factors to be considered. At high pressures we may have all gradations from an arc to a lightning discharge; at low pressures, everything from an arc to a glow discharge. Current and electrode fall of potential determine the amount of metallic vapor that will appear at a given distance from an electrode. Hence at constant current, pressure and distance apart of electrodes need not be considered, except in so far as they affect electrode fall. Since neither anode fall<sup>a</sup> nor cathode fall<sup>b</sup> varies greatly with pressure, at constant current the amount of metallic spectrum showing in a given part of a discharge is practically constant. Even in a low-pressure discharge tube 40 cm long, at a pressure of 0.06 mm, I have found the spectrum of the electrodes (aluminum and magnesium) to be a considerable portion of the canal ray glow spectrum.

The effects of varying other conditions than those above discussed appear to be vanishingly small if existent. Varying the e. m. f., for example, has little if any effect so long as current remains constant, account being taken of the very considerable effect on the form of the discharge of varying the control resistance. Nor does the form of the current wave in an intermittent discharge appear to affect the relative intensity of the metallic spectrum. Adding capacity increases the maximum current enormously, but, if the mean current is kept constant, does not appreciably increase the preponderance of the metallic spectrum. The same may be said of the auxiliary spark. Inductance changes only the form of the current wave. Resistance of course would change not only wave form but mean current, but at constant current does not affect metallic preponderance.

#### CONCLUSIONS.

The results of this investigation show that the same three general laws (p. 400) hold for the distribution of spectral energy between electrodes and gas as hold for other mixed gases. Relative spectral energy depends upon the relative amounts of the two gases or vapors present and, with moderate excitation, upon the atomic weight. The amount

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<sup>a</sup> Skinner: Wied. Ann., 68, p. 752; 1899.

<sup>b</sup> Warburg: Wied. Ann., 40, p. 1; 1890.

of electrode vapor present depends upon current and electrode fall of potential, being a surface effect proportional to the energy used in metal-gas conduction. This electrode fall is again a function of a number of other variables. It is nearly proportional to the current, varies slightly with the pressure on the gas and the temperature of the electrodes. The conditions necessary for obtaining a gas spectrum free from the spectrum of the electrodes used, or for obtaining the spectrum of a given metal as free as possible from the spectrum of the surrounding atmosphere, may then be mapped with some certainty. The general case is considered, namely that of a column of gas forming part of an electric circuit that is otherwise metallic.

The conditions favoring a pure gas spectrum are (1) small current density at the electrodes, (2) substance of electrodes to be of some metal like aluminum, copper, or platinum showing small vaporization, (3) electrodes to be at a considerable distance apart and only the middle portions of the discharge are to be used. Gas density, capacity, inductance, series spark, and other conditions have but little effect, and hence may be adapted to other requirements. All three conditions above stated are realized in the Plücker tube, the loss in luminosity caused by the use of small currents being compensated for by the central constriction. In the ordinary arc or spark under the most favorable conditions, it is difficult to obtain a gas spectrum more than 80 per cent pure, whereas by decreasing the current and separating the electrodes, practically 100 per cent purity may easily be realized.

To obtain a pure electrode spectrum, we must have (1) large current density and (2) electrodes near together. Oxygen appears to be the most favorable gas to use with the majority of metals, probably on account of the greater cathode and anode fall of potential in this gas. The above conditions are realized in the arc and spark, 95 per cent purity being easily obtainable. Pressure being of little consequence, it may be reduced to give narrow lines, a series spark being added if necessary. Other independent conditions may of course be varied in adaptation to experimental requirements.

March, 1905.

## USE OF WHITE WALLS IN A PHOTOMETRIC LABORATORY.

By EDWARD P. HYDE.

It is usually considered by photometricians that one of the essential requisites for accurate work in photometry is that the walls of the photometer room be made as nearly "dead black" as possible. It is surprising how long this tradition of photometry has persisted, particularly as it necessitates the exclusive use of a room for photometric measurements, since a room blackened in the customary manner is unfit for any other work. Perhaps it is due to this fact that one so often finds the photometric laboratory in a small room in the basement or garret, or some other place that is unsuited for any other purpose. In all open-flame work, however, much larger errors may result from poor ventilation in a small room than from light reflected from white walls of a larger room, if a proper set of screening diaphragms is used.

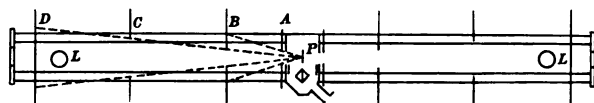


FIG. 1.—Horizontal section through center of photometer screen.

So far as I know nothing has been published as to the magnitude of the error due to white walls when the photometer is protected by black diaphragms, although this arrangement is at present in use to some extent in the photometric laboratory of the Physikalisch-Technische Reichsanstalt of Germany.

In the photometric laboratory of the Bureau of Standards the walls of one of the rooms, which is about 29 feet long, 19 feet wide, and 12 feet high, were left white for a year after the construction of the building. Recently they have been given a light terra-cotta finish for decorative purposes, but the following measurements were made while the walls were still white. The photometer is placed near the middle of the room and is supplied with a set of diaphragms covered with black velvet. These diaphragms may be so arranged for any position on the bar that no light, except that reflected from the lamps under test or from the diaphragms themselves can reach the photometer

screen, as shown by the accompanying sketches. Fig. 1 is a horizontal section of the photometer through the center of the Lummer-Brodhun sight-box, showing the arrangement of diaphragms *A*, *B*, *C*, and *D*. Elevations of these diaphragms perpendicular to the axis of the photometer are given in fig. 2. By reference to fig. 1 it is seen that an eye placed at *P* could see nothing but black velvet diaphragms and the lamps under test. Conversely, no light could reach the screen except from the lamps or from the diaphragms.

In case a Bunsen or Leeson disk is used, extra precautions would have to be taken to prevent light from penetrating to the screen from the front of the box. It is essential that this condition be fulfilled if the walls are to be left white, as any reflected light reaching the screen directly would materially affect the result. Furthermore, it is desirable that all outside light be excluded by the use of dark curtains, as under these circumstances the only light that can reach the photometer screen, apart from the direct light from the lamps, is that part of the light from the two lamps which is reflected by the walls back to the lamps and the diaphragms, and thence to the photometer.

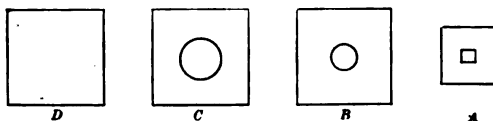


FIG. 2.—Front elevation of diaphragms.

In order to determine the magnitude of the errors resulting from this reflected light from the white walls of the photometer room of the Bureau of Standards, the measurements described below were made. The results of these measurements show conclusively that the error is entirely negligible.

Before giving the results of the measurements it may be well to say a word or two in regard to the method of measurement employed at the Bureau in the comparison of incandescent lamp standards. On the carriage on the right side of the photometer is placed a well seasoned incandescent lamp, though not ordinarily a standard. This lamp is raised to such a voltage that its color is approximately the same as the color of the lamps to be compared on the other side of the photometer, and it is kept at this voltage throughout the test. The carriage on which this comparison lamp is mounted is then rigidly connected by means of adjustable links to the carriage on which the photometer screen is mounted, and at a suitable distance to produce the desired illumination of the screen. The standard is then mounted on the carriage on the left side of the photometer, and is placed at such a position as to give a balance with the photometer at approximately the middle of the bar. The standard is brought to the proper

voltage and several readings are taken. The lamps to be standardized are then mounted successively on the same carriage and readings are made. At the end the standard is again measured. If the lamps that are compared with the standard are of the same nominal candle-power as the standard, the left-hand carriage is kept at the same position on the bar. If the candle-power is different, the left-hand carriage is placed so that the balance comes at about the same place on the bar.

This method of measurement, which is a substitution method, has many advantages in accurate work. It eliminates all dissymmetry on the two sides of the photometer, and thus avoids the necessity of reversing the photometer screen.

In making the measurements of the effect of reflection from the walls the right side of the photometer was entirely protected from reflected light from the walls by large screens, which nearly surrounded that half of the photometer bar, so that when the left side of the Lummer-Brodhun sight-box was closed by a shutter no illumination was visible in the eyepiece. The velvet-covered diaphragms and the lamp on the left side of the photometer, on the contrary, were left exposed to the reflected light from the walls, as in the ordinary method of measurement, except that the lamp on that side was not burning. Therefore the only light that penetrated to the screen was that which, reflected from the walls, fell upon the lamp and black velvet diaphragms and was reflected by them to the screen.

In order that the walls should be illuminated to the same extent as in the ordinary measurements, a 16 c. p. lamp was burned near the position of the comparison lamp, and another lamp was burned near the position of the standard lamp, but so shielded in each case that no direct light could reach the photometer. The lamp on the left near the unburned standard was changed from time to time from a 32 c. p. lamp to a 16 c. p. lamp, depending upon whether a 32 c. p. unburned lamp was in the left socket at the distance from the screen at which a 32 c. p. lamp is usually placed or a 16 c. p. lamp was in the socket at the position used with a 16 c. p. lamp. These precautions were taken in order to secure as nearly as possible working conditions.

Under these circumstances it was perfectly plain, on looking into the eyepiece of the photometer, that the left side of the sight-field was slightly illuminated, and it only remained to measure this illumination under the different conditions likely to occur. To this end a 1 c. p. lamp, which had been placed on the carriage on the right in the position of the comparison lamp, was brought to incandescence, and the light from this lamp was cut down by absorption strips of smoked glass having known coefficients of transmission. Thus successive absorption strips were added until a balance was obtained, and the

number of strips and the positions of the three carriages were recorded. This method of procedure was followed through a great number of measurements, the conditions being changed from time to time until all the conditions that are actually encountered were included in order to determine whether any error due to the walls could possibly be introduced in any comparisons that are ever made. What was actually measured each time of course was the illumination of the screen due to the reflected light, and it was merely a matter of computation to determine what the candle-power at any distance would be in order to produce the illumination found.

I shall not give the results of all the measurements that were made, but merely indicate the results for two typical cases which show clearly the order of magnitude of the effect. First, from the direct measurement of the illumination that was produced when a 32 c. p. standard (not burning) was placed at the position at which a 32 c. p. lamp is generally placed in standard measurements, and another 32 c. p. lamp was illuminating the walls, calculation showed that the illumination on the screen, due to the reflection from the walls, was that which would be produced by a source of 0.003 candle at the position of the 32 c. p. standard. In a similar way the correction to a 16 c. p. lamp was found to be about 0.001 or 0.002 candle. Thus we see that not only would the relative error in comparing a 16 c. p. lamp with a 32 c. p. lamp be negligible, but that the absolute error for either one would be negligible in the most refined measurements, being of the order of magnitude of 1 part in 10,000. To be sure, the determination of this correction may be considerably in error, but since the correction is so very small it could be increased to several times its value and still be beyond recognition.

All of these measurements were made with the windows covered with heavy curtains, the only sources of illumination of the walls being lamps similar to ones that would ordinarily illuminate the walls when making photometric measurements. When the curtains were raised the effect was greatly increased, so that the error in a 32 c. p. lamp was several tenths of a candle, varying with the brightness of the sky and with the distribution of objects about the room. Approximately the same relative error was found for the 16 c. p. lamp, so that in comparative measurements the errors in general would be negligible; but since the errors of measurement of these corrections were necessarily large, and since we found the corrections to vary considerably when an observer moved in front of a window, it is desirable to exclude daylight by curtains.

In conclusion, I desire to express my indebtedness to Mr. F. E. Cady for valuable assistance in making the observations.



## INFLUENCE OF WAVE FORM ON THE RATE OF INTEGRATING INDUCTION METERS.

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By E. B. ROSA, M. G. LLOYD, and C. E. REID.

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We give in this paper the results obtained with five integrating induction wattmeters, on which we have made a large number of tests, although further work remains to be done. These results may therefore be regarded as preliminary, illustrating the methods employed and the results obtained when changes are made in the wave form by altering the magnitude or phase of the harmonics present.

Two of the meters employed were sent to the Bureau of Standards for test by the makers. The others were meters which we happened to have in the laboratory when the tests were undertaken. The following is a list of the meters:

- No. 1, Stanley (magnetic suspension type), 50 amperes.
- No. 2, Stanley (magnetic suspension type), 50 amperes.
- No. 3, Fort Wayne, type "K," 50 amperes.
- No. 4, General Electric (1902 House type), 25 amperes.
- No. 5, Siemens & Halske, 25 amperes.

All the meters are made for 60 cycles, single phase. The first four are American instruments; the last is of German make. Each meter was tested at full load and at 110 volts, and at approximately unity power factor.

In order to determine the effect on the rate of an induction meter due to varying the wave form, it is necessary to eliminate carefully any effects due to variation in the temperature of the meter or changes in the frequency of current, or other alterations in the conditions of the meter or circuit. In most cases the effect of a moderate distortion of the wave is small, and unless all measurements are made with great care the effects looked for may be masked by other effects or by errors of measurement. The meters were tested alternately with current of sine wave form and with a distorted wave, the distortion being produced by adding a harmonic of three times the frequency

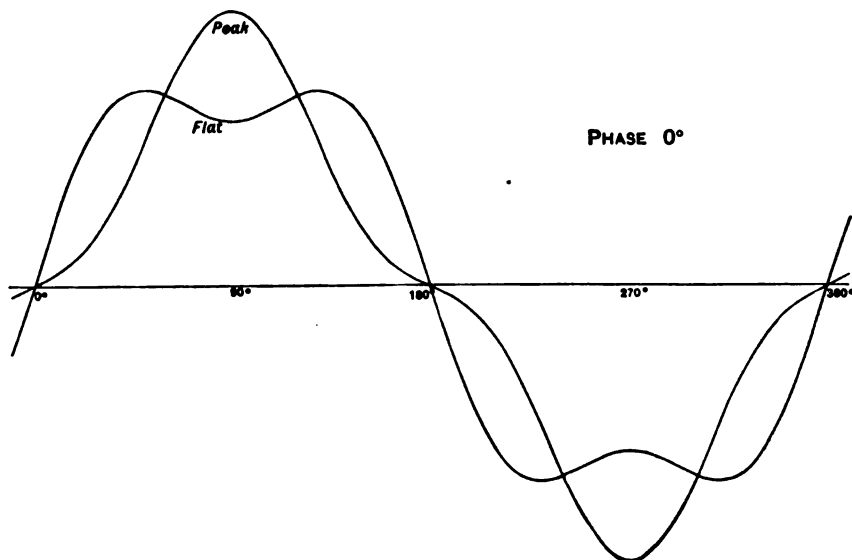


FIG. 1.—Showing the resultant of combining the fundamental and harmonic of three times the frequency and 25 per cent of the magnitude of the fundamental, giving first a peaked wave and second (when the phase of the harmonic is reversed) a flat or dimpled wave. Both fundamental and harmonic are of sine-wave form.

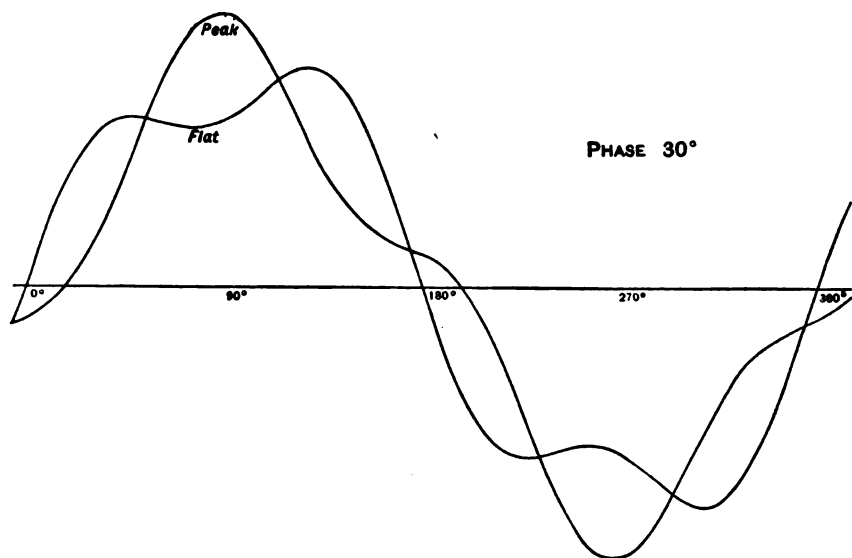


FIG. 2.—Showing the resultant of a fundamental and a harmonic, as in fig. 1, except that the phase of the harmonic has been shifted 30° by changing the coupling 5°.

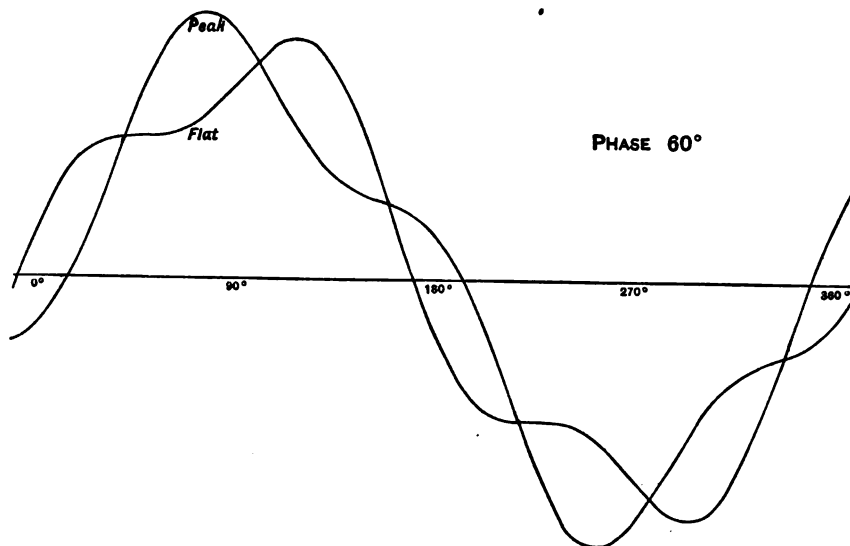


FIG. 3.—Showing the resultant of a fundamental and a harmonic as in fig. 1, except that the phase of the harmonic has been shifted  $60^\circ$ .

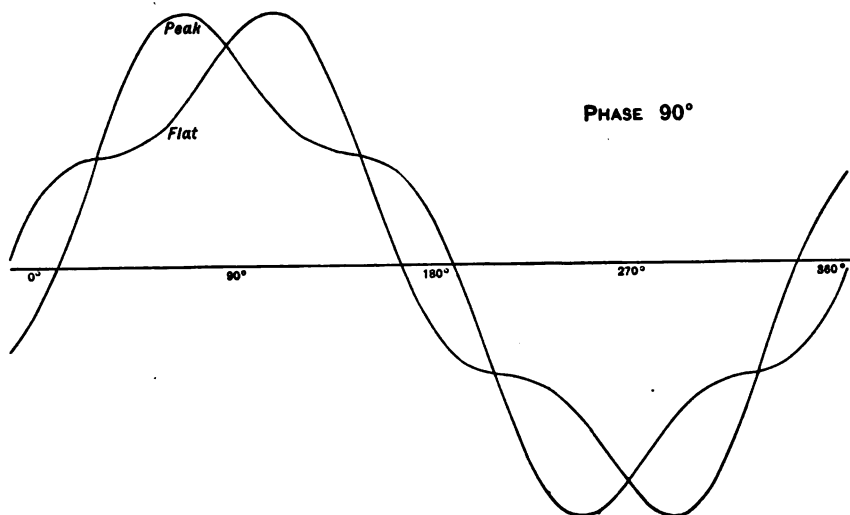


FIG. 4.—Showing the resultant of a fundamental and a harmonic as in fig. 1, except that the phase of the harmonic has been shifted  $90^\circ$  by changing the coupling  $15^\circ$ . The wave form for "peak" and "flat" are here alike, except that the steeper side is in advance in the "peak" and the more gradual slope is in advance on the "flat." "Peak" and "flat" are conventional terms, indicating the phase of the harmonic. If the coupling were shifted  $15^\circ$  more, the "flat" curve would become peaked.

of the fundamental, varying both the amplitude and phase of this harmonic. This was done by means of an alternating current generating set of three machines, two alternators and a direct connected driving motor, one alternator having four poles and the other twelve. The current from each machine is very nearly of sine wave form, and tests were made of the meters alternately with the fundamental only, and with the harmonic added. Three different relative values and four different phases of the harmonic have been employed. The three values of the harmonic are 10 per cent, 25 per cent, and 50 per cent, respectively, of the value of the fundamental. For example, since  $E = \sqrt{E_1^2 + E^2}$ , in the first case the addition of 11 volts of the harmonic to 110 volts of the fundamental gives a resultant of about 110.5 volts, the wave being more or less peaked than a sine wave, according to the phase of the harmonic. In the second case 108 volts of the fundamental plus 27 volts of the harmonic gives a resultant of 111.3 volts. (The voltage in each case was reduced to exactly 110 by resistance in series.) This resultant is shown in figs. 1 to 4, where the 25 per cent harmonic is in different phase in each of the four cases. This difference is produced by shifting the coupling of one of the generators to the driving motor, 5°, 10°, or 15° in the coupling corresponding to 10°, 20°, or 30° in the wave of the fundamental, and to 30°, 60°, or 90° in the phase of the harmonic. A shift of 30° in the coupling corresponds to 180° in the phase of the harmonic, and is the same as reversing the phase by reversing the connections at the terminals of the higher frequency generator. The latter is of course the more convenient, and was the usual method of changing from what we call a flat to a peaked wave. The curves shown in figs. 1 to 4 have been frequently verified by drawing the resultant waves by means of a curve tracer. This not only verifies the wave form, but serves to insure against errors in the connections.

The current, voltage, and power factor, as well as the temperature and frequency, were maintained as nearly constant as possible during a set of runs. A standard wattmeter which was calibrated by direct currents using two potentiometers to measure simultaneously the current and the voltage, was read by a telescope and scale. By means of a carbon rheostat the deflection of this instrument was maintained accurately constant while carrying alternating current during a set of runs on the meters. The wattmeter is of the dynamometer type and astatic. The fixed coils are stranded and wound on wooden spools, and very little metal is used in the region of the coils. The movable coils have very slight inductance, and every precaution is taken to avoid errors due to eddy currents or wave form. The instrument, being

carefully calibrated with direct current, is then correct for alternating current.

**Table I.—DETERMINATION OF THE TIMES OF REVOLUTION OF THE DISKS OF THREE METERS.**

RUN NO. 1, MAY 26, 1904.

| No.                                     | METER 1. |                                       |                                                                    | METER 2.                                 |                                       |                                                                    | METER 3. |                                         |                                                                    |  |         |
|-----------------------------------------|----------|---------------------------------------|--------------------------------------------------------------------|------------------------------------------|---------------------------------------|--------------------------------------------------------------------|----------|-----------------------------------------|--------------------------------------------------------------------|--|---------|
|                                         | Time.    | Interval between successive contacts. | Interval between 1st and 15th contacts, 2d and 16th contacts, etc. | Time.                                    | Interval between successive contacts. | Interval between 1st and 15th contacts, 2d and 16th contacts, etc. | Time.    | Interval between successive contacts.   | Interval between 1st and 15th contacts, 2d and 16th contacts, etc. |  |         |
|                                         | m. s.    |                                       |                                                                    | m. s.                                    |                                       |                                                                    | m. s.    |                                         |                                                                    |  |         |
| 1                                       | 1 5.30   | 1.55                                  |                                                                    | 0 45.96                                  | 1.60                                  |                                                                    | 0 45.70  | 1.80                                    |                                                                    |  |         |
| 2                                       | 6.85     | 1.55                                  |                                                                    | 47.56                                    | 1.68                                  |                                                                    | 47.00    | 1.81                                    |                                                                    |  |         |
| 3                                       | 8.40     | 1.65                                  |                                                                    | 49.24                                    | 1.57                                  |                                                                    | 48.81    | 1.82                                    |                                                                    |  |         |
| 4                                       | 10.05    | 1.61                                  |                                                                    | 50.81                                    | 1.61                                  |                                                                    | 49.63    | 1.87                                    |                                                                    |  |         |
| 5                                       | 11.66    | 1.64                                  |                                                                    | 52.42                                    | 1.63                                  |                                                                    | 51.00    | 1.80                                    |                                                                    |  |         |
| 6                                       | 13.30    | 1.52                                  |                                                                    | 54.05                                    | 1.54                                  |                                                                    | 52.30    | 1.80                                    |                                                                    |  |         |
| 7                                       | 14.82    | 1.63                                  |                                                                    | 55.59                                    | 1.66                                  |                                                                    | 53.60    | 1.87                                    |                                                                    |  |         |
| 8                                       | 16.45    | 1.60                                  |                                                                    | 57.25                                    | 1.65                                  |                                                                    | 54.97    | 1.83                                    |                                                                    |  |         |
| 9                                       | 18.05    | 1.67                                  |                                                                    | 58.90                                    | 1.60                                  |                                                                    | 56.80    | 1.80                                    |                                                                    |  |         |
| 10                                      | 19.72    | 1.58                                  |                                                                    | 1 0.50                                   | 1.55                                  |                                                                    | 57.60    | 1.40                                    |                                                                    |  |         |
| 11                                      | 21.30    | 48.60                                 |                                                                    | 2.05                                     | 48.25                                 |                                                                    | 59.00    | 26.60                                   |                                                                    |  |         |
| 12                                      | 2 9.90   | 32.20                                 |                                                                    | 50.80                                    | 32.10                                 |                                                                    | 1 25.60  | 39.88                                   |                                                                    |  |         |
| 13                                      | 42.10    | 32.22                                 |                                                                    | 2 22.40                                  | 32.10                                 |                                                                    | 2 5.48   | 39.87                                   |                                                                    |  |         |
| 14                                      | 3 14.32  | 32.33                                 |                                                                    | 54.50                                    | 32.20                                 |                                                                    | 45.35    | 39.95                                   |                                                                    |  |         |
| 15                                      | 46.65    | 1.55                                  | 161.85                                                             | 3 26.70                                  | 1.65                                  | 160.74                                                             | 3 25.30  | 1.80                                    | 159.60                                                             |  |         |
| 16                                      | 48.20    | 1.62                                  | 161.85                                                             | 28.35                                    | 1.60                                  | 160.79                                                             | 26.60    | 1.80                                    | 159.60                                                             |  |         |
| 17                                      | 49.82    | 1.59                                  | 161.42                                                             | 29.95                                    | 1.59                                  | 160.71                                                             | 27.90    | 1.80                                    | 159.59                                                             |  |         |
| 18                                      | 51.41    | 1.59                                  | 161.86                                                             | 31.54                                    | 1.56                                  | 160.73                                                             | 29.20    | 1.85                                    | 159.57                                                             |  |         |
| 19                                      | 53.00    | 1.61                                  | 161.84                                                             | 33.10                                    | 1.68                                  | 160.68                                                             | 30.55    | 1.40                                    | 159.55                                                             |  |         |
| 20                                      | 54.61    | 1.61                                  | 161.31                                                             | 34.78                                    | 1.60                                  | 160.73                                                             | 31.95    | 1.85                                    | 159.65                                                             |  |         |
| 21                                      | 56.22    | 1.68                                  | 161.40                                                             | 36.38                                    | 1.62                                  | 160.79                                                             | 33.80    | 1.83                                    | 159.70                                                             |  |         |
| 22                                      | 57.90    | 1.58                                  | 161.45                                                             | 38.00                                    | 1.60                                  | 160.75                                                             | 34.63    | 1.82                                    | 159.66                                                             |  |         |
| 23                                      | 59.48    | 1.55                                  | 161.43                                                             | 39.60                                    | 1.59                                  | 160.70                                                             | 35.95    | 1.40                                    | 159.65                                                             |  |         |
| 24                                      | 4 1.08   | 1.59                                  | 161.31                                                             | 41.19                                    | 1.61                                  | 160.69                                                             | 37.35    | 1.88                                    | 159.75                                                             |  |         |
| 25                                      | 2.62     |                                       | 161.32                                                             | 42.80                                    |                                       | 160.75                                                             | 38.68    |                                         | 159.68                                                             |  |         |
| Mean time for 100 revolutions .....     |          |                                       | 161.367                                                            | Mean time for 100 revolutions .....      |                                       |                                                                    | 160.733  | Mean time for 120 revolutions .....     |                                                                    |  | 159.636 |
| Time in seconds for one revolution..... |          |                                       | 1.6137                                                             | Time in seconds for one revolution ..... |                                       |                                                                    | 1.6073   | Time in seconds for one revolution..... |                                                                    |  | 1.3308  |

# **DETERMINATION OF THE RATE OF THE METERS AND THE FREQUENCY OF THE CURRENT.**

The rate of the meters was determined by means of a chronograph and chronometer, record being made at the end of every revolution of the disk or drum for the first ten revolutions at the beginning of a

run and the last ten at the end of the run, and in addition once in ten or twenty revolutions during the run. The runs average about three minutes each. The record on the chronograph sheet was read by means of a diagonal scale and gave the mean time of one revolution with great accuracy. An example is given in Table I. Eleven independent determinations of the time of 100 revolutions of the disk are given in the fourth and seventh columns for meters 1 and 2 and of 120 revolutions in the tenth column for meter 3. The average of these eleven values is used in deriving the mean time for one revolution. At the same time an electric circuit was closed by a contact point connected to the generator once in every hundred revolutions, and these contacts were recorded on the chronograph sheet. This gave the frequency of the current very exactly. A slight correction is applied to the rate of each meter for the small departure of the frequency from 60, which is the standard frequency for the meters tested. This correction is of course determined for each meter separately and may be taken from the curves in fig. 10. In Table II are given the readings from the chronograph record for determining the frequency of the current, which in this case averaged 59.95 for the period of the run.

Table II.—RECORD FOR DETERMINATION OF THE FREQUENCY OF THE CURRENT.

| Time of contacts at beginning of run. |           | Time of contacts at end of run. |           | Time for 5,100 revolutions. |
|---------------------------------------|-----------|---------------------------------|-----------|-----------------------------|
| <i>m.</i>                             | <i>s.</i> | <i>m.</i>                       | <i>s.</i> | <i>Seconds.</i>             |
| 1                                     | 3.25      | 3                               | 53.40     | 170.15                      |
|                                       | 6.60      |                                 | 56.73     | 170.13                      |
|                                       | 9.92      | 4                               | 0.06      | 170.13                      |
|                                       | 13.28     |                                 | 3.40      | 170.12                      |
|                                       | 16.60     |                                 | 6.75      | 170.15                      |

Mean = 170.136

$$\text{Frequency} = \frac{2 \times 5,100}{170.136} = 59.95 \text{ cycles per second.}$$

Before making a series of runs on the meters the load was applied and they were warmed up to an equilibrium temperature. In order to eliminate errors due to any changes in temperature that might subsequently occur, as well as any other constant errors due to changing conditions, and so obtain the effect of the varying wave form alone, the tests with distorted wave forms were interspersed with tests using sine wave forms. Any progressive change in the meters, or the standard wattmeter, would thus be eliminated. Table III gives the

results of 49 runs on three meters made May 26, 1904, using a sine wave and a distorted wave due to 25 per cent harmonic, peak and flat, as shown in figs. 1 to 4. The numbers of the runs show the order in which they were taken. For example, the first, fourth, and seventh were made with a sine wave and are grouped together in the table. The actual frequency for each run is given in the table, but the times of one revolution given in columns 6, 8, and 10 have been reduced to 60 cycles. The numbers given in columns 7, 9, and 11 are the means of the corresponding values in columns 6, 8, and 10.

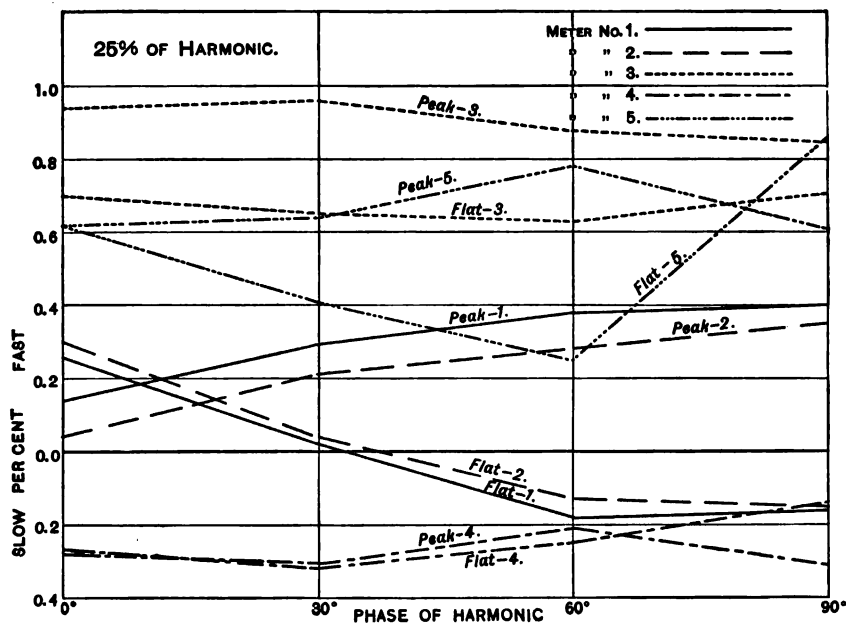


FIG. 5.—Showing the variation in the rate of five induction meters with 25 per cent harmonic in the current as the phase of the harmonic is changed from 0° to 30°, 60°, and 90°.

Runs 2 and 5 were made using a peaked wave and runs 3 and 6 using a flat wave. The phase of the harmonic was then shifted 30°, and seven runs made in the same order as before. The third set of seven runs was made with the harmonic at 60°, the fourth set at 90°, and then three more sets of runs were made in reverse order with respect to the phase of the harmonic, making in all seven sets of seven runs each. The difference in per cent obtained in Table IV, together with corresponding differences in other runs using 10 and 50 per cent harmonic, the detailed results of which are not here given, are shown in Table V. All these results are plotted in figs. 5 to 9. The results obtained in the runs of May 26, given in Tables III and IV, are plotted in fig. 5.

**Table III.**—TABLE OF RESULTS OF 49 RUNS ON THREE METERS USING SINE, PEAK AND FLAT WAVES, WITH DIFFERENT PHASES OF THE HARMONIC, WHICH IS 25 PER CENT OF THE FUNDAMENTAL.

| No. of run. | Phase of harmonic. | Lag of current. | Wave form. | Frequency. | Time in seconds for one revolution of disk. |        |              |        |              |        |
|-------------|--------------------|-----------------|------------|------------|---------------------------------------------|--------|--------------|--------|--------------|--------|
|             |                    |                 |            |            | Meter No. 1.                                |        | Meter No. 2. |        | Meter No. 3. |        |
|             |                    |                 |            |            | Time.                                       | Mean.  | Time.        | Mean.  | Time.        | Mean.  |
| 1           | .....              | 19.8            | Sine ....  | 59.95      | 1.6137                                      | 1.6154 | 1.6073       | 1.6084 | 1.3303       | 1.3306 |
| 4           | .....              | 18.9            | Sine ....  | 60.35      | 1.6178                                      |        | 1.6109       |        | 1.3321       |        |
| 7           | .....              | 18.6            | Sine ....  | 60.23      | 1.6148                                      |        | 1.6071       |        | 1.3300       |        |
| 2           | 0                  | 19.8            | Peak ...   | 59.89      | 1.6180                                      | 1.6188 | 1.6098       | 1.6099 | 1.3438       | 1.3431 |
| 5           | 0                  | 19.0            | Peak ...   | 60.31      | 1.6187                                      |        | 1.6105       |        | 1.3425       |        |
| 3           | 0                  | 17.0            | Flat ....  | 60.38      | 1.6203                                      | 1.6198 | 1.6141       | 1.6131 | 1.3407       | 1.3398 |
| 6           | 0                  | .....           | Flat ....  | 60.26      | 1.6184                                      |        | 1.6121       |        | 1.3390       |        |
| 8           | .....              | .....           | Sine ....  | 59.96      | 1.6121                                      | 1.6148 | 1.6051       | 1.6077 | 1.3289       | 1.3305 |
| 11          | .....              | 20.0            | Sine ....  | 60.08      | 1.6169                                      |        | 1.6099       |        | 1.3320       |        |
| 14          | .....              | 19.2            | Sine ....  | 59.95      | 1.6153                                      |        | 1.6082       |        | 1.3307       |        |
| 9           | 30                 | 19.1            | Peak ...   | 60.09      | 1.6197                                      | 1.6199 | 1.6116       | 1.6120 | 1.3440       | 1.3441 |
| 12          | 30                 | 19.3            | Peak ...   | 59.98      | 1.6202                                      |        | 1.6125       |        | 1.3443       |        |
| 10          | 30                 | 17.7            | Flat ....  | 60.12      | 1.6149                                      | 1.6152 | 1.6091       | 1.6092 | 1.3405       | 1.3404 |
| 13          | 30                 | 17.0            | Flat ....  | 59.99      | 1.6155                                      |        | 1.6092       |        | 1.3404       |        |
| 15          | .....              | 19.5            | Sine ....  | 60.14      | 1.6148                                      | 1.6156 | 1.6089       | 1.6088 | 1.3309       | 1.3309 |
| 18          | .....              | 18.9            | Sine ....  | 60.04      | 1.6158                                      |        | 1.6086       |        | 1.3306       |        |
| 21          | .....              | 18.3            | Sine ....  | 59.90      | 1.6162                                      |        | 1.6088       |        | 1.3311       |        |
| 16          | 60                 | 17.9            | Peak ...   | 60.10      | 1.6214                                      | 1.6216 | 1.6127       | 1.6133 | 1.3423       | 1.3426 |
| 19          | 60                 | 17.2            | Peak ...   | 60.12      | 1.6219                                      |        | 1.6140       |        | 1.3429       |        |
| 17          | 60                 | 16.6            | Flat ....  | 60.05      | 1.6127                                      | 1.6126 | 1.6063       | 1.6061 | 1.3398       | 1.3391 |
| 20          | 60                 | 15.9            | Flat ....  | 59.94      | 1.6125                                      |        | 1.6059       |        | 1.3390       |        |
| 22          | .....              | 20.2            | Sine ....  | 60.28      | 1.6151                                      | 1.6151 | 1.6076       | 1.6079 | 1.3302       | 1.3305 |
| 25          | .....              | .....           | Sine ....  | 60.10      | 1.6152                                      |        | 1.6084       |        | 1.3307       |        |
| 28          | .....              | 19.8            | Sine ....  | 60.05      | 1.6150                                      |        | 1.6077       |        | 1.3306       |        |
| 23          | 90                 | 18.9            | Peak ...   | 60.18      | 1.6211                                      | 1.6216 | 1.6133       | 1.6136 | 1.3417       | 1.3419 |
| 26          | 90                 | 18.3            | Peak ...   | 60.07      | 1.6222                                      |        | 1.6138       |        | 1.3422       |        |
| 24          | 90                 | 18.0            | Flat ....  | 60.10      | 1.6124                                      | 1.6125 | 1.6056       | 1.6054 | 1.3401       | 1.3400 |
| 27          | 90                 | .....           | Flat ....  | 60.04      | 1.6127                                      |        | 1.6052       |        | 1.3399       |        |
| 29          | .....              | 19.3            | Sine ....  | 59.86      | 1.6162                                      | 1.6178 | 1.6091       | 1.6103 | 1.3328       | 1.3328 |
| 32          | .....              | 19.1            | Sine ....  | 60.15      | 1.6183                                      |        | 1.6105       |        | 1.3328       |        |
| 35          | .....              | 19.3            | Sine ....  | 60.11      | 1.6190                                      |        | 1.6113       |        | 1.3333       |        |



**Table III.**—TABLE OF RESULTS OF 49 RUNS ON THREE METERS USING SINE, PEAK AND FLAT WAVES, ETC.—Continued.

| No. of run. | Phase of harmonic. | Lag of current. | Wave form. | Frequency. | Time in seconds for one revolution of disk. |        |              |        |              |        |
|-------------|--------------------|-----------------|------------|------------|---------------------------------------------|--------|--------------|--------|--------------|--------|
|             |                    |                 |            |            | Meter No. 1.                                |        | Meter No. 2. |        | Meter No. 3. |        |
|             |                    |                 |            |            | Time.                                       | Mean.  | Time.        | Mean.  | Time.        | Mean.  |
| 30          | 60                 | 18.1            | Peak ...   | 59.83      | 1.6234                                      | 1.6239 | 1.6145       | 1.6150 | 1.3447       | 1.3448 |
| 33          | 60                 | .....           | Peak ...   | 60.11      | 1.6244                                      |        | 1.6154       |        | 1.3450       |        |
| 31          | 60                 | 16.8            | Flat ....  | 60.22      | 1.6143                                      | 1.6149 | 1.6087       | 1.6089 | 1.3405       | 1.3414 |
| 34          | 60                 | 16.3            | Flat ....  | 60.10      | 1.6155                                      |        | 1.6091       |        | 1.3423       |        |
| 36          | .....              | 20.0            | Sine ....  | 60.12      | 1.6184                                      | 1.6179 | 1.6116       | 1.6106 | 1.3389       | 1.3332 |
| 39          | .....              | 20.1            | Sine ....  | 60.20      | 1.6177                                      |        | 1.6103       |        | 1.3390       |        |
| 41          | .....              | 20.1            | Sine ....  | 60.25      | 1.6176                                      |        | 1.6104       |        | 1.3326       |        |
| 37          | 30                 | 19.5            | Peak ...   | 60.12      | 1.6222                                      | 1.6220 | 1.6132       | 1.6132 | 1.3457       | 1.3455 |
| 40          | 30                 | .....           | Peak ...   | 60.22      | 1.6219                                      |        | 1.6131       |        | 1.3454       |        |
| 38          | 30                 | 17.5            | Flat ....  | 60.17      | 1.6169                                      | 1.6182 | 1.6104       | 1.6110 | 1.3414       | 1.3415 |
| 42          | 30                 | .....           | Flat ....  | 60.27      | 1.6195                                      |        | 1.6115       |        | 1.3417       |        |
| 44          | .....              | 20.2            | Sine ....  | .....      | 1.6178                                      | 1.6177 | 1.6101       | 1.6106 | 1.3326       | 1.3327 |
| 47          | .....              | 20.4            | Sine ....  | 60.01      | 1.6176                                      |        | 1.6112       |        | 1.3328       |        |
| 45          | 0                  | .....           | Peak ...   | .....      | 1.6191                                      | 1.6190 | 1.6104       | 1.6106 | 1.3458       | 1.3456 |
| 49          | 0                  | 22.0            | Peak ...   | 60.11      | 1.6190                                      |        | 1.6107       |        | 1.3460       |        |
| 43          | 0                  | .....           | Flat ....  | .....      | 1.6216                                      | 1.6220 | 1.6154       | 1.6154 | 1.3424       | 1.3426 |
| 46          | 0                  | 17.6            | Flat ....  | 60.04      | 1.6224                                      |        | 1.6154       |        | 1.3429       |        |

**Table IV.**—SUMMARY OF RESULTS SHOWN IN TABLE III.

| Meter No. | Phase of harmonic. | Sine.  | Peak.  | Per cent slow. | Flat.  | Per cent. |       |
|-----------|--------------------|--------|--------|----------------|--------|-----------|-------|
|           |                    |        |        |                |        | Slow.     | Fast. |
| 1         | 0                  | 1.6166 | 1.6186 | 0.12           | 1.6206 | 0.25      | ..... |
| 1         | 30                 | 1.6164 | 1.6210 | 0.28           | 1.6167 | 0.02      | ..... |
| 1         | 60                 | 1.6167 | 1.6228 | 0.38           | 1.6138 | .....     | 0.18  |
| 1         | 90                 | 1.6151 | 1.6216 | 0.40           | 1.6125 | .....     | 0.16  |
| 2         | 0                  | 1.6095 | 1.6102 | 0.04           | 1.6142 | 0.29      | ..... |
| 2         | 30                 | 1.6092 | 1.6126 | 0.21           | 1.6101 | 0.06      | ..... |
| 2         | 60                 | 1.6096 | 1.6142 | 0.29           | 1.6075 | .....     | 0.13  |
| 2         | 90                 | 1.6079 | 1.6136 | 0.35           | 1.6054 | .....     | 0.16  |
| 3         | 0                  | 1.3318 | 1.3444 | 0.95           | 1.3412 | 0.70      | ..... |
| 3         | 30                 | 1.3318 | 1.3448 | 0.98           | 1.3410 | 0.69      | ..... |
| 3         | 60                 | 1.3318 | 1.3437 | 0.89           | 1.3402 | 0.63      | ..... |
| 3         | 90                 | 1.3305 | 1.3419 | 0.85           | 1.3400 | 0.71      | ..... |

In Table IV the values given are the means of the two values found for the corresponding case and shown in Table III. Thus 1.6166 is the average of 1.6154, the mean found for runs 1, 4, 7 and

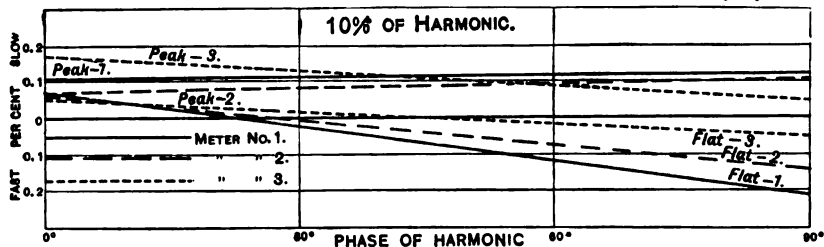


FIG. 6.—Showing the variation in the rate of three induction meters with 10 per cent harmonic in the current as the phase of the harmonic is changed from  $0^\circ$  to  $90^\circ$ .

1.6177, the mean of runs 44, 47. Likewise, 1.6186 is the average of 1.6183 (runs 2, 5) and 1.6190 (runs 45, 49). This eliminates effect of changing temperature during the course of the experiments.

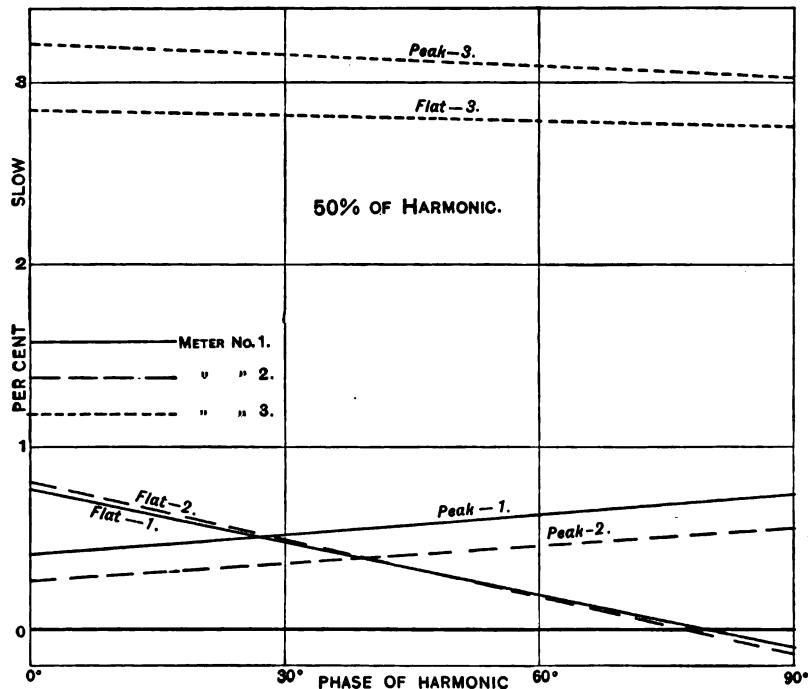


FIG. 7.—Showing the variation in the rate of three induction meters with 50 per cent harmonic in the current as the phase of the harmonic is changed from  $0^\circ$  to  $90^\circ$ .

The maximum variation due to 25 per cent harmonic is, in the case of meter 3, a little less than 1 per cent, being greater with the peak than the flat, but not varying much with the phase of the

harmonic. On the other hand, meters 1 and 2 show smaller errors due to the presence of the harmonic, but greater changes due to shifting the phase of the harmonic, both changing from slow to fast on the flat wave when the phase is shifted. Meter 4 runs faster for both peak and flat and at all phases than on a sine wave. It is the only meter of the five for which this is true.

Fig. 6 shows the effect of changing the phase of the harmonic from  $0^\circ$  to  $90^\circ$  when using a harmonic of 10 per cent, and fig. 7 shows the same for 50 per cent. Only three meters were used in these experiments.

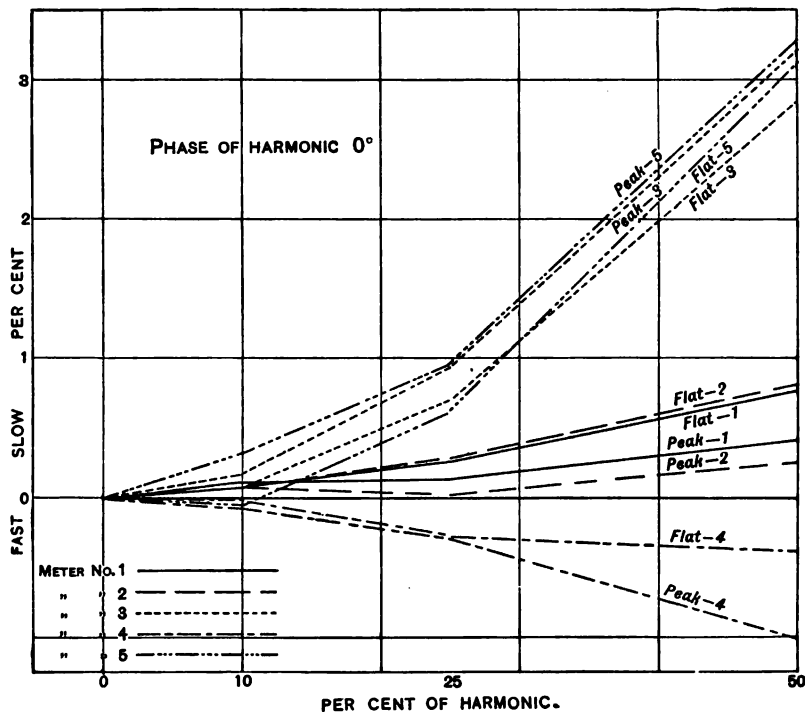


FIG. 8.—Showing the variation in the rate of five induction meters with 10, 25, and 50 per cent of harmonic, the phase of the harmonic in each case being  $0^\circ$ .

Fig. 8 shows the effect of changing the harmonic from 10 per cent to 25 per cent and 50 per cent, keeping the phase constant. Meters 1 and 2 show the least change in rate; meter 4 runs faster, and meters 3 and 5 run slower and show the greatest change in rate. Fig. 9 shows for three meters the same thing as fig. 8, except that the phase of the harmonic is  $90^\circ$  different. Meters 1 and 2 show relatively small changes, but both run faster on the flat than on the sine. Meter 3 runs nearly 3 per cent slower on the 50 per cent harmonic than on the sine.

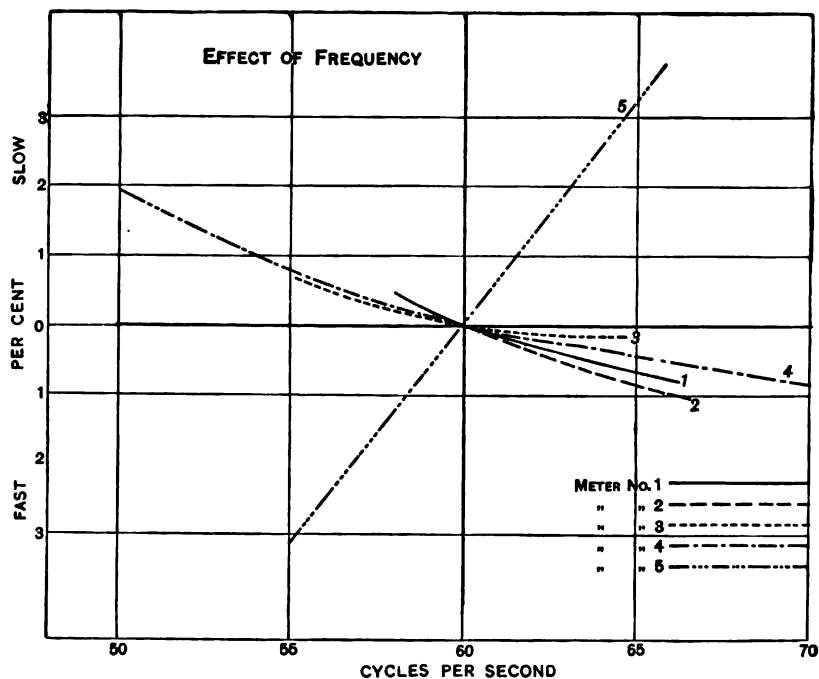
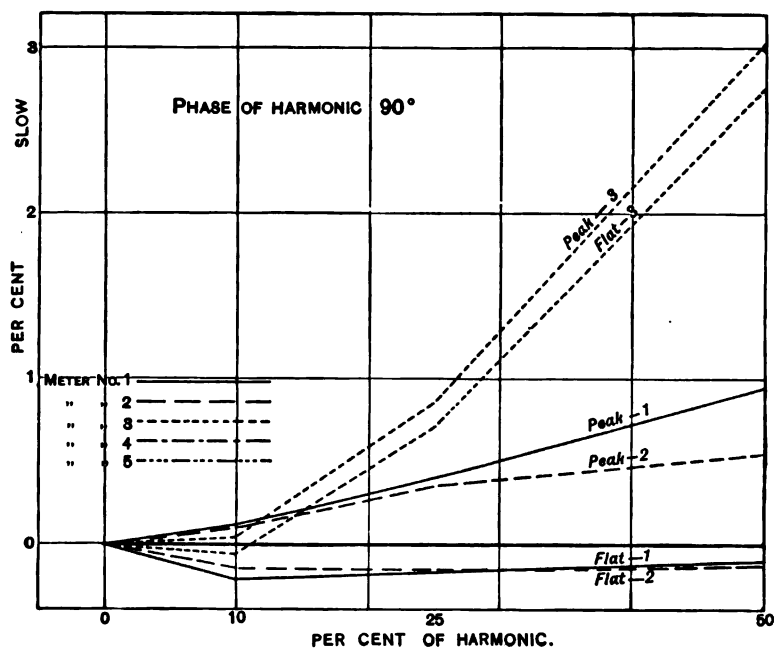


Table V.—EFFECT OF VARYING THE PHASE OF THE HARMONIC.  
[Change of rate of meters in per cent.]

| Percentage of harmonic.      | Phase 0°. |       |       |       | Phase 30°. |       |       |       | Phase 60°. |       |       |       | Phase 90°. |       |       |       |
|------------------------------|-----------|-------|-------|-------|------------|-------|-------|-------|------------|-------|-------|-------|------------|-------|-------|-------|
|                              | Flat.     |       | Peak. |       | Flat.      |       | Peak. |       | Flat.      |       | Peak. |       | Flat.      |       | Peak. |       |
|                              | Fast.     | Slow. | Fast. | Slow. | Fast.      | Slow. | Fast. | Slow. | Fast.      | Slow. | Fast. | Slow. | Fast.      | Slow. | Fast. | Slow. |
| <i>10 per cent harmonic.</i> |           |       |       |       |            |       |       |       |            |       |       |       |            |       |       |       |
| Meter No. 1.....             | 0.07      | ..... | ..... | 0.11  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | 0.21       | ..... | ..... | 0.12  |
| Meter No. 2.....             | 0.06      | ..... | ..... | 0.07  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | 0.14       | ..... | ..... | 0.11  |
| Meter No. 3.....             | 0.05      | ..... | ..... | 0.17  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | 0.05       | ..... | ..... | 0.05  |
| Meter No. 4.....             | 0.01      | ..... | ..... | 0.07  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... |
| Meter No. 5.....             | 0.05      | ..... | ..... | 0.32  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... |
| <i>25 per cent harmonic.</i> |           |       |       |       |            |       |       |       |            |       |       |       |            |       |       |       |
| Meter No. 1.....             | 0.25      | ..... | ..... | 0.12  | .....      | ..... | 0.02  | ..... | 0.18       | ..... | ..... | ..... | 0.16       | ..... | ..... | 0.40  |
| Meter No. 2.....             | 0.29      | ..... | ..... | 0.04  | .....      | ..... | 0.06  | ..... | 0.13       | ..... | ..... | ..... | 0.16       | ..... | ..... | 0.35  |
| Meter No. 3.....             | 0.70      | ..... | ..... | 0.95  | .....      | ..... | 0.69  | ..... | .....      | 0.63  | ..... | ..... | .....      | 0.71  | ..... | 0.85  |
| Meter No. 4.....             | 0.27      | ..... | ..... | 0.28  | .....      | ..... | 0.32  | ..... | 0.25       | ..... | 0.23  | ..... | 0.14       | ..... | 0.31  | ..... |
| Meter No. 5.....             | 0.62      | ..... | ..... | 0.95  | .....      | ..... | 0.41  | ..... | 0.25       | ..... | 0.78  | ..... | 0.86       | ..... | ..... | 0.61  |
| <i>50 per cent harmonic.</i> |           |       |       |       |            |       |       |       |            |       |       |       |            |       |       |       |
| Meter No. 1.....             | 0.77      | ..... | ..... | 0.41  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | 0.10       | ..... | ..... | 0.74  |
| Meter No. 2.....             | 0.81      | ..... | ..... | 0.26  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | 0.13       | ..... | ..... | 0.55  |
| Meter No. 3.....             | 2.84      | ..... | ..... | 3.21  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | .....      | 2.75  | ..... | 3.03  |
| Meter No. 4.....             | 0.39      | ..... | 1.01  | ..... | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... |
| Meter No. 5.....             | 3.10      | ..... | ..... | 3.26  | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... | .....      | ..... | ..... | ..... |

The effect of change of frequency on the rate of the meters is shown in fig. 10. It is relatively small in every case but one.

These results show that with suitable precautions induction meters may be made to repeat their readings very accurately, so that precision methods may be applied in studying them. They also show that the variations due to wave form depend not only on the harmonics which are present and their magnitudes, but also on their phases.

The Bureau of Standards is now having a generating set constructed which will give all the odd harmonics up to the fifteenth and any desired combination of them with the fundamental. When this is completed it will be used to study the effects of the higher harmonics on the rate of these meters. The results given here show that for commercial purposes all the meters so far studied may be considered accurate on any ordinary wave form where only the third harmonic enters appreciably, although two meters show variations of about 3 per cent when the harmonic amounts to as much as 50 per cent of the fundamental.

## DETECTOR FOR VERY SMALL ALTERNATING CURRENTS AND ELECTRICAL WAVES.

By L. W. AUSTIN.

A number of years ago I noticed that a copper sulphate cell with copper electrodes, one of which was considerably larger than the other, showed an apparent rectifying effect for alternating currents of the order of those produced by speaking with the mouth pressed against an ordinary telephone receiver. In the original experiment the rectifying cell, the telephone, and a d'Arsonval galvanometer of moderate sensibility were connected in series. The lowest tone spoken into the telephone-produced a deflection of the galvanometer, and a louder one drove it from the scale. Recently I have again taken the matter up, desiring, if possible, to find an explanation of the rectifying effect, and also to test the usefulness of the detector in the laboratory.

To test the sensibility of the detector for alternating currents, a known alternating current was passed through a bridge wire of about 1 ohm resistance, and from this, by means of sliding contacts, various potential differences were applied to the detector (see fig. 1). A form of key was used which kept the detector circuit short-circuited when not connected to the potential wire. This was necessary on account of the small electromotive forces always present in the cell, due to slight differences in the electrodes. In order to reduce these as much as possible, the electrodes were generally made from the same No. 30 wire, the small one varying from 1 mm to 3 or 4 cm in length, and the larger one from 10 to 30 cm.

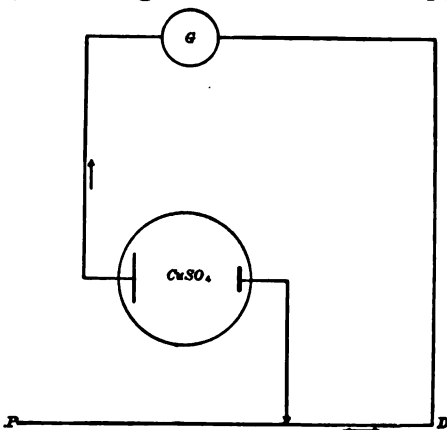


FIG. 1.—Connections for detector (form 1).

The sensibility did not seem to vary in any marked degree with the size of the small electrode within the limits indicated.

The first hypothesis entertained regarding the nature of the phenomenon was that it was a true rectification, the resistance being greater for a current entering the small electrode than for one leaving it. But the following experiment seems to indicate that the direct current is due, in part at least, to some chemical action produced by the alternating current and analagous, perhaps, to the solution of Pt in  $\text{H}_2\text{SO}_4$ , noticed by Margules<sup>a</sup> and Ruer<sup>b</sup>, the action being greater on the small electrode on account of the greater current density. It seems probable, however, that the cause of the phenomenon may contain more than one factor. This is indicated by the fact that a reversal of

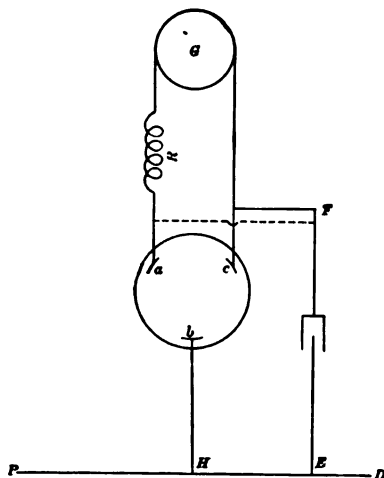


FIG. 2.—Connections for detector (form 2).

the usual direction of the current is occasionally observed, especially with fine point electrodes and high frequencies. A cell was constructed with three electrodes, *a*, *b*, *c* (see fig. 2), *b* and *F* being connected to the potential wire *P D* with a condenser inserted in *E F* and a high inductive resistance in the galvanometer circuit, the arrangement being such that most of the alternating current could be sent through either one of the electrodes of the galvanometer circuit, and very little through the other. It was found that either *a* or *c* could be made electropositive by connecting *F* to it, and that the direction of the current was now independent of the relative size of the electrodes, at least within wide limits.

<sup>a</sup> M. Margules: Wied. Ann. **65**, p. 629, 1898; **66**, p. 540, 1898.

<sup>b</sup> M. Ruer: ZS. f. phys. Chem. **44**, p. 81; 1903.



In the accompanying curve (fig. 3) the relation is shown between the direct current produced and the alternating potential difference applied to a two-electrode cell (fig. 1), connected in series with a galvanometer having a sensibility of about  $1 \cdot 10^{-8}$  amp. Above 0.015 volt (not shown in figure) the curve is nearly a straight line until in the neighborhood of 0.3 volt it bends suddenly and becomes nearly parallel to the horizontal axis. The lowest voltage observable under favorable circumstances with the instrument connected directly (i. e. without a transformer) and with a sensitive galvanometer is about 0.0001 volt, the sensibility being about the same as that of a good telephone receiver. The sensitiveness is in general limited by the difficulty of keeping a steady zero. By using a small transformer the sensibility can be much increased when the external circuit is of low resistance.

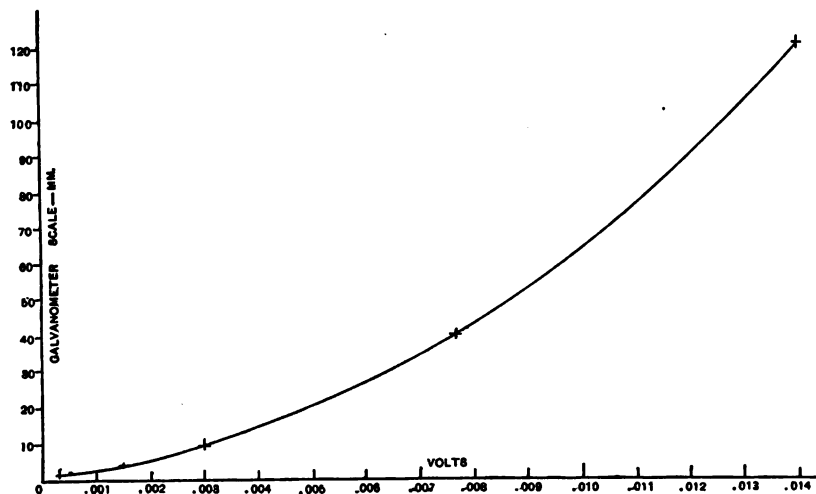


FIG. 3.—Relation between direct current and alternating P. D.

In this way I have sometimes obtained deflections of over 10 cm for 0.001 volt.

The action of the rectifier seems to be independent of frequency within the limits of ordinary laboratory practice. It responds equally well to a series of slow taps on the diaphragm of a telephone, or to a frequency of several thousand per second. It has also been found that it is sensitive to electrical waves, though less so and less regular in its action than the polarized platinum-point detector of Fessenden and Schlömilch. With a modified Blondlot apparatus it was found to be too sensitive to connect directly, but when connected to two small glass-tube condensers sliding on the wires of the secondary, as in the Rubens arrangement, the deflections increased or decreased, as the

two circuits were thrown in or out of resonance. Waves could also be detected when the receiving apparatus was moved into the next room, and about 2 meters of wire were hung up in the doorway, as an aerial, 3 or 4 meters from the source of the waves. In these experiments with waves it is necessary that the small electrode be merely a point of wire, best sealed in glass, most of the experiments being made with No. 40 wire or smaller.

To secure the best results, the small electrode must be polished every two or three days, and the solution made up with pure  $\text{CuSO}_4$  and distilled water. The constancy of deflection is not by any means perfect, and unless it is found possible to keep the electrodes in a uniform condition, its main use must be as a zero instrument. I have also tested the same arrangement of a large and a small electrode of a number of other metals, not only in solutions of their own salts, but also in other electrolytes, but have found them less sensitive than copper in copper sulphate.

## POSITIVE CHARGES CARRIED BY THE CANAL RAYS.

By L. W. AUSTIN.

Experiments have been made by W. Wien<sup>a</sup>, Ewers<sup>b</sup>, and Villard<sup>c</sup> for detecting the positive charges carried by the canal rays of Goldstein by methods similar to those of Perrin<sup>d</sup> for detecting the negative charges of cathode rays. Wien and Ewers conclude that the charges observed in a Faraday cylinder placed behind a perforated cathode were carried by the canal rays, while Villard holds that they are due to the slow diffusion of positive ions into the Faraday cylinder, basing his opinion on the observation that the positive charge first appeared some time after the canal rays began entering the cylinder, and in some cases only after the discharge had ceased. As has been pointed out by J. J. Thomson,<sup>e</sup> this effect is very probably due to the fact that on account of the great conductivity of the gas produced by canal rays, the charge can not accumulate in the cylinder, but after the discharge ceases the gas recovers its insulating power and the cylinder can retain any charges which diffuse into it. If this view is correct, the positive charges in the cylinder, at least when observed electrometrically, are due mainly to diffusion and not to convection by the canal rays.

While almost all physicists must be satisfied from the work of W. Wien on the magnetic and electric deflections of canal rays that these rays carry a positive charge, the prominence given to the objections of Villard in recent publications<sup>a</sup> make it desirable to publish the following direct experimental demonstration of the positive charges of the canal rays, which seems to be free from the objections just cited.

The plan of the experiment is somewhat similar to that of Ewers. The form of the tube is shown in figure 1. The discharge takes place

<sup>a</sup> W. Wien: Wied. Ann. 65, p. 445; 1898.

<sup>b</sup> Ewers: Wied. Ann. 69, p. 167; 1899.

<sup>c</sup> Villard: J. de Phys. (3) 8, pp. 5 and 140; 1899.

<sup>d</sup> Perrin: Comptes Rendus, 121, p. 1130; 1895.

<sup>e</sup> J. J. Thomson: Conduction of Electricity Through Gases, p. 521.

entirely in the part *A*, which is 7 cm long and 2.5 cm in diameter. The diaphragms *d*, with openings 2 mm in diameter, form the cathode, and the canal rays pass through this into the part of the tube *B*. This is lined with brass netting, and at its center is placed a brass cylinder *c*, 1.5 cm in diameter, with a 4 mm opening. This cylinder can be turned back out of the way, and is connected to earth through a galvanometer having a sensibility of  $4 \times 10^{-8}$  amperes. The anode *e* is con-

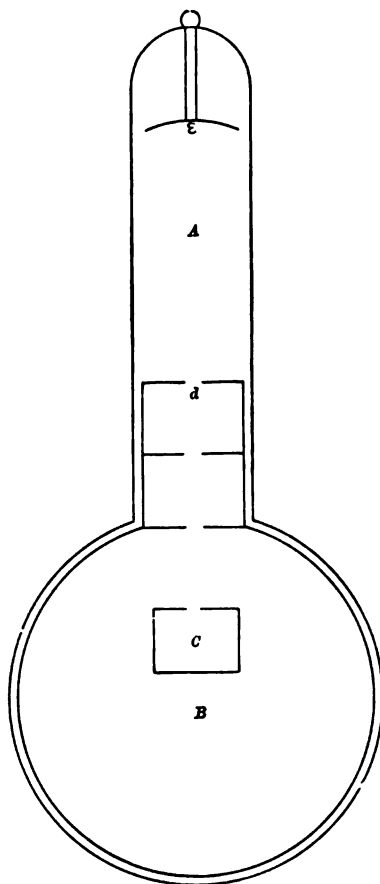


FIG. 1.—Plan of tube.

nected to one pole of an influence machine giving a current of about 0.0003 amp., the other pole of which, as well as the cathode *d* and the brass net, are connected to earth. By this arrangement practically the whole of any charge imparted to the cylinder by the canal rays which enter it passes through the galvanometer to earth, as the resistance of the gas, even when highly ionized, is always very much greater than that of the galvanometer.

With the cylinder in position to receive the rays, the galvanometer showed a positive deflection of about 300 mm as soon as the discharge was started; the deflection dropped at once to 5 mm when the cathode rays were deflected by a magnet, so that the canal rays could not pass the diaphragms. The original deflection appeared again as soon as the magnet was removed. When the cylinder was turned back so that the canal rays just grazed it as they passed through *B*, the deflection was about 15 mm, probably due to diffuse rays from the main stream. The exact position of the rays could be easily determined by the bright fluorescence of the gas.

This experiment was performed with a potential difference between the electrodes of the tube of 8,000 volts, as measured on a Braun electrometer.

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\*J. J. Thomson: loc. cit. E. Rutherford: Address before the International Congress of Arts and Sciences, St. Louis, 1904. G. C. Schmidt: *Die Kathodenstrahlen*, p. 110.



## RADIATION FROM PLATINUM AT HIGH TEMPERATURES.

By G. K. BURGESS

The study of the emissive properties of substances which can be brought to high temperatures without undergoing chemical or physical changes of surface is important in optical pyrometry for the practical realization and measurement of high temperatures; for a knowledge of the emissive properties of platinum, for instance, at various temperatures, gives a ready means of obtaining true temperatures from observations of the "black-body temperatures" as measured by an optical pyrometer. A platinum strip may replace to advantage an experimental black body, especially at temperatures above 1,500° C., and may conveniently serve as a luminous source for the comparison or calibration of optical pyrometers.

It is the object of this paper to interpret the observations<sup>a</sup> of Dr. Waidner and the author on the departure of platinum from black-body radiation for red, green, and blue light, in terms of the now better known values of the higher temperatures involved, and as expressed by Wien's law as applied in a form first suggested by Lucas.<sup>b</sup>

For a black body, Wien's law for the distribution of energy in the spectrum may be written

$$\log E = \log C_1 - 5 \log \lambda - \frac{C_2 \log \epsilon}{\lambda} \theta_1 \quad (1)$$

where  $E$  is the energy radiated between wave lengths  $\lambda$  and  $\lambda + d\lambda$ ,  $\theta_1$  the reciprocal of the absolute temperature,  $C_1$  and  $C_2$  constants, and  $\epsilon$ , the logarithmic base.

For any other substance at the same photometric brightness we have

<sup>a</sup> C. W. Waidner and G. K. Burgess: Optical pyrometry; Bull. Bureau of Standards, 1, p. 189; 1904.

<sup>b</sup> R. Lucas: Phys. ZS., 6, pp. 19, 418; 1905.

$$\log E = \log C_1' - n \log \lambda - \frac{C_2^1 \log \epsilon}{\lambda} \theta_2 \quad (2)$$

in which  $n > 5$ ,  $C_2^1 > C_2$  and  $\theta_2 > \theta_1$ . Thus, for platinum  $n = 6.42$  according to Paschen, and  $n = 6$  from Lummer and Pringsheim's work. The largest value of  $C_2$  for the visible spectrum is that deduced from the measurements of Lummer<sup>a</sup> and Pringsheim:  $C_2 = 5\lambda_m T = 5 \times 2940 = 14700$ , and the smallest value of  $C_2^1$  is  $C_2^1 = 6\lambda_m^1 T^1 = 6 \cdot 2600 = 15600$ .

That  $\theta_2 > \theta_1$ , follows from the definition of a black body.  $C_1$  and  $C_1^1$  are both small and the difference of their logarithms is negligible in most of what follows.

We may shorten the expressions by grouping the various constants, as follows:

$$(A) \begin{cases} \log C_1 - 5 \log \lambda = K_1 \\ \log C_1' - n \log \lambda = K_1' \end{cases} \quad (B) \begin{cases} C_2 \frac{\log \epsilon}{\lambda} = K_2 \\ C_2^1 \frac{\log \epsilon}{\lambda} = K_2' \end{cases}$$

Whence, by equating (1) and (2),

$$K_1 - K_2 \theta_1 = K_1' - K_2' \theta_2$$

Or, more simply,

$$\theta_1 = \alpha \theta_2 + \beta \quad (3)$$

Where

$$\alpha = \frac{K_1'}{K_2'} = \frac{C_2^1}{C_2} \text{ by } (B) \quad (4)$$

And

$$\beta = \frac{K_1 - K_1'}{K_2} = \frac{\lambda (\log C_1 - \log C_1' + (n-5) \log \lambda)}{C_2 \log \epsilon} \text{ by } (A) \text{ and } (B) \quad (5)$$

or

$$\beta = K \lambda \log \lambda \quad (5a)$$

where  $K$  is a constant, characteristic of the substance.

Furthermore, the constants  $\alpha$  and  $\beta$  must lie within well-defined limits, as follows:

$$\alpha > 1 \text{ since } C_2^1 > C_2 \quad (6)$$

That is,  $\alpha$  is always greater than unity, but approaches unity as the substance examined is more nearly a black body;  $\alpha = 1.06$  from

<sup>a</sup>A discussion of the values of these constants is given by Lummer in Reports to Congress of Physics, Paris, 1900, vol. 2, p. 41.



the data cited above. Again,  $\alpha$  is independent of the wave length  $\lambda$  to a first approximation, by (4).

Since experiment shows  $n > 5$ , we have

$$\beta > 0 \quad (7)$$

or  $\beta$  approaches zero as the black body condition is approached, and equation (5a) shows the dependence of  $\beta$  upon the wave length, namely, that  $\beta$  increases as the product of the wave length into its logarithm. The constant  $n$  can evidently be computed by (5) if  $\beta$  and  $C_1$  are known, or conversely,  $\beta$  may be calculated; but then, in these computations  $\log C_1 - \log C_1'$  is no longer negligible.

Equation (3) was deduced by Lucas,<sup>a</sup> although he does not show completely the interrelations of the constants involved nor the necessary limitations as to the numerical values of  $\alpha$  and  $\beta$ .

Equation (3) with the above corollaries is fundamental and completely expresses the departure of any substance from a black body in terms of the temperature and two constants whose values are a measure of its emissive properties, and states that *the reciprocals of the temperatures of a black body and any substance having the same photometric brightness are directly proportional.*

As the equation is linear, a knowledge of the black body temperature and true temperature at two points only is sufficient to completely define the departure of the radiation of a given substance from that of a black body throughout the entire temperature scale, provided the substance undergoes no chemical change; and since  $\alpha$  is a constant independent of  $\lambda$ , when equation (3) is determined for any single value of  $\lambda$ , that for any other value of  $\lambda$  in the visible spectrum is had by an observation of  $\theta_1$  at a single temperature. These facts are expressed by writing (3) in the form:

$$\theta_1 = \alpha \theta_2 + K \lambda \log \lambda \quad (8)$$

Experimental verification of the above conclusions is had in the observations<sup>b</sup> of Dr. Waidner and the author on the relation between the black body temperature and the true temperature of platinum for wave lengths  $\lambda = 0.651\mu$  (red),  $\lambda = 0.550\mu$  (green), and  $\lambda = 0.474\mu$  (blue). The observations were taken on a platinum strip mounted on a Joly maldometer, and the scale defined by melting points, that of platinum<sup>c</sup> being assumed as  $1,715^\circ \text{C.}$  and palladium (slightly impure) as  $1,525^\circ \text{C.}$

<sup>a</sup> Lucas, l. c.

<sup>b</sup> Waidner and Burgess, l. c.

<sup>c</sup> Holborn and Henning: *Sitzungsber., Berlin Akad.*, March 2, 1905, p. 311. Har-ker: *Chem. News*, 91, pp. 262, 274; 1905.

The accompanying table gives  $S$ , the black body temperatures (absolute) of the platinum as measured by a Holborn-Kurlbaum optical pyrometer, corresponding to the true absolute temperatures  $T$ .  $\theta_1$  and  $\theta_2$  are the reciprocal temperatures, i e.,  $\theta_1 = \frac{1}{S}$  and  $\theta_2 = \frac{1}{T}$ , and the number of determinations of  $S$  is given in the column headed  $p$ .

| <i>Radiation from platinum.</i>                                                |           |            |            |                |                      |     |
|--------------------------------------------------------------------------------|-----------|------------|------------|----------------|----------------------|-----|
| RED LIGHT: $\lambda = 0.651\mu$ . $\theta_1 = 1.0256 \theta_2 + 0.0000357$ .   |           |            |            |                |                      |     |
| $T_{obs}$                                                                      | $S_{obs}$ | $\theta_1$ | $\theta_2$ | $T_{computed}$ | $T_{obs} - T_{calc}$ | $p$ |
| 996                                                                            | 939       | 0.0010650  | 0.0010040  | 996.3          | -0.3                 | 6   |
| 1055                                                                           | 990       | 10101      | 9479       | 1052.5         | +2.5                 | 2   |
| 1223                                                                           | 1145      | 8734       | 8177       | 1224.3         | -1.3                 | 5   |
| 1337                                                                           | 1246      | 8026       | 7480       | 1337.9         | -0.9                 | 10  |
| 1500                                                                           | 1392      | 7184       | 6667       | 1502.2         | -2.2                 | 2   |
| 1606                                                                           | 1482      | 6748       | 6227       | 1604.7         | +1.3                 | 2   |
| 1798                                                                           | 1647      | 6072       | 5562       | 1794.5         | +3.5                 | 9   |
| 1988                                                                           | 1814      | 5513       | 5030       | 1989.1         | -1.1                 | 14  |
| GREEN LIGHT: $\lambda = 0.550\mu$ . $\theta_1 = 1.0320 \theta_2 + 0.0000218$ . |           |            |            |                |                      |     |
| 1337                                                                           | 1258      | 0.0007949  | 0.0007480  | 1334.9         | +2.1                 | 5   |
| 1500                                                                           | 1409      | 7097       | 6667       | 1500.3         | -0.3                 | 2   |
| 1606                                                                           | 1506      | 6640       | 6227       | 1607.0         | -1.0                 | 2   |
| 1798                                                                           | 1676      | 5967       | 5562       | 1795.2         | +2.8                 | 4   |
| 1988                                                                           | 1850      | 5405       | 5030       | 1989.6         | -1.6                 | 10  |
| BLUE LIGHT: $\lambda = 0.474\mu$ . $\theta_1 = 1.034 \theta_2 + 0.0000107$ .   |           |            |            |                |                      |     |
| 1606                                                                           | 1529      | 0.0006540  | 0.0006227  | 1607.3         | -1.3                 | 2   |
| 1798                                                                           | 1699      | 5886       | 5562       | 1789.3         | +8.7                 | 2   |
| 1988                                                                           | 1885      | 5305       | 5030       | 1989.2         | -1.2                 | 8   |

The observations for the red ( $\lambda = 0.651\mu$ ) satisfy the equation, see (3),

$$\theta_1 = 1.0256 \theta_2 + 0.0000357$$

the constants being determined by least squares.

Similarly, for the green ( $\lambda = 0.550$ )

$$\theta_1 = 1.0320 \theta_2 + 0.0000218$$

and for the blue ( $\lambda=0.474$ )

$$\theta_1 = 1.034 \theta_2 + 0.0000107.$$

The values of  $T$  as computed from the formula  $T = \frac{\alpha}{\theta_2 - \beta}$  are given in the fifth column of the table and  $\Delta T$  in the sixth. The differences between the observed and computed values of  $T$  are well within the errors of observation.

It will be seen, moreover, that within the same limits the value of  $\alpha$  (coefficient of  $\theta_2$ ) satisfies the necessary conditions of being slightly greater than unity (equation 6) and a constant nearly independent of the wave length (equation 4). The values of  $\beta$  also satisfy the condition expressed in equations (5a) and (8).

Lucas<sup>a</sup> discusses Holborn and Kurlbaum's<sup>b</sup> observations on platinum for red radiation ( $\lambda=0.643$ ), and finds they satisfy the relation

$$\theta_1 = 0.9857 \theta_2 + 0.0000598$$

with an average deviation of about  $4^\circ$  C. Holborn and Henning<sup>c</sup> find empirically for the same observations

$$\theta_1 = \theta_2 + 0.000507.$$

The preceding deductions from Wien's law are general and apply to any substance conserving the optical properties of its surface with continued heating. The comparative ease with which  $\alpha$  and  $\beta$  may be determined experimentally, and the simplicity of the linear relation connecting them, would seem to indicate the desirability of expressing mono-chromatic radiation properties in terms of these two constants  $\alpha$  and  $\beta$ . To the same order of approximation, Planck's equation gives the same solution.

<sup>a</sup> Lucas l. c.

<sup>b</sup> Holborn and Kurlbaum: Ann. d. Phys. 10, p. 225; 1903.

<sup>c</sup> Holborn and Henning, l. c.



## FIVE-THOUSAND VOLT GENERATOR SET.

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By P. G. NUTTING.

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At the Pittsburg meeting of the American Association for the Advancement of Science in 1902, Professor Moler described a set of generators for producing continuous current at a high potential which he had installed at Cornell University. This set consisted of twenty-four 500-volt Crocker-Wheeler units, connected in series and giving a maximum current of one-fifth ampere. It was used for several months by the writer and served as a basis for the design of the set here described.

The generation and control of a high potential current from multiple units is a comparatively simple matter. But the destructive breakdowns caused by the breaking of belts and brushes suggested the advisability of mounting the set as compactly as possible on a table provided with castors, so that it might be kept under the close supervision of the person using it.

One or two thousand volts is a sufficiently high potential for much of the work pertaining to gas conduction, while it is seldom that a voltage of more than 5,000 is desired. On the other hand, it is only above 8,000 volts that insulation becomes troublesome, surface leakage occurring over oiled, painted, or lacquered surfaces, and across switches. Hence, 5,000 volts was chosen as the most suitable potential, requiring no special precautions as to insulation and yet ample for most work.

Units of 500 volts were chosen as a compromise between reducing the number of machines to the least possible amount on the one hand, and minimizing commutator sparking and difficulties of insulation by increasing the number of machines on the other. The old standard form of bipolar, shunt-wound, 500-volt Crocker-Wheeler motor was chosen as the most suitable form of unit on the market. The commutators of these machines, containing sixteen segments, are easily accessible while the machines are in operation and require but little polishing to be kept free from sparking.

As shown in the accompanying reproduction of a photograph of the set, five units are arranged on each side of the top of a specially designed table of massive oak. The 2 h. p. driving motor (General Electric, type "C. A.") is mounted in the middle of one end of the table and is belted to the countershaft to which each generator is separately belted.

The generators are mounted directly upon the varnished oak plank forming either side of the top of the table, so that the bodies of the units are insulated from one another only by this wood. Brush holders and field terminals are, however, carefully insulated from the frames of the machines by heavy mica bushings. Three wires run from each unit to the next; one wire connects the armatures in series, while two others serve to connect the fields in parallel.

These wires are covered throughout with thick-walled rubber tubing. Where they project above the table they are supported by three-fourth-inch ebonite tubing, having one-eighth-inch wall. Thus there is but little strain on the insulation, either between the separate machine frames or between the field and armature circuits. During a year of heavy service there was no case of breakdown of insulation, nor any trace of a leak developed.

The generator fields are excited by being connected directly to a 120-volt circuit but where possible to a different circuit from the one used by the driving motor. A speed controller of wide range, used in connection with the driving motor, permits a wide variation in the voltage of the output. Between 3,000 and 5,000 volts the voltage may easily be held constant to within 1 per cent for some time.

The accurate control of the high voltage circuit is a matter of considerable difficulty. The armatures have a resistance of but 250 ohms each, while the maximum current is but 0.22 ampere. Hence, a minimum resistance of at least 25,000 ohms must always remain in circuit, while gas conductivity work, with a current of about 1 milliampere, requires a continuously variable resistance of 5,000,000 ohms. I have been using tubes of amyl alcohol solution of cadmium iodide for this purpose, but this fluid resistance is to be replaced by a rheostat of wire and graphite, now being constructed by the Gebr. Ruhstrat.

Detailed specifications relating to the set are tabulated below:

#### GENERATORS.

|                                      |                                      |
|--------------------------------------|--------------------------------------|
| Output, 110 watts.                   | Armature resistance, 250 ohms.       |
| Voltage, 500 volts.                  | Commutator, 16 segments.             |
| Maximum current, 0.22 ampere.        | Pulley, 1½ in. face, 2 in. diameter. |
| Speed, 2,500 revolutions per minute. | Greatest length (spindle), 9.5 in.   |
| Field, 500 ohms, 0.2 ampere.         | Greatest width (field), 7.5 in.      |

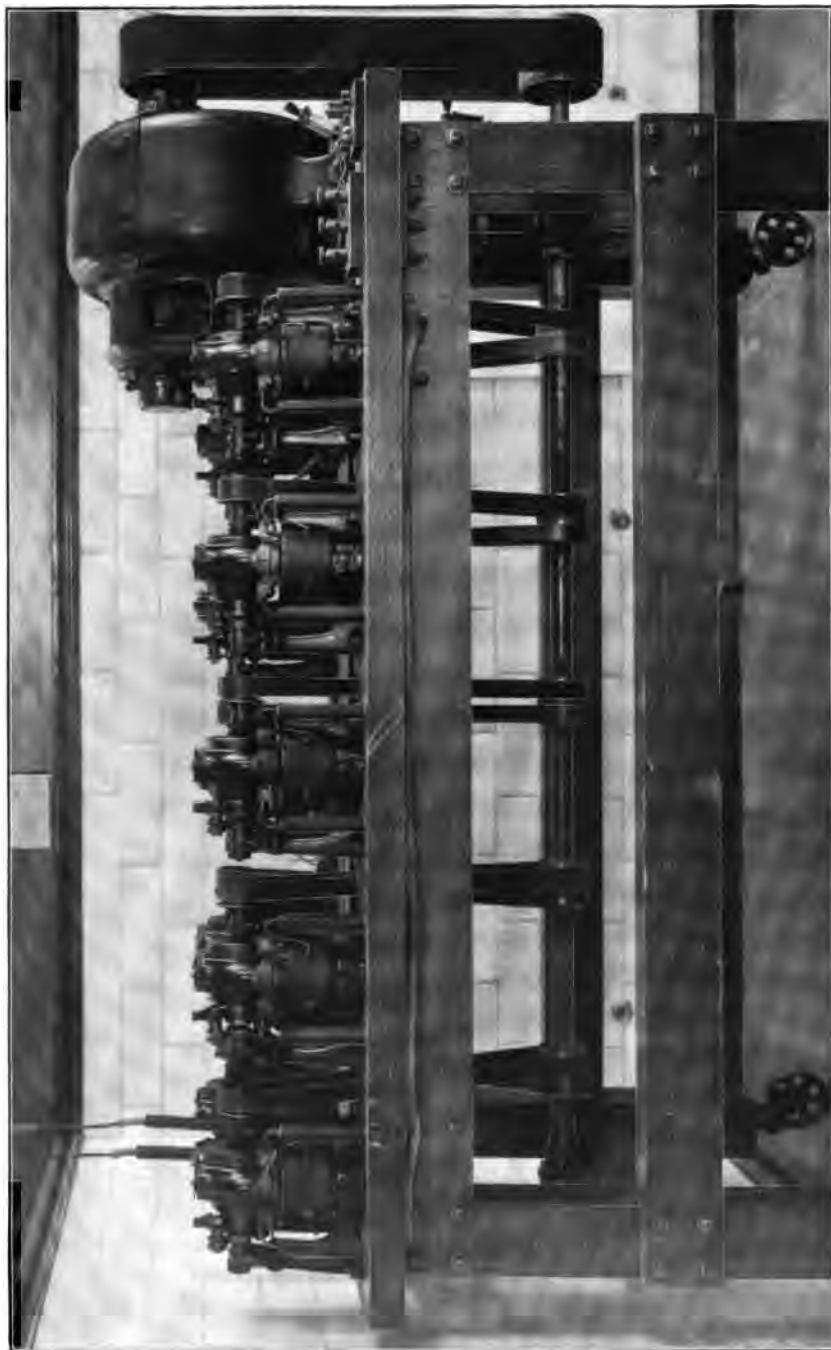


FIG. 1.—FRONT VIEW OF THE 5,000-VOLT GENERATOR SET.





## DRIVING MOTOR.

2 h. p. General Electric.  
Type, "C. A." 15 ampere, 115 volts.  
Speed, 1,800 revolutions per minute.  
Pulley, 3.5 in. face, 4.5 in. diameter.

## TABLE.

Top, 26 by 62 in.  
Space per generator, 8 by 10.5 in.  
Belts: Motor shaft, 3 in.; shaft generators, 1 in.

The generator set here described is calculated to make unnecessary the use of batteries of storage cells for all kinds of vacuum tube work, and even for accurate conductivity tests. It is, however, no competitor for the small high potential transformer for use in spectroscopic work with sparks and Plücker tubes.

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